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Z_2 GAUGE MATTER COUPLED TO 4D SIMPLICIAL QUANTUM GRAVITY

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Abstract

Employing Monte-Carlo simulation we study the phase diagram of a Z_2 gauge field coupled to simplicial quantum gravity. We localize a critical point of the model where both the matter and gravity sectors have a second order phase transition. We found the value of the critical index $\gamma_g = 0.16(4)$ of the gravity susceptibility at the critical point.

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Introduction

Recently much effort was put into the understanding of a theory that combines the basic principles of both General Relativity and Quantum Theory. The dynamical triangulation method, which was successfully applied in two dimensions (see [1] for a review, and references therein) was extended to the case of three [2–5] and four dimensions [6–15] to define a discretized version of the Euclidean path integral.

In the continuum approach the geometry of the manifold is represented by the metric tensor field. Direct summation over all metric tensors would introduce overcounting to the integration measure since the tensors differing by the diffeomorphism transformation describe the same geometry. In two dimensions the problem of dividing out the orbits of diffeomorphism was exactly solved and the measure is known. This is not the case for $d > 2$ gravity. Moreover, for $d > 2$ the theory is not renormalizable. A possible solution of this difficulty goes back to the idea of asymptotic safety which assumes the existence of the safe subspace in the coupling space [16]. For the couplings on the safe surface a renormalization flow leads to the safe nonperturbative UV fixed point. The sign of evidence for the fixed point which realizes this scenario first came from the perturbation theory in $2 + \epsilon$ dimensions [17]. The idea behind the $2 + \epsilon$ expansion is to make first a resummation in terms of well defined integrals of the 2d-theory, which is renormalizable, and then rearrange them as a series in ϵ . For four dimensions ϵ equals two, and it is not clear if one can make any conclusive statements from few first orders of the ϵ expansion. Moreover, the Einstein–Hilbert action is a topological invariant in 2d which makes the interpretation of the nonperturbative fixed point found in this way difficult.

In four dimensions a candidate for the asymptotically safe point was found in the dynamical triangulation approach [7]. In this approach one conjectures that the sum over geometries on a manifold can be approximated by the sum over equilateral triangulations. In two dimensions, where the summation can be performed analytically, the results of the continuum and discretized approaches are in perfect agreement. This suggested extending the discretized method to higher dimensional gravity (called simplicial quantum gravity) with the hope of finding a nonperturbative fixed point. In fact, the numerical simulations of 4d quantum simplicial gravity show the existence of a second order phase

transition opening up the possibility of defining a continuum theory at this point. It is not clear what is the underlying continuum theory since no continuum quantum gravity theory exists yet in four dimensions. Among the problems which still require clarification is the numerical observation that the average curvature of the discretized manifold does not vanish at the critical point [9]. Some of the problems are expected to be cured by enlarging the space of the couplings of the model. In this way one can possibly reach the critical safe point with well defined physical quantities. It is for example known that including the higher derivative terms in the action suppresses the curvature fluctuations. The system with the action containing the term proportional to the curvature square was considered, but it was found to belong to the same universality class as the pure gravity without such a term [9].

New geometrical terms in the action can be induced dynamically by coupling the pure gravity to the matter fields. Integrating out the matter fields we get the effective action, which typically contains a whole spectrum of higher derivative terms. Some of the new terms may modify the critical behaviour of the gravity sector. In two dimensions, for instance, the matter changes the critical exponent of the entropy [1,20]. Four dimensional models of Ising and gaussian fields interacting with gravity have already been considered in [10,11].

In this paper we present a new model of this type, in which the 4d gravity sector is coupled to the Z_2 gauge field. The model can be viewed as a first step towards more physical gauge models. Contrary to the models discussed before we have a direct coupling between the plaquettes (i.e. triangles) of the gauge field and the scalar curvature, (i.e. the order of the triangle). In effect we may get new interaction terms in the effective action of the gravity sector. The Z_2 gauge model on a regular lattice in four dimensions has a first order phase transition. It is very interesting to check how the order of the transition is modified by the presence of the dynamical geometry. One can expect non-trivial effects here based on the experience from two dimensions. The matter field works as a tool to measure the effective dimensionality of the system. A similar study for 3d-system coupled to the $U(1)$ gauge field was presented in [5].

