Gluon Decay as a Mechanism for Strangeness Production in a Quark-Gluon Plasma.

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Abstract

A calculation of thermal gluon decay shows that this process contributes significantly to strangeness production in a quark-gluon plasma. Our analysis does not support recent claims that this is the dominant process. In our calculations we take into account the resummed form of the transverse and longitudinal parts of the gluon propagator following the Braaten-Pisarski method. Our results are subject to the uncertainty concerning the estimate of the damping rate entering the effective gluon propagator.
1 Introduction

A possible signal for quark-gluon plasma formation in heavy-ion collisions is the enhancement of the production of strange particles. The original proposal by Rafelski and Müller [1] was followed by extensive discussion in the literature [2, 3, 4, 5, 6, 7]. In this context thermal gluon decay has been recently discussed [5, 6, 7]. It has been claimed recently that the process $g \rightarrow q\bar{q}$ dominates for a wide range of quark masses [6, 7]. Normally, the gluon cannot decay into a strange quark-antiquark pair because its thermal mass is too low. Even for the optimistic case where one takes the coupling constant $g = 2$ in a plasma with two massless quarks, the gluon mass is given to lowest order in perturbation theory by

$$m_g = \frac{2}{3} g T.$$  \hspace{1cm} (1.1)

For a temperature of $T = 200$ MeV this gives $m_g = 267$ MeV which is below the threshold for the production of strange quarks. The important observation by Altherr and Seibert is that in addition to acquiring a thermal mass, gluons also acquire a width, determined by the large damping rate, of the order $g^2 T$ [8]. This is the reason why thermal gluon decay into a heavy quark-antiquark pair is allowed, even though the gluon mass is below the threshold for strange pair production.

In this paper we present a systematic re-evaluation of the production rate of massive quarks in a quark-gluon plasma due to the processes of quark-antiquark annihilation, gluon fusion and thermal gluon decay in the spirit of Altherr and Seibert. Since the production rate depends strongly on the damping rate we take a more conservative approach in its estimation. Our main point is that even with the parameters chosen in Ref. [6, 7] we cannot support the claim that gluon decay is the dominant mechanism for strange quark production in a quark-gluon plasma. To the best of our knowledge, the gluon fusion mechanism, originally proposed by Rafelski and Müller [1], remains the leading process.

The paper is organized as follows. In section 2 we briefly review the properties of thermal-gluon propagators and the damping rate. In section 3 we calculate the production rates. In section 4 we present results and concluding remarks.

2 Gluon Propagator and the Damping rate

The effective gluon propagator at finite temperature in the Feynman gauge is given by [9]

$$iD_{\mu\nu}^{ab}(q_0, q) = -i \delta^{ab}[P_{\mu\nu}^T \Delta_T(q_0, q) + P_{\mu\nu}^L \Delta_L(q_0, q)],$$  \hspace{1cm} (2.1)

where $P_{\mu\nu}^T$ and $P_{\mu\nu}^L$ are transverse and longitudinal projectors respectively, and

$$\Delta_{T,L}(q_0, q) = \frac{1}{Q^2 - \Pi_{T,L}(q_0, q)},$$  \hspace{1cm} (2.2)

where $Q^2 \equiv q_0^2 - q^2$. The real transverse and longitudinal parts of the gluon self energy in the high temperature limit are respectively given by

$$\text{Re} \Pi_T(q_0, q) = \frac{3}{2} m_g^2 \left[ \frac{q_0^2}{q^2} + \left(1 - \frac{q_0^2}{q^2}\right) \frac{q_0}{2q} \ln \frac{q_0 + q}{q_0 - q} \right],$$  \hspace{1cm} (2.3)
and
\[
\text{Re}\Pi_L(q_0, q) = \frac{3}{2} m_g^2 \left( 1 - \frac{q_0^2}{q^2} \right) \left[ 2 - \frac{q_0}{q} \ln \frac{q_0 + q}{q_0 - q} \right].
\]  
(2.4)

The positions of the poles in the propagator (2.2) are determined by the dispersion relations
\[
q_0^2 = q^2 + \Pi_{T,L}(q_0, q).
\]  
(2.5)

