ONE-LOOP CORRECTIONS TO LEPTON-PAIR PRODUCTION BY TRANSVERSELY POLARIZED HADRONS⁽⁺⁾

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Abstract: A complete calculation of QCD higher order corrections (HOC) at one loop for Drell-Yan lepton pair production by transversely polarized hadrons is presented. Production via both γ and Z is taken into account. For $p_{\uparrow}p_{\uparrow} \rightarrow \ell^{-}\ell^{+} + X$, K-factors well exceeding unity are found and, in particular near the Z peak, significant rates may be anticipated. The dominance of HOC by soft, collinear and virtual gluons is also studied.

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Chirality-odd structure functions and the related transversity distributions $\Delta_T F_{q/p}(x, Q^2)$ of quarks in a proton have recently attracted much attention. The process most appropriate to study them is Drell-Yan lepton pair ($\ell^- \ell^+$) production by transversely polarized nucleons [1–6]; and experiments at RHIC in polarized p - p collisions have been proposed [5,7].

The proposals [7] are based on the Born term of the above process. Knowledge, however, of higher order QCD corrections (HOC), or K -factors, is essential; for, if HOC are large and of opposite sign to the Born term the predicted cross sections will be very small making experiments inconclusive.

This work presents a complete calculation of one-loop HOC. In view of RHIC experiments, with c.m. energy $\sqrt{s} = 100$ –500 GeV, we consider $\ell^- \ell^+$ production via both a virtual γ and Z .

To regulate the singularities present in one-loop calculations we work in $n = 4 - 2\epsilon$ dimensions. Dealing, however, with polarized particles and quarks implies the presence of the Dirac matrix γ_5 , and this requires the choice of a proper scheme. We use dimensional reduction, i.e. calculate traces in 4 dimensions, but carry integrations in $n \neq 4$ [8]. The satisfaction of the Ward identities, which often poses a problem, is discussed later. We work in the Feynman gauge and use the universal definition of corrections.

Consider, in general, the subprocess:

$$q(p_1, s_1) + \bar{q}(p_2, s_2) \rightarrow V^* + g \rightarrow \ell^-(p_3) + \ell^+(p_4) + g \quad (1)$$

where $V = \gamma$ or Z , s_1, s_2 transverse spin 4-vectors and $p_i, i = 1, \dots, 4$, 4-momenta. In the c.m. of q and $\bar{q}$ let $\hat{\theta}_3, \phi_3$ define the direction of $\vec{p}_3$. With $M^2 \equiv (p_3 + p_4)^2$ and $d\hat{\Omega}_3 = \sin \hat{\theta}_3 d\hat{\theta}_3 d\phi_3$ we shall first determine the differential cross section

$$\Delta_T \frac{d\hat{\sigma}}{dM^2 d\hat{\Omega}_3} \equiv \frac{1}{2} \left\{ \frac{d\hat{\sigma}(s_1, s_2)}{dM^2 d\hat{\Omega}_3} - \frac{d\hat{\sigma}(s_1, -s_2)}{dM^2 d\hat{\Omega}_3} \right\} \quad (2)$$

The coupling of Z to the fermion $f(= \ell, q)$ is taken $-ie\gamma_\mu(\alpha_f - \beta_f\gamma_5)$ where the constants α_f, β_f are well-known in the standard model. With Γ_Z the width of Z it is convenient to introduce the factor

$$F_q(M) \equiv e_q^2 + (\alpha_\ell^2 + \beta_\ell^2)(\alpha_q^2 - \beta_q^2) \frac{M^4}{(M^2 - M_Z^2)^2 + (M^2 \Gamma_Z / M_Z)^2} \\ - 2e_q \alpha_\ell \alpha_q \frac{M^2(M^2 - M_Z^2)}{(M^2 - M_Z^2)^2 + (M^2 \Gamma_Z / M_Z)^2} \quad (3)$$

where e_q =quark charge; the last term arises from γ - Z interference.