In this work we explore the phase diagram of the model. We locate the point on the coupling-plane, where both systems are critical and estimate the scaling behavior of the

curvature at this point. We discuss the effective action and present the measurements of critical exponents to find evidence for a nontrivial coupling between the two sectors.

The model

In this section we briefly recall the 4d simplicial quantum gravity formulation, first introduced in [6, 7]. We also discuss the problem of the coupling of the Z_2 gauge field to the gravity.

The simplicial manifold (or triangulation) is constructed from equilateral 4d simplices glued along the three-dimensional faces (3d sub-simplices) shared by the neighbors. A crude characteristic of the manifold can be given by specifying the numbers N_i of (sub)simplices of dimension $0 \leq i \leq 4$. The neighborhood of a sub-simplex with dimension i formed by 4d simplices sharing this sub-simplex, has the topology of a $(4-i)$ -dimensional ball, which in particular means that the triangulation is locally homeomorphic to R^4 . This imposes relations between the numbers N_i , known as the Dehn-Sommerville relations. They have the form :

$$5N_4 = 2N_3, \quad 5N_4 - 4N_3 + 3N_2 - 2N_1 = 0. \quad (1)$$

The numbers N_i are also related by the Euler formula :

$$N_4 - N_3 + N_2 - N_1 + N_0 = \chi \quad (2)$$

where χ is the Euler characteristic of the manifold. Thus, in the set of manifolds with fixed topology, only two of the numbers of (sub)simplices, say N_4 and N_2 , are independent.

Following Regge [18] one can define discretized counterparts of concepts and expressions used in the continuum approach on the simplicial manifold discretization of the Einstein-Hilbert action :

$$S_{EH}[g] = \lambda \int d^4\xi \sqrt{g} - \frac{1}{16\pi G} \int d^4\xi \sqrt{g} R \quad (3)$$

leads to :

$$S_{EH}[T] = \kappa_4 N_4[T] - \kappa_2 N_2[T] \quad (4)$$

where T stands for a triangulation, which in this approach specifies the internal geometry of the manifold, and κ_4, κ_2 are two independent couplings related by linear combinations to the couplings in formula (3). The second term comes from the integration over the scalar curvature which for the simplicial manifold can be written as a sum of deficit angles of the triangles t of the triangulation :

$$\int d^4\xi \sqrt{g} R \rightarrow \sum_{t \in T} (2\pi - o(t)\Theta) \propto N_2[T] - (2.097...) N_4[T]. \quad (5)$$

The number $o(t)$ is the order of the triangle t , ie the number of 4d simplices sharing this triangle and $\Theta = \arccos 1/4$ is the angle between two triangles in the equilateral 4d simplex. The average lattice curvature density $R_{latt} = N_2/N_4 - 2.097...$ up to an unimportant multiplicative constant absorbed in the definition of R_{latt} .

In the quantum theory one must define the integration measure on the space of geometries. In the simplicial quantum gravity approach it is conjectured that the formal integral over the metric of the manifold can be represented as a sum over triangulations :

$$\int \mathcal{D}[g_{\mu\nu}] e^{-S[g]} \rightarrow \sum_{T \in \mathcal{T}} e^{-S[T]} \quad (6)$$

The sum is usually restricted to manifolds with fixed topology since otherwise the integral is undefined due to the factorial dependence of the entropy on the manifold volume N_4 . The triangulations are weighted by the exponent of the action obtained by the Regge method. The resulting model can be accepted as a quantum theory of gravity if the sum over triangulations properly samples the integral over geometries and the Regge action provides a correct weight for the triangulations. This is known to be the case in two dimensions, where as mentioned above the critical behaviour predicted by the dynamical triangulation method based on the integration measure (6) agrees completely with the continuum theory. No corresponding proof exist in higher dimensions.

The numerical study of the pure gravity model was performed in [6-8]. The main result is the existence of two distinct phases : a "hot" phase, corresponding to small κ_2 with large Hausdorff dimension and negative or small positive average curvature R_{latt} , and the "cold" phase for bigger κ_2 , with small Hausdorff dimension, dominated by the

branched polymer type structures with large R_{lat} . This two phases are separated by a phase transition of second or higher order (see ref. [7] for details).

