**DESIGN OF A RF QUADRUPOLE RESONATOR FOR LANDAU DAMPING IN HL-LHC**

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**Abstract**

The design and optimization of a quadrupole resonator for transverse Landau damping in the High Luminosity Large Hadron Collider (HL-LHC) is presented. Two different cavity types are considered whose shape is determined by the quadrupolar strength, surface peak fields, and beam coupling impedance. The lower order and higher order mode spectra of the optimized cavities are investigated and different approaches for their damping are proposed. Furthermore, the required RF power and optimal external quality factor for the input coupler are derived.

**INTRODUCTION**

The betatron frequency spread in circular accelerators yield a natural effect of suppressing transverse collective instabilities, the so-called Landau damping [1]. The incoherent frequency or tune spread is caused by non-linearities in the machine. To ensure this mechanism dedicated non-linear elements, typically octupole magnets, also known as Landau Octupoles (LO) are installed in the accelerator [2]. However, adiabatic damping and increased beam rigidity reduce their efficiency at higher energies. Future accelerators may call for more efficient devices in order to satisfy the beam requirements at higher energies.

Recently in [3], a superconducting RF quadrupole resonator was proposed as an alternative to the LOs. Its performance is affected likewise by beam rigidity but not by the adiabatic damping since in contrast to magnets, a RF quadrupole resonator introduces a longitudinal instead of a transverse betatron tune spread. The stabilization mechanism of a quadrupole resonator has been proven experimentally by using $Q''$ which likewise introduces a longitudinal betatron tune spread [4].

The variation of the betatron frequency due to quadrupolar focusing (so-called detuning) is proportional to the integrated quadrupolar strength $b_2$ which again is correlated to the transverse kick $\Delta p_i$ that a particle $i$ experiences while traversing the cavity (Fig. 1).

In [3], it has been demonstrated that an RF quadrupole resonator can result in a significantly more compact solution than a comparable set of octupole magnets providing the same tune spread. In this paper we present the first detailed design and optimization studies of a RF quadrupole resonator for Landau damping based on two types of cavities: the elliptical and the so-called four-vane cavity whose cross sections are shown in Fig. 1.

**CAVITY DESIGN PARAMETERS**

The parameters used to optimize the SC cavity can be differentiated into two categories. First, parameters that are derived from eigenmode simulations such as $b_2$ and second, parameters derived from wakefield simulations such as the transverse and longitudinal impedance. The latter is considerably more time-consuming than eigenmode calculations, hence, it is desirable to minimize the number of wakefield simulations. We propose, first, to optimize the designs independently of the impedance but with varying aperture which has the main influence on the impedance and, second, to calculate the impedance as a subsequent step for each pre-optimized design.

### Integrated Quadrupolar Strength $b_2$

The integrated quadrupolar strength $b_2$ [5] should be as high as possible to minimize the total number of cavities. The values shown in Table 1 provide an equivalent stabilization effect as the LOs in HL-LHC at a specific chromaticity [6]. They are obtained from macro particle tracking simulation comprising $6 \times 10^5$ turns. Though only for a specific case of instability, we use these results as a reference for the cavity design in this paper.

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Projects/Facilities

New proposals/concepts
Table 1: Required $b_2$ Yielding the Same Stabilization as Provided by LOs for HL-LHC at a chromaticity $Q_{x,y}^{\prime}=10$[6]

<table>
<thead>
<tr>
<th>Frequency [MHz]</th>
<th>$b_2$ [Tm/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.35</td>
</tr>
<tr>
<td>800</td>
<td>0.27</td>
</tr>
<tr>
<td>1200</td>
<td>0.40</td>
</tr>
</tbody>
</table>

**Frequency**

The maximum coherent tune shift the quadrupole resonator is able to stabilize by Landau damping depends on the frequency as the curvature of the cosine wave along the bunch introduce the non-linearity. The frequency is limited to harmonics of the main RF system (e.g. for LHC 400, 800 or 1200 MHz). According to the results shown in Table 1, a frequency of 800 MHz is favored which is consistent with [3].

**Field Limitation**

The strength is limited by either the peak magnetic or electric surface field. Therefore, it is reasonable to maximize the quantities $b_2/B_{pk}$ and $b_2/E_{pk}$ while a ratio $B_{pk}/E_{pk} = 2.13 \text{ mT}/(\text{MV/m})$ represents a good balance between both, the electric and magnetic surface peak fields with respect to the current technical limitations [7].

**Impedance**

The limited impedance budget guaranteeing a stable beam operation in the accelerator asks for a minimized narrow and broadband impedance. We use the definition from [8] to quantify the effective longitudinal and transverse impedance of the quadrupole resonator denoted as $(Z_{/\parallel}/n)_{eff}$ and $(Z_{/\perp})_{eff}$. All impedance calculations assume a bunch length with RMS sigma of $\sigma_z = 80$ mm. The total effective impedance of the required quadrupole resonators for LHC should be far below the limits shown in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(Z_{/\parallel}/n)_{eff}$</td>
<td>93 mΩ</td>
</tr>
<tr>
<td>$(Z_{/\perp})_{eff}$</td>
<td>20 MΩ</td>
</tr>
</tbody>
</table>

**Mechanical Restrictions**

Finally, a few mechanical limitations should be mentioned:

- The minimal aperture is 20 mm.
- Due to the bending process of niobium sheet, the minimal curvature radius is 10 mm.
- The maximum length of the entire system including the cryomodule(s) is 10 m (LHC layout constraint).