With the subprocess variables:

$$\hat{s} = (p_1 + p_2)^2 \quad \hat{t} = (p_3 - p_1)^2 \quad \hat{u} = (p_3 - p_2)^2 \quad (4)$$

we introduce

$$T \equiv \{\hat{t}\hat{u}(s_1 \cdot s_2) + 2\hat{s}(p_3 \cdot s_1)(p_3 \cdot s_2)\} \frac{1}{M^4}; \quad (5)$$

In $n = 4 - 2\epsilon$ dimensions and denoting $\Delta_T d\hat{\sigma}/dM^2 d^{n-2}\hat{\Omega}_3 \equiv \hat{\sigma}^n$, the Born contribution is:

$$\hat{\sigma}_B^n = B(\epsilon)\delta(1 - \hat{\tau}) \quad B(\epsilon) \equiv \frac{\alpha^2 \mu^{2\epsilon}}{N_c} \left(\frac{4\pi\mu}{M}\right)^{2\epsilon} \frac{T}{\hat{s}^2} F_q(M) \quad (6)$$

where $\hat{\tau} \equiv M^2/\hat{s}$, $N_c = 3$ in color SU(3) and μ =regularization scale. In the c.m. of q and $\bar{q}$ with ϕ_1, ϕ_2 the azimuthal angles of $\vec{s}_1, \vec{s}_2$ we obtain [2],[6]:

$$T = \frac{1}{4} \sin^2 \hat{\theta}_3 \cos(2\phi_3 - \phi_1 - \phi_2) \quad (7)$$

The virtual HOC arise from a vertex graph and the quark self-energy graphs, all of the type $q\bar{q} \rightarrow V^* \rightarrow \ell^- \ell^+$. Introducing

$$K(\epsilon) \equiv \left(\frac{4\pi\mu^2}{M^2}\right)^\epsilon \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \quad (8)$$

we find that the vertex graph gives:

$$\hat{\sigma}_V^n = C_F \frac{\alpha_s}{2\pi} \left\{ K(\epsilon) \left(-\frac{2}{\epsilon^2} - \frac{4}{\epsilon} + \frac{2\pi^2}{3} - 9 \right) B(\epsilon) + K(\epsilon') \left(\frac{1}{\epsilon'} + 2 \right) B(\epsilon') \right\} \delta(1 - \hat{\tau}) \quad (9)$$

where $\epsilon < 0$ and $\epsilon' > 0$. The term $\sim 1/\epsilon^2$ arises from the combination of an infrared and a collinear singularity. The term $\sim 1/\epsilon'$ is an ultraviolet (UV) singularity.

Each self energy graph introduces an UV singularity, and, due to the masslessness of q , a collinear one. They contribute:

$$\hat{\sigma}_{se}^n = C_F \frac{\alpha_s}{2\pi} \left\{ K(\epsilon) \left(\frac{1}{\epsilon} + 2 \right) B(\epsilon) - K(\epsilon') \left(\frac{1}{\epsilon'} + 2 \right) B(\epsilon') \right\} \quad (10)$$

We see that, adding (9) and (10), the UV singular terms $\sim 1/\epsilon'$ cancel.

Now it is in order to briefly discuss the satisfaction of the well-known Ward identity between vertex and self-energy graphs, which is relevant to our problem. Straightforward application of dimensional reduction leads to an extra term from

the vertex graph, which, after taking traces, produces a finite contribution $\sim s_1 \cdot s_2$. This term, however, violates the Ward identity, and we have eliminated it by adding a proper counterterm. Such counterterms are discussed in [9].

We turn to the gluon Bremsstrahlung contributions, Eq. (1). Here we introduce the transverse split function [6]:

$$\Delta_T P_{qq}(z) = \frac{2}{(1-z)_+} - 2 + \frac{3}{2}\delta(1-z) \quad (11)$$

We also introduce:

$$R_{\pm} \equiv [1 + \hat{\tau} \pm (1 - \hat{\tau}) \cos \hat{\theta}_3]^{-1} \quad (12)$$

and

$$F(R) = R\{1 + \hat{\tau}^{-3} + 3R(1 + \hat{\tau}^{-2}) + 12R^2(1 + \hat{\tau}^{-1}) - 88R^3 - 24R^3 \ln(4\hat{\tau}R^2)\}, \quad (13)$$