The next step in constructing the model is to include correctly a matter sector. The general way to couple matter to simplicial gravity is described in ref. [19]. Here we consider the matter field with a Z_2 local symmetry which is coupled to gravity by an additional term given by the Wilson action :

$$S_W[\{\sigma\}_T] = -\beta \sum_{t \in T} o(t) \{\sigma\sigma\sigma\}_t \quad (7)$$

$$S_{EH+W}[T, \{\sigma\}_T] = S_{EH}[T] + S_W[\{\sigma\}_T] \quad (8)$$

The sum runs over triangles, $\{\sigma\sigma\sigma\}_t$ is the product of the Z_2 spin variables on the border links of the plaquette (triangle).

It is very important that the plaquette $\{\sigma\sigma\sigma\}_t$ is multiplied by the order of a triangle (the dual "volume" associated with the triangle). It ensures that the action is extensive and does not introduce an extra coupling to curvature. To see this let us replace the plaquette in (7) by the mean field value $P = \langle \sigma\sigma\sigma \rangle$. From the geometric considerations one has $\sum o(t) = 10N_4$ and therefore $\langle S_W \rangle = -10\beta N_4 P$ is proportional to the volume. Without the $o(t)$ factor in the action we would have $\langle S_W \rangle = -\beta N_2 P = -\beta N_4 P R$ introducing an extra coupling of the average plaquette to the average curvature R .

The partition function for the model now reads :

$$Z = \sum_{T \in \mathcal{T}} \sum_{\{\sigma\}_T} \exp -(S_{EH}[T] + S_W[\{\sigma\}_T]) = \sum_{T \in \mathcal{T}} \exp -S_{eff}[T, \beta] \quad (9)$$

where $S_{eff}[T, \beta]$ is the effective action obtained after performing summation over $\{\sigma\}$. We shall be interested in the properties of the effective action to find the possible new interaction terms introduced by the gauge field.

In the large and small β limits the gauge field decouples and the effective action S_{eff} is equal to the pure gravity Hilbert-Einstein action S_{EH} (4) with renormalized coupling constants :

$$\begin{aligned} S_{eff}[T, \beta] &= -\kappa_{2,eff} N_2 + \kappa_{4,eff} N_4 = \\ &= \begin{cases} -(\kappa_2 + 1/2 \log 2) N_2 + (\kappa_4 + \log 2 - 10\beta) N_4 & \text{for } \beta \rightarrow \infty \\ -(\kappa_2 + 3/2 \log 2) N_2 + (\kappa_4 + 5/2 \log 2) N_4 & \text{for } \beta = 0 \end{cases} \end{aligned} \quad (10)$$

The shifts of the couplings come from the energy and the entropy of the spins. For $\beta = 0$ the entropy factor is 2^{N_1} and corresponds to the number of states of the decoupled spins. For $\beta \rightarrow \infty$ the plaquettes get aligned and the entropy factor 2^{N_0} is the volume of the gauge orbit. For other values of β the effective action contains also other operators. For example, in the first few orders of the high temperature expansion (small β), one obtains the terms $\sum o(t)^2$ where the sum runs over triangles on the triangulation, $\sum o(t1)^2 o(t2)^2$ where the sum runs over pairs of triangles and $\sum o(t1) o(t2) o(t3) o(t4)$ where the four triangles $t1, t2, t3, t4$ form a tetrahedron on the triangulation, etc. These terms have purely geometrical meaning and may affect the properties of the geometrical critical transition in the gravity sector.

The Z_2 gauge model on a regular lattice has a first order phase transition for $d \geq 4$, second order for $d = 3$ and no transition for $d \leq 2$ [25]. One expects that this will have strong influence on the properties of the phase diagram of the composite gauge + gravity system : in the "cold" phase of the gravity sector, where the Hausdorff dimension is typically 2 or less, no phase transition should appear in the gauge sector. On the other hand in the "hot" phase the Hausdorff dimension is large, approximately equal to four near the phase transition and growing for smaller values of κ_2 [6, 7]. Accordingly for small κ_2 we expect the gauge sector to have the first order phase transition. For κ_2 in the transition region close to the gravity phase transition the order of the gauge transition may be changed by the dynamics of the random lattice. This was observed for instance in two dimensions [1], where the gravity sector dresses the matter field operators and for the Ising model the order of the phase transition changes from second to third [21]. If the same mechanism is present in four dimensions the order of the gauge sector phase transition can be modified and become second or higher in the transition region of the gravity sector.

