AD

SW 9433

Preprint PHYSIN 9405

Unitarity mass bounds due to massive spin-1 fields

E. H. LEMKE

*Otto-Hahn-Institut für Partikelforschung
Institut für Theoretische Physik
Bielefeld, Jöllenbecker Heide 5, Germany 33739*

Unitarity mass bounds due to massive spin-1 fields

E. H. Lemke

Otto-Hahn-Institut für Partikelforschung, Institut für Theoretische Physik

33739 Bielefeld, Jöllenbecker Heide 5a, Germany

Abstract. If the principle of renormalizability is applied to massive spin-1 fields, then the mechanism of spontaneous symmetry breaking makes possible a minimal particle content. The drawbacks of such a model could possibly be avoided by a doubling of all the states of leptons, quarks and weak bosons. Mass could then be generated without spontaneous symmetry breaking. The couplings of the partner states can be determined by that requirement of renormalizability. Lowest-order processes give rise to upper unitarity bounds on the partner masses. The effect is demonstrated for the decays of the neutrino partner $N \rightarrow W + e$ or $Z + \nu$. It is shown that unitarity is saved at *any* partner mass (M) if all orders of perturbation theory are taken into account. The corresponding form factor of the decay amplitude is estimated. It is suppressive at $M > m_W/\sqrt{\alpha}$ so that the specific decay width Γ/M even decreases with increasing M . The form factor exhibits a substantial imaginary part, which opens up a new approach to the issue of CP-violation.

PACS 11.10. Jj – Asymptotic problems and properties.

PACS 11.30. Er – Charge conjugation, parity, time reversal, and other discrete symmetries.

PACS 12.15. Ji – Applications of electroweak models to specific processes.

THE history of pointlike massive spin-1 fields in quantum theory began with the discovery that the electromagnetic interaction of that field is not renormalizable like the cases of spin $\frac{1}{2}$ and spin 0. It became clear that it must be difficult to implement renormalization. First success came with the mechanism of spontaneous symmetry breaking called after Higgs today. The mechanism is to be applied to a massless Yang-Mills field. A model description of particle phenomena resulted that is still not in variance with the experimental data. Nevertheless, the model has drawbacks. Two of them will be described subsequently. Then an alternative description will be outlined and applied to the decays of lepton partners and to the weak self-energies of leptons. The results will suggest that those drawbacks can be removed.

An electroweak Yang-Mills-type model is characterized by the technique of dimensional regularization, which was adopted from solid state theory. Though the phase transitions of solid state theory can be treated consistently without that technique, models of a broken Yang-Mills symmetry must use it, which seems to be a fundamental drawback. A sound theory should be able to hide possible underlying structures by a cut-off procedure.

From the practical point of view, that model has the following drawback. It is unable to describe the prominent violation of CP-invariance in K-decays without introducing fields extra to this purpose. These fields would be Higgs-type fields, which are unfortunately not seen in experiment.

Theoretical advances were sought by tackling the feature of the imaginary mass of the hypothetical Higgs particle [1]. Another avenue chose as starting point the need to understand the fourth spectroscopy of the lepton and quark masses and their mixing angles first [2]. It led to the

hypothesis that all the weak bosons, leptons and quarks, i.e. all the fundamental particles possessing a pointlike low-energy limit, occur together with heavy partners in nature. In contrast to supersymmetric phenomenologies, the tree-level transition between a particle and its partner must be allowed. The coupling is uniquely determined by the principle of renormalizability, since it requires a cancellation of leading terms in tree-level amplitudes between a particle and its partner at high energies [3].