so that for $\hat{\tau} = 1$: $F(R_{\pm}) = 0$. Then we find

$$\begin{aligned} \hat{\sigma}_{Br}^n = C_F \frac{\alpha_s}{2\pi} 4^{1-\epsilon} K(\epsilon) B(\epsilon) & \{4\hat{\tau}(R_+^{4-2\epsilon} + R_-^{4-2\epsilon})[(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon})\delta(1-\hat{\tau}) - \frac{1}{\epsilon}\Delta_T P_{qq}(\hat{\tau})] \\ & - 2(1-\hat{\tau})R_+R_- - \frac{1-\hat{\tau}}{\hat{\tau}}R_+^2R_-^2[(1+\hat{\tau})^4 - (1-\hat{\tau})^4 \cos^2 \hat{\theta}_3] \\ & + 16\hat{\tau}(R_+^{4-2\epsilon} + R_-^{4-2\epsilon})[(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}})_+ - \ln(1-\hat{\tau})] + \frac{1}{3}\frac{\hat{\tau}^2}{1-\hat{\tau}}(F(R_+) + F(R_-)) \\ & + \frac{1}{\sin^2 \hat{\theta}_3}(\frac{1-\hat{\tau}}{\hat{\tau}} + \frac{4}{1-\hat{\tau}}\frac{\ln(4\hat{\tau}R_+R_-)}{\sin^2 \hat{\theta}_3})\}; \end{aligned} \quad (14)$$

note that for $\hat{\tau} \rightarrow 1$: $4\hat{\tau}R_+R_- \rightarrow 1$.

The term $\sim 1/\epsilon^2$ cancels the corresponding singularity of (9). The term $\sim 3/2\epsilon$ is canceled by the corresponding singularities of (9) and (10), and the term $\sim \frac{1}{\epsilon}\Delta_T P_{qq}(\hat{\tau})$ is canceled by adding a factorization counterterm. Application of procedures, well-known from HOC calculations of unpolarized and longitudinally polarized processes, gives:

$$\hat{\sigma}_{ct}^n = \frac{1}{\epsilon} 16C_F \frac{\alpha_s}{2\pi} 4^{-\epsilon} K(\epsilon) B(\epsilon) \hat{\tau}(R_+^{4-2\epsilon} + R_-^{4-2\epsilon}) \Delta_T P_{qq}(\hat{\tau}) (\frac{\hat{s}}{M_f^2})^\epsilon \quad (15)$$

where M_f = factorization scale.

Adding (9), (10), (14) and (15) to the Born term we obtain:

$$\Delta_T \frac{d\hat{\sigma}}{dM^2 d\hat{\Omega}_3} = B(0) \{ [1 + \frac{\alpha_s}{2\pi} C_F (\frac{2\pi^2}{3} - 7)] \delta(1-\hat{\tau}) +$$

$$\begin{aligned}
& + \frac{\alpha_s}{2\pi} C_F \cdot 4 \left\{ -\frac{1-\hat{\tau}}{\hat{\tau}} R_+ R_- [2\hat{\tau} + R_+ R_- ((1+\hat{\tau})^4 - (1-\hat{\tau})^4 \cos^2 \hat{\theta}_3)] \right. \\
& + 16\hat{\tau} \left[\left(\frac{\ell n(1-\hat{\tau})}{1-\hat{\tau}} \right)_+ - \ell n(1-\hat{\tau}) \right] (R_+^4 + R_-^4) + \frac{1}{3} \frac{\hat{\tau}^2}{1-\hat{\tau}} (F(R_+) + F(R_-)) + \\
& \left. \frac{1}{\sin^2 \hat{\theta}_3} \left[\frac{1-\hat{\tau}}{\hat{\tau}} + \frac{4\ell n(4\hat{\tau} R_+ R_-)}{(1-\hat{\tau}) \sin^2 \hat{\theta}_3} \right] + 4\hat{\tau} \Delta_T P_{qq}(\hat{\tau}) (R_+^4 + R_-^4) \ell n(\hat{s}/M_f^2) \right\}. \quad (16)
\end{aligned}$$

Integrating over the angle $\hat{\theta}_3$ and defining

$$\tilde{B}(s) \equiv \frac{\alpha_s^2}{3N_c} \frac{\cos(2\phi_3 - \phi_1 - \phi_2)}{s^2} F_q(M) \quad (17)$$

we obtain the total contribution

$$\begin{aligned}
\Delta_T \frac{d\hat{\sigma}}{dM^2 d\phi_3} = \tilde{B}(\hat{s}) \{ & [1 + \frac{\alpha_s}{2\pi} C_F (\frac{2\pi^2}{3} - 7)] \delta(1-\hat{\tau}) + \\
& + \frac{\alpha_s}{2\pi} C_F \cdot 2 \left[\frac{1}{\hat{\tau}} \Delta_T P_{qq}(\hat{\tau}) \ell n \frac{\hat{s}}{M_f^2} + \frac{4}{\hat{\tau}} \left(\frac{\ell n(1-\hat{\tau})}{1-\hat{\tau}} \right)_+ - \frac{4}{\hat{\tau}} \ell n(1-\hat{\tau}) - 3 \frac{\ell n^2 \hat{\tau}}{1-\hat{\tau}} + 2 \frac{1-\hat{\tau}}{\hat{\tau}} \right] \} \quad (18)
\end{aligned}$$