The numerical method and the simulation

The algorithm used for the simulations is based on a set of the so called (p, q) -moves introduced in [4]. These moves are known to be ergodic in the grand canonical space of triangulations [22, 23]. The measurements, however, are taken for the canonical en-

semble, when the volume N_4 of the manifold is equal to the canonical volume N_4^0 . The cosmological coupling constant κ_4 is fine tuned so that the volume of the system performs a kind of random walk with average $\bar{N}_4 = N_4^0$. For practical purposes in computer simulations one has to constrain the walk. It is useful to introduce an additional volume dependent term [24] of the form $\delta|N_4^0 - N_4|^\alpha$ to the action. The term does not modify the canonical ensemble but controls the magnitude of the volume fluctuations and suppresses the effect of small miss-tunings of κ_4 . δ should be chosen large enough to reduce the number of moves necessary to wait for a measurement until $N_4 = N_4^0$ after a sweep. On the other hand δ should not be too large because this could kill the mobility of the algorithm. In our simulations we used $\delta = 0.05$, $\alpha = 1$.

Another non-conventional problem related to the grand canonical simulations is that during the run one has either to create or delete matter degrees of freedom. One should do this in a way to optimize the acceptance rate, which can be fairly low, if one assigns new degrees of freedom randomly. We propose to proceed as follows : The first step is to accept or reject a move with the Metropolis probability which instead of the usual ratio of exponents of the action contains the ratio of integrals over the space of new fields, which can be produced in the move. The probability depends on the type of the move and on the state of the field variables in the vicinity of the place where the move is performed. Once the geometrical change is accepted, the heat bath algorithm is used to assign the new matter fields. This scheme has much larger acceptance rate than the Metropolis one. For a more elaborated discussion on this point, the details of the algorithm and the description of an automatic tuning procedure, which we used to prepare the whole set of parallel jobs, we refer to [12].

We performed high precision numerical experiments for the model on a parallel machine with the MIMD architecture. This is well suited to cover the phase diagram in the two-dimensional coupling space. First, to have a rough idea how the phase diagram looks like, we submitted on each node a job with different values of (β, κ_2) . From this information we could choose the regions of interest and then submit there massive runs. More precisely, we organized the runs along the β or κ_2 directions. In the average for each point (β, κ_2) we performed 2000 measurements, which were taken every 10-th sweep. By a sweep we mean a number of *accepted* moves equal to the volume of the system. We

kept the balance between the numbers of accepted moves of each type. In some regions of the phase diagram some moves had very low acceptance rate, so to balance them with other moves we had to attempt to perform them more often. Altogether the acceptance rate was very low. On the average it was of the order 15%. This means roughly speaking that one of our sweeps is equal to 6.5 sweeps defined in [13].

Results

Formulas (10) provide information about the shape of the phase diagram of the gravity sector in the (β, κ_2) plane. One expects that the critical line between the cold and hot phases of gravity goes along $\kappa_{2,EH} - 3/2 \log 2 \approx 0.0$ for small β , comes through a kind of a crossover region and ends up at the line going along $\kappa_{2,EH} - 1/2 \log 2 = 0.7$ for large betas. By $\kappa_{2,EH}$ we denoted a critical coupling for pure gravity, which is known to be around 1.0. To find the exact shape of the critical line we traced a position of the maximum of the curvature susceptibility

$$\chi_{gg} = \frac{\langle N_2^2 \rangle - \langle N_2 \rangle^2}{N_4} \quad (11)$$

as a function of κ_2 for fixed β (cf. e.g. [9]). In the figure 1 we show χ_{gg} vs. κ_2 for $\beta = 0$ and $\beta = 0.16$. Within the error bars both curves have the same shape and are relatively shifted by approximately $\log 2$. This means that already for $\beta = 0.16$ the model is well described by the large beta limit. Indeed, for β larger than 0.16 we did not observe any further shift of χ_{gg} .