For example, the amplitude of the decay $N \rightarrow e + W$, where N denotes the partner of the electron-neutrino, would be given by

$$i\mathcal{M}_\mu = \frac{-ieg}{2\sqrt{2}} \bar{u}(p_e) \gamma^\mu (1 - \gamma_5) (\varepsilon_W)_\mu U(P_N) \quad (1)$$

in the lowest order of perturbation theory. One can see in (1) the conventional weak interaction vertex with $g = 1/\sin \theta_W$. The heavy-light mixing angle, whose natural value is about 1, is approximated by 1. Amplitude (1) leads to the decay width

$$\Gamma(N \rightarrow e + W) = \frac{1}{8} \alpha g^2 |p_e| \left[\frac{M_N^2}{m_W^2} + 1 - \frac{2m_W^2}{M_N^2} \right] \quad (2)$$

with M_N the mass of the neutrino partner. The electron mass is put equal to zero. The first of the three terms on the right is the leading term provided that $M_N \gg m_W$. Let us confine attention to this scenario. It is based on the absence of new physics in the $g-2$ data and the conjecture that the neutrino partner be as heavy as the electron partner.

That leading term is solely due to the longitudinal polarization state of the W . Unitarity requires that $\Gamma/M_N < 1$, which means $M_N < m_W/\sqrt{\alpha} \sim 1 \text{ TeV}$, since $|p_e| \sim \frac{1}{2}M_N$. A similar bound is known for the Higgs scalar [4]. Such a bound does not exist in QED, where the width remains relatively small and of the order of α . The heavy partners are always electroweak objects and never purely electromagnetic ones, as one

could think in view of an electron partner.

The nature of the W can be interpreted by a novel underlying compositeness structure. It is characterized by massless spin- $\frac{1}{2}$ spinors, which couple to the photon in the conventional minimal manner [3]. These spinors flow from the lepton to the W in the ν - W - e transition so that they are basic building blocks of both the leptons, quarks and weak bosons. The doubling of states can be regarded as an outgrowth of the existence of two different representations of $SL(2, \mathbb{C})$ for half-integer spin. The pure left-handedness of the W -fermion coupling is indicative of the existence of fundamental laws of such a kind. An essential point is that the compositeness structure is of a GUT scale so that it does not abolish the duty to be renormalizable.

In particular, antiparallel alignment of two such spinors engenders weak spin-0 states. They will be called Φ . Their properties are defined by the feature of regularizing the bad high-energy behaviour of the longitudinal W -state. The states W and Φ are always exchanged simultaneously, which is described by the W - Φ propagator

$$(-i) \left[\frac{g_{\mu\nu}}{k^2 - m_W^2} - \frac{1}{m_W^2} k_\mu k_\nu \left\{ \frac{1}{k^2 - m_W^2} - \frac{1}{k^2 - m_\Phi^2} \right\} \right] \quad (3)$$

The cancellation between the two longitudinal terms at high energies is of the same type as that provided by a partner state. Let us assume that m_Φ is about as large as m_W . Then, we have the decay $N \rightarrow e + \Phi$ besides that in (2) and a doubling of the total decay width.

For a better understanding of massive spin-1 fields it is important to know whether unitarity can be saved at $M > m_W/\sqrt{\alpha}$ by taking into account the higher orders of perturbation theory. To this end, self-energies will be computed in the following.

The W - Φ - e self-energy $-i\Sigma(p)$ can be split up into the sum $-i\Sigma(p) =$

$-i\Sigma_{\text{Trans}}(p) - i\Sigma_{\text{Long}}(p)$ according to the sum of the transversal $g_{\mu\nu}$ term in (3), which can also be called heavy-photon term, and the longitudinal m_W^{-2} term. We intend to substitute the self-energy into the decaying N line. Then, there is an absorptive part because a real W-e pair can be created. The self-energy can be represented as a sum of a dispersive part ("Dp") and an absorptive part ("Ap"). The results for $-i\Sigma_{\text{Long}}(p)$ are

$$\begin{aligned} \text{Dp}[-i\Sigma_{\text{Long}}^e(p)] &= \frac{i\alpha g^2}{16\pi} \not{p} (1-\gamma_5) \int_0^1 dz \left[\frac{1}{2}(1-\varphi) - \right. \\ &\quad \left. - [\varepsilon_W(1+3z) - \kappa_W^{-1}(1-z)(1+4z)] \cdot z \cdot \ln|d_\Phi/d_W| - \right. \\ &\quad \left. - (1-z)(1+3z) [\ln(\Lambda^2/|d_W|) - \varphi \cdot \ln(\Lambda^2/|d_\Phi|)] \right] \end{aligned} \quad (4a)$$