**ELLIPICAL CAVITY**

The standard non-reentrant elliptic cavity provides a good reference for other cavity types which are typically more complicated to fabricate [Fig. 2(a)]. In the frame of the optimizations, we observed a correlation between the optimal transition angle $\alpha$ (common tangential angle of both ellipses and the straight part) and $b_2$ that motivated the extension of the analyses towards reentrant profiles [Fig. 2(b)]. Depending on the iris radius $r_{ir}$, either a non-reentrant or a reentrant elliptical cavity provides the higher quadrupolar kick.

Figure 2: Profile of elliptical half cells with equator radius $r_{eq}$, iris radius $r_{ir}$ and length $l_{hc}$. The dotted lines subscribe the corresponding ellipses as well as the connecting straight with the concerted tangential angle $\alpha$. (a) Non-reentrant cavity ($\alpha < 90$ deg) and (b) reentrant cavity ($\alpha > 90$ deg).

It is obvious to choose either the TM$_{210}$ or the TE$_{211}$ mode, both, resulting in the quadrupolar field distributions as qualitatively shown in Fig. 1. However, the latter achieves approximately 40% less quadrupolar strength for the same amount of stored energy in the cavity due to its longitudinal dependency. Therefore we focus our considerations on the TM$_{210}$ (Fig. 3).

Figure 3: The fields distribution of the TM$_{210}$ mode.

**Optimization**

The prime objective that defines the “best” cavity shape is given by the maximum possible quadrupolar strength $b_2$ with respect to the limiting surface peak fields (either $B_{pk}$ or $E_{pk}$). Throughout all considered designs provided a limitation by the magnetic field ($B_{pk}/E_{pk} > 2.13$). A further major constraint is given by the frequency, typically adjusted via the equator radius $r_{eq}$. Likewise, the iris radius
is not a free parameter for the optimization because the quadrupolar strength $b_2$ monotonously increase with decreasing cavity aperture. Instead, we optimize several designs with varying iris radius and subsequently investigate their impedance. The remaining free parameter shown in Fig. 2 have been optimized using a constrained BFGS algorithm\textsuperscript{1} together with SLANS2\textsuperscript{9} as 2D eigenmode solver. The implementation is largely parallel allowing a very efficient optimization of even more than five parameters.

The optimizations was carried out with varying iris radii $r_{ir}$ between 25 – 60 mm for the reentrant cavities and between 50 – 175 mm for the non-reentrant cavities. The transition between both cavity types appears at $r_{ir}$ = 60 mm (Fig. 4). The properties of the designs are summarized in Figs. 5 and 6. Reentrant elliptical cavities provide a maximum of $b_2 = 0.16\, \text{Tm}/\text{m}$ for the smallest apertures while their counterpart do not achieves more than $b_2 = 0.12\, \text{Tm}/\text{m}$. With increasing aperture the number of required cavities to provide the same betatron tune spread increases monotonically, and thereby the length and impedance of the whole system. Due to a limited space of 10 m length for HL-LHC and due to economic reasons, the iris radius of the elliptical cavity must not be greater than $r_{ir}$ = 125 mm.

\textsuperscript{1}Broyden Fletcher Goldfarb Shanno algorithm, a group of hill-climbing optimization techniques.

Figure 4: Optimal tangential angle versus the iris radius $r_{ir}$.

A system of non-reentrant 800MHz elliptical cavities that satisfy the requirements of Table 1 provides an effective longitudinal impedance of at least 4mΩ, almost 5% of the entire LHC impedance budget (Fig. 6). Reentrant cavities provide more than 8% of the budget. The effective transverse impedance [Fig. 6(b)] is not of concern since the results are throughout marginal against the given impedance value in Table 2.

**LOMs and HOMs**

The potentially harmful lower and higher order modes (LOMs and HOMs) are listed in Table 3 for an optimized cavity with $r_{ir}$ = 90 mm. Taking advantage of the field distribution of the quadrupole mode, it is reasonable to extract their power by placing couplers around the equator where the fundamental mode vanishes (Fig. 7). The ports can have a diameter up to 100 mm without notably modifying the field distribution or enhancing the surface field.

Figure 5: (a) Quadrupolar strength of the optimized cavities distinguished by their aperture $r_{ir}$ assuming a magnetic peak field at the surface of $B_{pk} =100\, \text{mT}$. (b) The corresponding ratio of the surface peak fields.