Considering the physical process $A_\uparrow + B_\uparrow \rightarrow \ell^- \ell^+ + X$ let $\eta_3 = \ell n \cot \theta_3 / 2$ the pseudorapidity of ℓ^- in the c.m. of the hadrons A and B . Then the contribution of eq. (16) to the corresponding inclusive cross section is:

$$\begin{aligned}
\Delta_T \frac{d\sigma}{dM^2 d\eta_3 d\phi_3} = 4\tau \sum_q \int_{\tau}^1 \frac{dx_a}{x_a} \int_{\hat{\tau}_1}^1 \frac{d\hat{\tau}}{\hat{\tau}^2} \Delta_T F_{q/A}(x_a) \Delta_T F_{\bar{q}/B}(x_b) \frac{\Delta_T d\hat{\sigma}/dM^2 d\hat{\Omega}_3}{(x_a e^{-\eta_3} + x_b e^{\eta_3})^2} \\
+ q \leftrightarrow \bar{q} \quad (19)
\end{aligned}$$

where $\tau \equiv M^2/s$ and

$$\hat{\tau}_1 = \tau/x_a \quad x_b = \hat{\tau}_1/\hat{\tau} \quad (20)$$

similarly for the contribution of (18) to $\Delta_T d\sigma/dM^2 d\phi_3$.

For valence quarks in the proton, nonrelativistic quark models predict that the transversity distributions equal the corresponding helicity ones:

$$\Delta_T F_{q/p}(x, Q_0) = \Delta_L F_{q/p}(x, Q_0) \quad q = u_v, d_v; \quad (21)$$

this is also in fair agreement with the relativistic MIT bag model. We accept (21) and for $\Delta_L F_{q/p}(x, Q_0)$ we use the set 1 of [10] ($Q_0^2 = 4 \text{ GeV}^2$). Then the evolution with $Q^2 = M_f^2$ is determined via the split function (11).

The above would be sufficient to give the approximate magnitude of cross sections in $\bar{p}p$ collisions. However, experiments at RHIC involve pp collisions; then sea transversity distributions are required, and these are largely unknown. We use

$$\Delta_T F_{\bar{q}/p}(x, Q_0) = -0.24x^{0.1}(1-x)^{9.5} \quad \bar{q} = \bar{u}, \bar{d}, s, \bar{s} \quad (22)$$

which has the following properties: (i) It satisfies $|\Delta_T F_{\bar{q}/p}(x, Q)| < F_{\bar{q}/p}(x, Q)$ where $F_{\bar{q}/p}$ the unpolarized $\bar{q}$ distribution [11]; (ii) It is in fair accord with Regge requirements as $x \rightarrow 0$ and, more generally, it has a shape similar to $\Delta_L F_{\bar{q}/p}(x, Q_0)$ [10]; (iii) It has a shape similar to $F_{s/p}(x, Q_0)$ of [12]; (iii) It gives $\int_0^1 \Delta_T F_{\bar{q}/p}(x, Q_0) dx \simeq -0.015$ which is about the same as the EMC and SMC value of $\int_0^1 \Delta_L F_{s/p}(x, Q_0) dx$ [13]. It should be stressed that in particular the magnitude of (22) is fairly arbitrary, and RHIC experiments will provide useful information.

Subsequently for $\alpha_s = \alpha_s(\mu)$ we use $\Lambda = 0.2$ GeV, 4 flavours, and fix the scales at $\mu = M_f = M$; variation of μ, M_f away from this value is discussed below. In the expressions of α_f, β_f , Eq. (3), we use $\sin^2 \theta_W = 0.2325$. We present results at $\sqrt{s} = 100, 200$ and 500 GeV for $2\phi_3 - \phi_1 - \phi_2 = 0$ which maximizes T , Eq. (7).