where $\varphi = m_\Phi^2/m_W^2$ and $d_W = (m_W^2 - p^2 z)(1-z) + m_e^2 z$. Polynomial d_Φ follows from d_W by $m_W \rightarrow m_\Phi$. Symbol $\varepsilon_W = m_e^2/m_W^2$ denotes the dimensionless squared electron mass. Further,

$$\begin{aligned} \text{Ap}[-i\Sigma_{\text{Long}}^e(p)] &= \frac{-\alpha g^2}{16} \not{p} (1-\gamma_5) [\kappa_W^{-1} - 2 + \kappa_W] \cdot O(1 - \kappa_W) + \\ &\quad + O(\not{p} \cdot m_e^2/m_W^2) \end{aligned} \quad (4b)$$

up to higher-order-terms, where $\kappa_W = m_W^2/p^2$. Result (4b) follows from (4a) by applying the formula $\ln(1/c) = \ln(1/|c|) + i\pi\Theta(-c)$ and carrying out the z integration. The ratio d_Φ/d_W in (4a) as well as the "-" between the two logarithms in the third term are due to the "-" between the longitudinal terms of W and Φ in (3). This "-" can however not mean that the absorptive parts are also to be subtracted. Because the width $\Gamma(N \rightarrow e + W)$ is to be added to $\Gamma(N \rightarrow e + \Phi)$, the absorptive part due to W is to be added to that of Φ . As a consequence, the κ_W^{-1} term appears in (4b), which is large at $p^2 = M_N^2$. There is also a κ_W^{-1} term in (4a). It is however suppressed by the logarithm at large p^2 .

The self-energy Σ_{Trans} that is due to the transversal $g_{\mu\nu}$ term of the W- Φ propagator does not exhibit κ_W^{-1} terms or other large terms. It contributes on the level of conventional QED self-energy corrections and will not be of interest in the following.

Next, let us insert $-i\Sigma_W^e(p)$ into the in-going line of the decay in (2). Two possibilities show up. Either the self-energy operator effects an N- ν transition before the final decay or an N-N transition. The N-N transition leads to a real N-state and is certainly part of the in-going state, so that only intermediate normal neutrinos are to be taken into account. This means that $U(P_N)$ in (1) is to be replaced by

$$\left[1 + i(\not{P}_N - m_\nu)^{-1} [-i\Sigma_W^e(P_N)] + \dots \right] U(P_N) \quad (5)$$

Let us substitute (4) and take into account only the leading term, which is given by κ_W^{-1} in (4b). Summing up all chains of self-energy insertions in (5) and abbreviating $y = \frac{1}{4}\alpha M_N^2/m_W^2$, we obtain

$$\left[\dots \right] = \left[1 - \frac{iy}{1+2iy} (1-\gamma_5) + \dots \right] \quad (6)$$

where use is made of $g^2 \sim 4$. Factor "2" is due to $(1-\gamma_5)^2 = 2(1-\gamma_5)$. The three dots stand for both the terms small in the ratios m_e/m_W , m_W/M_N and higher-order self-energy graphs *not* representing simple chains. On substituting (5)-(6) into amplitude (1), we see that $1-\gamma_5$ can be factored out to get the left-handed form factor $F_L = 1/(1+2iy)$. The decay amplitude (2) is modified and becomes

$$\frac{1}{2}(\Gamma_W + \Gamma_\Phi)/M_N = y/(1+4y^2) + \text{h.o.t.} \quad (7)$$

which is behaving as M_N^{-2} at large M_N . Unitarity is no longer violated! The self-energy operator modifies the N-state such that it is partly in the

ν -state. As a consequence, the decay amplitude is suppressed. The longitudinal W mode effects that the suppression is the stronger, the larger the mass of the neutrino partner.