If a pole is located at
\[
q_0 = \omega_{T,L} + i\gamma_{T,L}
\]  
(2.6)

then the imaginary shift of the pole \(\gamma_{T,L}\) is related to the imaginary part of the self energy through
\[
\gamma_{T,L} = \text{Res}(\Delta_{T,L}) \text{Im}\Pi_{T,L},
\]  
where \(\text{Res}(\Delta)\) is the residue of the propagator given by
\[
\text{Res}(\Delta_{T,L})^{-1} = \left. \frac{\partial \Delta_{T,L}^{-1}}{\partial q_0} \right|_{\omega_{T,L}},
\]  
(2.8)

or, explicitly in terms of \(\omega_{T,L}\)
\[
\text{Res}(\Delta_T)^{-1} = -\omega_T + \frac{q^2}{\omega_T} + \frac{3m_g^2\omega_T}{\omega_T - q^2},
\]  
(2.9)

\[
\text{Res}(\Delta_L)^{-1} = -\omega_L + \frac{q^2}{\omega_L} + \frac{3m_g^2}{\omega_L}.
\]  
(2.10)

Thus we can write (2.2) as
\[
\Delta(Q) = \frac{Q^2 - \text{Re}\Pi}{(Q^2 - \text{Re}\Pi)^2 + \text{Res}(\Delta)^{-1} \gamma^2} + \frac{i\text{Res}(\Delta)^{-1} \gamma}{(Q^2 - \text{Re}\Pi)^2 + \text{Res}(\Delta)^{-1} \gamma^2},
\]  
(2.11)

where we suppressed subscripts \(T, L\), i.e. \(\Delta, \Pi\) and \(\gamma\) are either transverse or longitudinal. This expression will be used to replace the mass-shell \(\delta\)-function for thermal gluons.

The imaginary part of the pole in (2.6) gives the damping rate of the plasma oscillations \([8, 10]\). In the following we consider transverse gluons and we set \(\gamma \equiv \gamma_T\). The damping rate is related to the so-called gluon magnetic mass \(m_{mag}\), or the inverse magnetic screening length at high temperature. Unfortunately, the exact relation between \(\gamma\) and \(m_{mag}\) is not known. A closed expression for the damping rate has been derived by Pisarski in the limit where \(m_{mag} \gg \gamma\) [8]:
\[
\gamma = \frac{g^2 NT}{8\pi} \left[ \ln \left( \frac{m_g^2}{m_{mag}^2 + 2m_{mag}\gamma} \right) + 1.1 \right],
\]  
(2.12)

where the thermal gluon mass is given by
\[
m_g^2 = (N_c + \frac{N_f}{2}) \frac{g^2 T^2}{9}.
\]  
(2.13)

The magnetic mass at high temperature is of the form
\[
m_{mag} = c_N g^2 T
\]  
(2.14)

where $c_N$ is a number depending on the gauge group and cannot be calculated by a perturbation expansion. Lattice estimates [11, 12] for SU(2)

$$c_2 = 0.27 \pm 0.03$$

(2.15)

have been confirmed by recent semiclassical calculations [13]. So far, no reliable estimate exists for SU(3) [14]. The best one can do is extrapolate the SU(2) value by [15]

$$c_3 = \frac{3}{2} c_2$$

(2.16)

Expanding the log in (2.12) in powers of $\gamma/m_{mag}$ and retaining only the leading terms one finds

$$\gamma = (1 + \eta) - i \frac{g^2 N T}{8 \pi} \left[ \ln \left( \frac{m_q^2}{m_{mag}} \right) + 1.09681 \ldots \right].$$

(2.17)

where $\eta = 0$ if we keep the leading log term only. If the next-to-leading term is included then

$$\eta = \frac{N}{4 \pi c_N}.$$  

(2.18)

To check the consistency of Pisarski’s approximation we plot in Fig 1 the damping rate for both values of $\eta$ as well as the damping rate used in [6, 7] and compare with $m_{mag}$. One can observe the poor validity of the approximation for small values of $g$. The approximation is well justified if one uses the expression (2.17) in the range $1 < g < 2.5$.