To investigate the behaviour of the gauge sector we measured the Wilson action $\langle S_W \rangle / N_4$ and the heat capacity :

$$\chi_{\sigma\sigma} = \frac{\langle S_W^2 \rangle - \langle S_W \rangle^2}{N_4} \quad (12)$$

measured as a function of β for fixed κ_2 . We studied the volume dependence of the heat capacity peak reflected in the ratio :

$$r_\sigma = \frac{\chi_{\sigma\sigma}(8000)}{\chi_{\sigma\sigma}(4000)} \quad (13)$$

It is known, that the deeper in the hot phase the larger is the Hausdorff dimension of the typical quantum geometry. Therefore starting from a certain value of κ_2 , the Z_2 sector is expected to undergo a first order phase transition and as a consequence $r = 2$. To check this scenario we ran some simulations for $\kappa_2 = -1.0$. In the range of β between 0.029 to 0.031 we observed jumps in the time history of $\langle S_W \rangle / N_4$. This can be interpreted as a signature of the first order phase transition. A typical run history is shown in figure 2. The time series is obtained from a 70000 sweeps. For the $N_4 = 8000$ lattice it was much more difficult to observe jumps in the time history, because the sweep takes longer and the flip flops seem to be more suppressed. In the run performed on 16 processors in parallel, with around 5000 sweeps on each, we observed 3 jumps. This again is a clear confirmation that deep in the "hot" phase the gauge system undergoes a first order phase transition.

A first order phase transition means that in the transition region we have typically two very different configurations of the matter sector. The jump between the two matter configurations affects very strongly the effective couplings of the gravity sector. The geometry tries to adjust to the changed couplings and in effect we observe a relaxation process which increases very much the autocorrelation time. Sometimes, a too large change of the effective couplings resulted in mistunings of the parameters and blowups or collapses of the universe. This happens more frequently for large negative values of κ_2 because there the dimensionality of the lattice is larger and hence the transition sharper. The increase of the autocorrelation induced by the above mentioned effect of geometry relaxations is further enhanced in this parameter region by the following mechanism. Deep in the "hot" phase the lattices have large connectivity and therefore it is difficult to find a (sub)simplex with a small order. Thus it becomes very likely that the elementary moves performed by the algorithm, which rely on adding a new subsimplex with a low order, are undone in the next step. In effect the geometry of the manifold changes very little and the system becomes trapped in the vicinity of metastable points.

Deep in the "cold" phase, the simplicial manifolds have the elongated structure with the Hausdorff dimension two or less. Therefore, no phase transition for the Z_2 system is expected there. To check this we performed measurements for $\kappa_2 = 1.6$, for two volume sizes. The heat capacity has a maximum, but the height of the peak does not grow with

the volume, in fact $r_\sigma < 1$. This can be interpreted as a reflection of a finite crossover effect. There is no phase transition there.

The most interesting region is the crossover region for κ_2 between 0.0 and 0.7. As we explained before in this region we expect a drastic change of the order of the matter sector phase transition, due to interactions between the matter and gravity sectors. The transition becomes weaker lowering the instabilities in κ_2 which finally disappear in this region. A typical behaviour of the heat capacity $\chi_{\sigma\sigma}$ for $\kappa_2 = 0.4$ and for system sizes 4000 and 8000 is shown on the figure 3. The height of the peak grows slowly with the volume indicating a weak second order phase transition.

In the crossover region we also observe a transition in the gravity sector when changing β for fixed κ_2 . In figure 4 we display the behavior of the susceptibility χ_{gg} , for $\kappa_2 = 0.4$.

The phase diagram is summarized in figure 5. The filled triangles represent the position of the maximum of the susceptibility χ_{gg} . The solid line connecting these points is the border between the "cold" and "hot" phases of gravity. For small and large β it approaches asymptotically the vertical lines representing the limiting values (10). The positions of the peaks in the heat capacity $\chi_{\sigma\sigma}$ are represented by circles. To indicate transitions we connected the points by the dashed line. This line separates the two phases of the Z_2 system in the "hot" phase of gravity. This line is continued as a dotted line in the cold phase, where it goes over the positions of the maxima of $\chi_{\sigma\sigma}$ which indicate a kind of non-critical crossover in the Z_2 gauge sector.