Table 3: Trapped Lower and Higher Order Modes for a 800 MHz Non-reentrant Elliptical Cavity with $r_{ir}$ = 90 mm

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f$ [MHz]</th>
<th>Mode</th>
<th>$f$ [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM$_{010}$</td>
<td>417.7</td>
<td>TM$_{110}$</td>
<td>594.5</td>
</tr>
<tr>
<td>TM$_{020}$</td>
<td>896.2</td>
<td>TE$_{111}$</td>
<td>778.2</td>
</tr>
<tr>
<td>TE$_{011}$</td>
<td>1107.5</td>
<td>TE$_{121}$</td>
<td>1281.0</td>
</tr>
<tr>
<td>TM$_{011}$</td>
<td>1137.9</td>
<td>TM$_{131}$</td>
<td>1586.7</td>
</tr>
<tr>
<td>TE$_{021}$</td>
<td>1430.9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 7: Electric and magnetic field distribution of the quadrupole mode in the transverse plane. The cavity is equipped with ports at the low field regions.

These ports serve also for detuning the other polarization of the TM210 mode.

**FOUR-VANE CAVITY**

The disadvantages of the previously discussed elliptical cavities can be summarized as follows:

- The maximum achievable $b_2$ is relatively low.
- The large cavity size ($r_{eq} \sim 300$ mm).
- The limitation by the B-field may cause quenching.
- The high effective longitudinal impedance.

All these drawbacks can be resolved by a cavity design whose contour in the transverse plane forms a quadrupole-like electric field [Fig. 1(b)]. The actual cavity is obtained by a smooth transition from the quadrupolar contour to the beam pipe (Figs. 8 and 9). As for the elliptical cavity, we define the transverse contour by two conjugated arcs connected with a straight line. The transition from the quadrupolar contour to the circular beam pipe is defined by the tapering angle $\alpha_{tap}$.

![Four-vane cavity design](image)

**Optimization**

The optimization of the four-vane shape relies on 3D eigenmode solvers using HFSS or CST MWS. A parametric model has been developed for both codes with the difficulty of ensuring a correct construction for any reasonable ensemble of design parameters. The optimization was carried out in a brute force manner with subsequent interpolations applied on the results of numerous parameter sweeps.

The properties of the optimized designs are summarized in Fig. 10 for various apertures (iris radius $r_{ir}$) and tapering angles $\alpha_{tap}$. Depending on its aperture a four-vane cavity is able to provide between two to five times more quadrupolar strength than a non-reentrant elliptical cavity with an acceptable effective longitudinal impedance.

![Graphs of optimization results](image)
iris radius of \( r_{ir} = 60 \text{ mm} \). This exceeds clearly the performance gain obtained by reentrant elliptical cavities.

The ratio of the surface peak fields was throughout for all designs smaller than \( 2.13 \text{ mT/(MV/m)} \) indicating a limitation by the electric field.

The effective longitudinal impedance of an entire system of four-vane cavities to satisfy the requirements in Table 1 is by a factor between two to four smaller than an adequate system of elliptical cavities provides. The effective transverse impedance is moderately larger but it is minor with respect to the LHC impedance budget.

**LOMs and HOMs**

Due to the large apertures at the ends of the cavity, only a few modes are trapped inside the cavity: a TE\textsubscript{111}-like mode with a frequency between 700 and 750 MHz and a TM\textsubscript{111}-like mode at around 1 GHz. The transverse wake spectrum is largely dominated by only the first dipole mode which can be easily extracted if the beam pipe radius becomes slightly larger by 20-30 mm. Based on this modification it is reasonable to apply LOM and HOM damping outside of the cryomodule using lossy materials in the beam pipe rather than couplers close to the cavity.

**Multipole Field errors**

These errors mainly arise from higher order multipoles \( b_m \) which satisfy the azimuthal periodicity of the fundamental multipole \( b_0 \) (we have \( n = 2 \)). They are correlated via the condition \( m/n = 3, 5, 7, \ldots \) [10]. Table 4 shows the corresponding values of one optimized cavity.

Table 4: Different multipoles of the four-vane cavity with \( r_{ir} = 30 \text{ mm} \) and \( \alpha_{tap} = 30 \text{ deg} \)

<table>
<thead>
<tr>
<th>Multipole</th>
<th>Unit</th>
<th>Value for ( E_{pk} = 50 \text{ MV/m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_2 )</td>
<td>[Tm/m]</td>
<td>( 3.47 \times 10^{-1} )</td>
</tr>
<tr>
<td>( b_6 )</td>
<td>[Tm/5/m]</td>
<td>( 9.62 \times 10^{4} )</td>
</tr>
<tr>
<td>( b_{10} )</td>
<td>[Tm/7/m]</td>
<td>( 1.98 \times 10^{10} )</td>
</tr>
</tbody>
</table>

The elliptical cavity do not provide any higher order multipoles due to the exact sinusoidal azimuthal dependency of the field components. However, there may appear so-called random multipole errors due to fabrication errors.

**RF-Power**

The optimal coupling for the fundamental mode coupler is derived from the input power required to compensate the beam loading assuming an off-centered beam. Fig. 11 shows the required input power for an optimized four-vane cavity as a function of the external quality factor for different transverse positions of the bunch. The needed input power is very moderate and by almost one order of magnitude smaller than required for the HL-LHC crab cavities [11], even for a bunch position as much as few mm from the cavity axis. This is only true if \( Q_{ext} > 1 \times 10^6 \) can be used in operating the cavity.

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**REFERENCES**