Besides the W- Φ -e insertion, there is a Z- Φ^0 - ν insertion. Does it spoil the suppression? The vertices ν -Z- ν and N-Z- ν are given by $-ie g_A \cdot \gamma^\mu (1 - \gamma_5)$ with $g_A^2 \sim 1/3$. The Z- Φ^0 - ν insertion is also purely left-handed. It gives rise to a self-energy of the type of (4). The insertion enhances the W- Φ -e effect. Using the approximation $3m_Z^2 \sim 4m_W^2$, we get that the form factor takes the form $F_L = 1/(1 + 3iy)$. The new M_N behaviour of (7) is

$$\frac{1}{2}(\Gamma_W + \Gamma_\Phi)/M_N = y/(1 + 9y^2) + \text{h.o.t.} \quad (8)$$

Finally, we investigate the case that the electron partner is lighter than the neutrino partner. Then, the third absorptive channel $N \rightarrow E + W$ is open. The analysis of the self-energy $-i\Sigma_W^E(p)$ leads to the absorptive part

$$\text{Ap}[-i\Sigma_{\text{Long}}^E(p)] = \frac{-\alpha g^2}{16} \not{p} (1 - \gamma_5) [\kappa_W^{-1} - E_W(3 - 3\kappa_E + \kappa_E^2)] \cdot \Theta(1 - \kappa_E) + O(\not{p} \cdot 1) \quad (9)$$

where $\kappa_W^{-1} = p^2/m_W^2$, $E_W = M_E^2/m_W^2$ and $\kappa_E = M_E^2/p^2$. It exhibits both a large κ_W^{-1} term and a large E_W term. We insert also (9) into the in-going line of the decay $N \rightarrow e + W$ or $e + \Phi$ to find the complete form factor $F_L = 1/[1 + (2\mu + 1)iy]$ where $\mu = 1 - (1 - R)^3$ with $R = M_E^2/M_N^2$ and $0 < R \leq 1$. The $(1 - R)^3$ term is due to (9). It correctly disappears at $R = 1$, where the absorptive channel of the E contribution is closed. The "—" in front of $(1 - R)^3$ in μ is due to the compensation between the propagators of e and E in the self-energy loop due to W- Φ . That "—" is the reason for the behaviour $\mu \rightarrow 0$ when $R \rightarrow 0$. In this limit of both $m_e \ll M_N$ and $M_E \ll M_N$, the leading-term effects of the insertions W- Φ -e and W- Φ -E

cancel each other out. Only the Z- Φ^0 - ν insertion survives. The numerical factor that is equal to 4 in (7) and equal to 9 in (8) lies between 1 and 9 in dependence of the ratio M_E/M_N .

The same suppressions result for the decay $N \rightarrow Z + \nu$, since the tree-level decay vertex is purely left-handed so that $1 - \gamma_5$ can again be factored out.

The decays of the electron partner are characterized by both left-handed and right-handed couplings and correspondingly two form factors. This complicates the analysis. It is nevertheless possible to derive the form factors rigorously. They are of the suppressive type of F_L !

In summary, electroweak theories of massive spin-1 particles seem to exist that do not make use of spontaneous symmetry breaking and do not give rise to unitarity masss bounds. This result is important for the interpretation of precise $g - 2$ data. The disappearance of the bounds can be described by form factors. They exhibit substantial imaginary parts which provides new insights into the nature of the violation of the CP symmetry. No extra fields or interactions besides the experimentally established ones and those required by renormalizability are to be introduced. Another novel source of CP violation is given by γWW interaction terms [3,5]. Both sources are heavily characterized by quark partners.

References

- [1] E. H. Lemke, *Phys. Rev. D* **22** 1342 (1980); *Inter. J. Theor. Phys.* **16** 307 (1977).
- [2] E. H. Lemke, *J. Phys. G: Nucl. Phys.* **13** 439 (1987); *J. Phys. G: Nucl. Part. Phys.* **17** 791 and 809 (1991); *Modern Phys. Lett. A* **7** 1175 (1992)
- [3] E. H. Lemke, *Inter. J. Modern Phys. A* **8** 3883 (1993); *Nuovo Cimento A* **106** 497, 637 and 1175 (1993); *Modern Phys. Lett. A* **8** 757 (1993).
- [4] B. W. Lee, C. Quigg and H. B. Thacker, *Phys. Rev. D* **16** 1519 (1993).
- [5] E. H. Lemke, *Nuovo Cimento A*, in press.