Asymptotically, for large and small β the only effect of the gauge matter is to renormalize the bare parameters of the pure gravity (10). Due to the direct coupling between the plaquettes $\sigma\sigma\sigma$ and local geometry $\sigma(t)$, different behavior can be expected in the cross-over region. Let us assume for the moment that the description in terms of the effective pure Einstein-Hilbert action is valid also in the intermediate region of β . If the assumption is right for arbitrary value of β we should be able to define the effective coupling $\kappa_{2,eff}$:

$$\kappa_{2,eff} = \kappa_2 + f(\beta). \quad (14)$$

We now define a function $f_1(\kappa_2, \beta)$ by demanding that the average curvature obtained in the pure Hilbert-Einstein model with the effective coupling $\kappa_{2,eff}$ is the same as

the average curvature in the full model parametrized by κ_2 and β . This permits us to determine the function $f_1(\kappa_2, \beta)$ numerically. In general this function will depend on N_4 for which we perform our analysis. If there is complete factorization of the matter and the gravity sector, f_1 should be a function of β only.

The function $f_1(\kappa_2, \beta)$ is plotted in figure 6 for fixed β along the κ_2 direction. For the large and small values of β the function is, within errors, independent of κ_2 . For $\beta = 0.04$ one can see a clear signal in the crossover region indicating appearance of new interactions in the effective theory. Outside the crossover region the curve flattens and approaches a constant.

The next step is to check how good the effective description in terms of the effective pure gravity action is. Let us take any operator O measured for the pure gravity and gravity interacting with the Z_2 gauge system and measure its averages in both cases. Define

$$\frac{\Delta O}{O} = \frac{O(\beta, \kappa_2) - O_{EH}(\kappa_2 + f_1(\beta, \kappa_2))}{O(\beta, \kappa_2)} \quad (15)$$

where $O()$ and $O()_{EH}$ are the averages of the operator O in the Z_2 + gravity system and in pure gravity, respectively. By definition $\Delta R/R = 0$. Among different operators we used the average geodesic distance d between $4d$ simplices, and the fluctuation of the curvature: $R_f^2 = \int \sqrt{g} R^2 - (\int \sqrt{g} R)^2$. We show them in the figure 7. In the crossover region we observe large differences between the measurements and values obtained from the effective action description. This difference is approximately 30 percent for the average distance between points on the lattice, d , and 10 percent for R_f^2 and disappears outside the transition region.

In principle this way we can identify other operators in the effective action. A complete identification is not possible because of the large number of terms which can be potentially included in the effective action. Moreover to identify relevant operators one should perform for them the volume analysis. We have tried several observables and found for example, that the behavior of the coupling between plaquettes and gravity $\langle \sigma\sigma\sigma \rangle - \langle \sigma \rangle \langle \sigma\sigma \rangle$ along a fixed $\beta = 0.04$ line where the interactions between the matter and geometry are strongest, has a similar shape and a maximum at the same point where R_f^2 (fig. 8). Therefore, one can speculate that the effective coupling between the

two sectors involves higher derivative terms, although to check which are the relevant operators would require much bigger computational effort.

For three volumes ($N_4 = 4000, 6000, 8000$) we localized the point, where both sectors are critical, or more precisely, where χ_{gg} (eq. 11) and $\chi_{\sigma\sigma}$ (eq. 12) have simultaneously maxima. We localized this point at $\kappa_2 = 0.40(2)$ and $\beta = 0.0395(5)$ for $N_4 = 4000$, $\beta = 0.038(1)$ for $N_4 = 8000$. The critical exponent γ_g defined by $\chi_g \approx N_4^{\gamma_g}$ is equal $\gamma_g = 0.16(4)$ which agrees within error bars with the value $\gamma_g = 0.14(4)$ we obtained from simulations for $\beta = 0.0$ corresponding to pure gravity. Our value of γ_g is smaller than 0.257(7) found in [13], where also smaller lattice sizes, starting with $N_4 = 500$, were included.