### 3 Production rates

Consider a quark-gluon plasma in which the gluons and the light quarks ($u, d$) are in thermal and chemical equilibrium. The strange quarks too are in thermal equilibrium but away from chemical equilibrium having very large and negative chemical potential $\mu \equiv \mu_s = \mu_s$. The chemical reactions

$$q + \bar{q} \rightarrow s + \bar{s}, \quad (3.1)$$

$$g + g \rightarrow s + \bar{s}, \quad (3.2)$$

$$g \rightarrow s + \bar{s}, \quad (3.3)$$

will then take place until chemical equilibrium is reached. The total production rate due to (3.1-3.3), including the reversed processes, is given by [4]

$$\delta R = (1 - e^{3\beta \nu})(R_{q\bar{q} \rightarrow s\bar{s}} + R_{gg \rightarrow s\bar{s}} + R_{g \rightarrow s\bar{s}})$$

(3.4)

where

$$R_{q\bar{q} \rightarrow s\bar{s}} = \int \frac{d^3 p_\alpha}{(2\pi)^3 E_{\alpha}} \frac{d^3 p_\beta}{(2\pi)^3 E_{\beta}} \frac{d^3 p_s}{(2\pi)^3 E_s} \frac{d^3 p_{\bar{s}}}{(2\pi)^3 E_{\bar{s}}} (2\pi)^4 \delta(P_\alpha + P_\beta - P_s - P_{\bar{s}}) \times f_{FD}(E_\alpha)f_{FD}(E_\beta)(1 - f_{FD}(E_s))(1 - f_{FD}(E_{\bar{s}})) \sum |M(q\bar{q} \rightarrow s\bar{s})|^2.$$  

(3.5)
In our notation the four-momenta are denoted by capitals and change variables where the integration space is restricted by the following kinematical constraints:

After trivially eliminating integrals over \( n_5b/4/n_5d \) and we shall use their expressions for \( n_28/3/./5/3/./6/n_29/ \). The thermal gluon decay, also discussed by Altherr and Seibert \( n_5b/6/, n_5d/, \) can be calculated similarly. We first replace the integrations over \( q \), \( p_s \) and \( p_5 \) by

\[
\frac{d^3q}{2E_g} = d^4Q \delta(Q^2 - m_s^2) \theta(q_0)
\]

\[
\frac{d^3p_s}{2E_s} \frac{d^3p_5}{2E_5} = d^4P_s \delta(P_s^2 - m_s^2) \theta(p_0^0) d^4P_5 \delta(P_5^2 - m_s^2) \theta(p_0^5)
\]

and change variables

\[
Q' = P_s + P_5
\]

\[
P = \frac{1}{2}(P_s - P_5).
\]

After trivially eliminating integrals over \( d^4Q' \) and \( d^3p \) we find

\[
R_{g \to s\bar{s}} = \frac{1}{4(2\pi)^4} \int d^4Q'(Q^2 - m_2^2) \frac{1}{4} f_{BE}(q_0) 
\]

\[
\times \int dp_0 (1 - f_{FD}(vq_0 + p_0))(1 - f_{FD}(\frac{1}{2}q_0 - p_0)) \sum |M(g \to s\bar{s})|^2
\]

where the integration space is restricted by the following kinematical constraints:

\[
g_0 > 2m_s, \quad 0 < q < (q_0^2 - 4m_s^2)^{1/2}, \quad p_0^2 < \frac{q_0^2}{4} \left( 1 - \frac{4m_s^2}{Q^2} \right)
\]
It immediately follows that $R_{g \rightarrow s\bar{s}} = 0$ if $m_s < 2m_g$. At the relevant temperatures the thermal gluon mass is not high enough to allow for decay into a strange quark pair. It is only because of its width that the gluon can decay. To take this into account the $\delta$-function is replaced by a function, similar to the Breit-Wigner resonance. In the case of a narrow resonance the width of the resonance is related to the imaginary shift of the pole in the propagator in the complex $\phi_0$ plane

$$\frac{1}{q_0^2 - (\sqrt{q^2 + m^2} + i\gamma)^2} \approx \frac{1}{Q^2 - m^2} + \frac{i2\sqrt{q^2 + m^2}\gamma}{(Q^2 - m^2)^2 + 4(q^2 + m^2)\gamma^2}$$

(3.14)

which in the limit $\gamma \rightarrow 0$ yields the standard free particle propagator

$$\frac{1}{Q^2 - m^2 - i\epsilon} = \mathcal{P}\frac{1}{Q^2 - m^2} + i\pi\delta(Q^2 - m^2).$$

(3.15)

Thus for a Breit-Wigner resonance with width $\Gamma = \gamma/2$ the mass-shell $\delta$-function should be replaced by

$$\delta(Q^2 - m^2) \rightarrow \frac{1}{\pi} \frac{\sqrt{q^2 + m^2}\Gamma}{(Q^2 - m^2)^2 + (q^2 + m^2)\Gamma^2}.$$  

(3.16)

This simple prescription cannot be directly applied to the case of thermal gluons because the location of the pole is determined by complicated dispersion relations (2.5) for transverse (T) and longitudinal (L) gluons. Due to (2.11), instead of (3.16), we use

$$\delta(Q^2 - m_g^2) \rightarrow \frac{1}{\pi} \frac{\text{Res}(\Delta_{T,L})^{-1}\gamma_{T,L}}{(Q^2 - \text{Re}\Pi_{T,L})^2 + \text{Res}(\Delta_{T,L})^{-2}\gamma_{T,L}^2}.$$  

(3.17)

The matrix element is simply given by

$$M(g \rightarrow s\bar{s}) = g\epsilon_\mu(\zeta) \bar{u}(P_s)\gamma_\mu\lambda_a v(P_s),$$

(3.18)

where $\epsilon_\mu(\zeta)$ is the polarization vector of the decaying gluon and $\lambda_a$ are the SU(3) matrices.

Summing over colors and all polarizations of the gluon leads to

$$\sum_{a,\zeta} |M(g \rightarrow s\bar{s})|^2 = -4g^2 \text{Tr}[(P_s + m_s)\gamma_\mu(P_s - m_s)\gamma^\mu]$$

$$\quad = 16g^2(2m_s^2 + Q^2).$$

(3.19)

Since the frame of the quark-gluon plasma introduces a preferred direction, it is furthermore necessary to distinguish between the transverse and the longitudinal components of the gluons. If the sum is taken over transverse or longitudinal polarization only, we find

$$\sum_T |M(g \rightarrow s\bar{s})|^2 = 8g^2[4m_s^2 + Q^2(1 + 4\frac{P_0^2}{q^2})],$$

$$\sum_L |M(g \rightarrow s\bar{s})|^2 = 8g^2Q^2(1 - 4\frac{P_0^2}{q^2}).$$

(3.20)

(3.21)
By making use of (3.17) and (3.20,3.21) we find from (3.12)

\[
R_{g-s}^T = \frac{g^2}{3\pi^2} \int_{2m_s}^{\infty} dq_0 f_{BE}(q_0) \int_0^{\sqrt{q_0^2-4m_s^2}} dq \int_{-\sqrt{q_0^2-4m_s^2}}^{\sqrt{q_0^2-4m_s^2}} dp_0 \times \left(1 - f_{FD}(\frac{1}{2}q_0 + p_0)(1 - f_{FD}(\frac{1}{2}q_0 - p_0)) \times \frac{\text{Res}(\Delta_T)^{-1} \gamma_T}{(Q^2 - \text{Re}\Pi_T)^2 + \text{Res}(\Delta_T)^{-2} \gamma_T^2}[4m_s^2 + Q^2(1 + 4\frac{R_0^2}{q_0^2})], \right) \quad (3.22)
\]

and a similar expression for \(R_{g-s}^L\). The production rate due to the gluon decay is given by the sum

\[
R_{g-s} = R_{g-s}^T + R_{g-s}^L. \quad (3.23)
\]

If we neglect the Pauli blocking factors the integral over \(p_0\) can be done explicitly, leading to

\[
R_{g-s}^T = \frac{2g^2}{3\pi^4} \int_{2m_s}^{\infty} dq_0 f_{BE}(q_0) \int_0^{\sqrt{q_0^2-4m_s^2}} dq q^2 \sqrt{1 - \frac{4m_s^2}{Q^2}}(Q^2 + 2m_s^2) \times \frac{\text{Res}(\Delta_T)^{-1} \gamma_T}{(Q^2 - \text{Re}\Pi_T)^2 + \text{Res}(\Delta_T)^{-2} \gamma_T^2} \quad (3.24)
\]

and a similar expression for \(R_{g-s}^L\).

We use the full high temperature expressions for \(\text{Re}\Pi_{T,L}\) given by (2.14,2.15) and numerically solve the dispersion relations (2.5) in order to determine \(\text{Res}(\Delta_{T,L})\) from (2.9,2.10). The temperature dependent gluon mass is given by (2.13) and the damping rate \(\gamma = \gamma_L\) is estimated using (2.17).