We tried to estimate the size dependence of the mean curvature R_{latt} at a critical point. As mentioned in the introduction, one would like this quantity to vanish for $N_4 \rightarrow \infty$. If it approaches a non-zero constant the average curvature in physical units $R_{cont} = R_{latt} a^{-2}$ blows up when the UV cut off, a , approaches zero. In fact one naively expects that at large volumes (= large numbers of $4d$ simplices N_4) the lattice curvature scales like $R_{latt} \approx (N_4)^{-1/2}$ and as a consequence the continuum curvature depends only on the physical volume $V_4 = N_4 a^4$, $R_{cont} \approx (V_4)^{-1/2}$, being independent of the cut off.

Comparing the curvature at this point we found that $R \propto N_4^{-0.09 \pm 0.04}$. It is clear that this exponent is far from $-1/2$ needed to make the above naive scaling work. It cannot be ruled out that the volume range studied is too small to reach a real asymptotics. We are studying this problem currently.

Discussion

In the paper we determined the phase diagram of the model of the Z_2 gauge field interacting with gravity. In the "hot" phase of gravity the gauge system undergoes a first order phase transition. It weakens when approaching the critical region where it is second order. In the cold phase there is no transition in the matter sector. We localized the critical point where both the gravity and gauge sectors are critical. The volume scaling of the gravity susceptibility is governed by the exponent γ_g which has the value 0.16(4) which seems to coincide numerically with the one for pure gravity model 0.14(4)

suggesting that the critical behaviour of the Z_2 + gravity model describes the theory which belongs to the same universality class as pure simplicial gravity.

This paper is a first step towards more realistic cosmological models of quantum gravity interacting with the gauge fields, like for example the $SU(2)$ or $SU(3)$ gauge field.

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Figure captions

- Fig. 1 The curvature susceptibility χ_{gg} for $\beta = 0.0(\circ)$ and $\beta = 0.16(\Delta)$, $N_4 = 4000$.
- Fig. 2 The run history for $\langle S_W \rangle$ at $\kappa_2 = -1$, $\beta = 0.030$, $N_4 = 4000$.
- Fig. 3 The heat-capacity $\chi_{\sigma\sigma}$ for $\kappa_2 = 0.4$.
- Fig. 4 The susceptibility χ_{gg} for $\kappa_2 = 0.4$. $N_4 = 4000(\circ)$, $N_4 = 8000(\Delta)$.
- Fig. 5 The phase diagram. The solid line separates the hot (crumpled) phase from the cold (elongated) phase of the gravity sector. In the measurements the peaks of the susceptibility $\chi_{\sigma\sigma}$ were localized at (Δ) . The $(\circ)$ represent the position of the peaks in the heat capacity $\chi_{\sigma\sigma}$. ($N_4 = 4000$). In the crumpled phase of the gravity sector we observed transitions in the spin system (depicted by the dashed line) while in the elongated phase scaling predicts no transitions of the spin system in this region. The dotted line connects the peaks of $\chi_{\sigma\sigma}$ in finite volumes, here $N_4 = 4000$.
- Fig. 6 The function $f_1(\beta, \kappa_2)$ at $\beta = -0.02(\circ)$, $0.04(\Delta)$, $0.2(+)$. On the same picture, the function $f_1(\beta, \kappa_2)$ for $N_4 = 8000$, $\beta = 0.04(\times)$ is shown.
- Fig. 7 The function $\Delta d/d(\circ)$ and $\Delta R_f^2/R_f^2(\Delta)$, where $R_f^2 = \int \sqrt{g} R^2 - (\int \sqrt{g} R)^2$, for $\kappa_2 = 0.4$, $N_4 = 4000$.
- Fig. 8 The correlation function $\langle \sigma\sigma\sigma\sigma \rangle - \langle \sigma \rangle \langle \sigma\sigma\sigma \rangle$ for $\beta = 0.02(\circ)$, $0.04(\Delta)$, $0.16(\times)$. The small diagram displays the behavior of the integrated curvature fluctuation $R_f^2 = \int \sqrt{g} R^2 - (\int \sqrt{g} R)^2$ for $\beta = 0.04$ along κ_2 .

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

$$f_1 = \kappa_2 - \kappa_2^{eff}$$

Figure 6

Figure 7

Figure 8