## 4 Results and Conclusion

The rates for different processes are depicted in Fig 2 and Fig 3. Our numerical calculation of the thermal gluon decay is done using equations (3.22,3.23) with (2.3,2.4,2.9,2.10) and (2.17). The rates for quark-antiquark annihilation and gluon fusion we calculate by making use of equations (3.23-3.25) in ref [4]. We fix the QCD coupling constant at the value \(g = 2\) because the temperature during the time evolution is almost constant and the running coupling effect is negligible. In Fig 2 the quark mass is kept fixed while in Fig 3 it varies with temperature according to (3.8). In both cases we find that the gluon fusion together with the quark-antiquark annihilation, dominates almost everywhere. The gluon decay process is as large as the gluon fusion in the narrow region around \(m_s(0)/T = 1\) only if we choose the optimistic [7] parameterization of the damping rate.

It has been shown that the time dependence of the strange-quark density can, to a great degree of accuracy, be described by the approximate evolution equation [4]

\[
n_s(t) = n_s^eq\tanh\left(\frac{t}{2\tau} + \text{const}\right), \quad (4.1)
\]

where the relaxation time is defined as

\[
\tau = \frac{1}{2\beta R} \left| \frac{\partial n_s}{\partial \mu} \right|_e \quad (4.2)
\]
with
\[ R = R_{q\bar{q} \rightarrow ss} + R_{gg \rightarrow ss} + R_{g \rightarrow ss}. \]  

The derivative of \( n_s \) with respect to \( \mu \) at fixed energy density is given by
\[ \frac{\partial n_s}{\partial \mu} = \frac{\partial n_s}{\partial \mu} - \frac{\partial n_s}{\partial T} \left( \frac{\partial \epsilon}{\partial T} \right)^{-1} \frac{\partial \epsilon}{\partial \mu} \]  

where
\[ n_s = 2N_f \int \frac{d^3 p}{(2\pi)^3} f_{FD}(E_s, \mu) \]  
and
\[ \epsilon = \frac{(N_f - 1)\pi^2 T^4}{15} + 4N_fN \int \frac{d^3 p}{(2\pi)^3} E_q f_{FD}(E_q, 0) + 4N \int \frac{d^3 p}{(2\pi)^3} E_s f_{FD}(E_s, \mu). \]  

All the quantities in (4.2) are to be evaluated at \( \mu = 0 \). In Fig 4 we plot the relaxation time for the saturation of the strange-quark density for the massive quarks with the zero temperature mass \( m_s(0) = 0.2 \) GeV along with the classical approximation. In this approximation Pauli blocking factors \( (1 - f_{FD}) \) are eliminated and the remaining Fermi-Dirac and Bose-Einstein distributions are replaced by the Boltzmann distribution. For comparison we also plot the relaxation time for the massless quarks.

We comment here on the various approximations made in the gluon decay calculation. First of all, use was made of the Braaten-Pisarski resummation scheme. This is strictly valid only when \( gT \ll T \) which is clearly not the case here. This is the case with most applications of QCD at finite temperatures. Secondly the magnetic mass has been introduced although only very limited knowledge is available. It has been used to calculate the damping rate of a thermal gluon inside a plasma. In comparison to the calculations of references [6, 7] we keep the standard form of the Breit-Wigner distribution. The consequences of this is that the rate for high masses is reduced, while for low masses it is enhanced. We also avoid a rather heuristic assumption that the thermal quark mass is generated by gluons only. Since our rates are defined near equilibrium our thermal mass includes both thermal gluon and thermal quark contribution. Finally, a more accurate calculation of the gluon fusion process shows that the high mass approximation used in [7] underestimates its contribution.

Our main point has been that even with the parameters chosen in Ref.[6, 7] we do not support the claim that the gluon decay process is the dominant mechanism for strange quark production inside a quark-gluon plasma. To the best of our knowledge, the gluon fusion mechanism is the leading process.
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References

Figure Captions:

1. The damping rate and the magnetic mass versus the coupling constant.

2. The quark production rate for thermal gluon decay for different damping rates (short dashed, long dashed and dot-dashed lines correspond to the damping rates depicted in Fig 1) compared to the production rate for gluon fusion and quark antiquark annihilation (solid line). The mass $m_s$ is temperature independent.

3. Same as Fig 2 with the thermal mass $m_s$ given by (3.8).

4. Relaxation times for the density of massive (solid line) and massless (long dashed line) quarks. Corresponding relaxation times in the classical approximation are plotted with dashed and dotted lines respectively.