Fig. 1(a) presents the K -factors $K_\eta = (\sigma_{\eta, \text{Born}} + \sigma_{\eta, \text{HOC}})/\sigma_{\eta, \text{Born}}$, where $\sigma_\eta \equiv \Delta_T d\sigma/dM^2 d\eta_3 d\phi_3$, for $\eta_3 = 0$ and 1 ; and Fig. 1(b) presents $K = (\sigma_{\text{Born}} + \sigma_{\text{HOC}})/\sigma_{\text{Born}}$, where $\sigma \equiv \Delta_T d\sigma/dM^2 d\phi_3$. The important point is that all K_η, K well exceed 1 , i.e. the HOC enhance the Born contribution. The local peaks and dips are due to the Z pole, to Z - γ interference and to the fact that the sum of Eq. (19) contains more than one quark flavor, with different values of e_q, α_q and β_q . For $\sqrt{s} = 100$ GeV the pole of Z lies outside the considered range of $\sqrt{\tau}$ and the K -factors are smooth. As $\sqrt{\tau} \rightarrow 1$ (kinematic endpoint) all K -factors show a typical increase.

Fig. 2 presents our inclusive cross sections. Fig. 2(a) shows σ for M near M_Z at $\sqrt{s} = 200$ and 500 GeV. Subsequently denote by $F_u(M)$ the function of Eq. (3) for the u -quark. Fig. 2(b) shows the quantity $-M^4 \sigma_\eta / F_u(M)$ versus $\sqrt{\tau}$ for $\eta_3 = 0$ and 1 ; by dividing with $F_u(M)$ we somewhat smoothen σ_η . The point to remark is that in much of the range of $\sqrt{\tau}$ (away from peaks and dips) the plotted quantity is smooth and almost scales. The dips are due to a change of sign caused by Z . With a luminosity $\mathcal{L} = 2 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$, for $\sqrt{s} = 500$ GeV our cross section at the Z peak corresponds to $\sim 250 \ell^- \ell^+$ events/GeV·rad·year.

The quantity $M^4 \sigma_\eta / F_u(M)$ also gives a measure of the effect of changing the scales μ, M_f away from $\mu = M_f = M$. This is obtained by considering the

variation with $\sqrt{s}$ of this quantity at fixed τ ; the reason is that, in the $\sqrt{\tau}$ range of smoothness, apart from $\alpha_s(\mu)$ and $\Delta_T F_{q/p}(x, M_f)$, $\Delta_T F_{\bar{q}/p}(x, M_f)$, the rest is a function of τ only. Fig. 2(b) shows that, in the range of smoothness, $M^4 \sigma_\eta / F_u(M)$ varies by factors 1.5~2 for $100 < \sqrt{s} < 500$ GeV.

Finally we ask to what extent HOC are dominated by soft, collinear and virtual gluons. Such gluons determine in (16) and (18) the terms $\sim \delta(1 - \hat{\tau}), (\ell n(1 - \hat{\tau}) / (1 - \hat{\tau}))_+$ and $1 / (1 - \hat{\tau})_+$. Denoting by σ_S the contribution of such terms to $\Delta_T d\sigma / dM^2 d\phi_3$ and by σ_H the contribution of the remaining terms ($\sigma_H = \sigma_{\text{HOC}} - \sigma_S$), Fig. 3(a) presents the ratio $L \equiv \sigma_H / \sigma_S$. Similarly for the corresponding contributions to $\Delta_T d\sigma / dM^2 d\eta_3 d\phi_3$. Fig. (3b) presents the ratio $L_\eta \equiv \sigma_{\eta, H} / \sigma_{\eta, S}$ for $\eta_3 = 0$ and 1. As for the K -factors, the local peaks and dips are due to Z . Now the point is that with increasing $\sqrt{\tau}$, on the average, all L, L_η decrease fast, in particular for large η_3 . Thus at sufficiently large $\sqrt{\tau}$, HOC are dominated by soft, collinear and virtual gluon contributions. This is in common with unpolarized or longitudinally polarized processes [14].

We remark that one-loop HOC to $\ell^- \ell^+$ production in transversely polarized pp collisions have also been studied in [15]. The work considers production by γ only and is restricted to low M ; thus the peaks and dips due to Z are absent. Also, [15] focuses mostly on different observables. Furthermore, unlike our work that employs popular dimensional methods, [15] uses a different regularization procedure (assigning a mass to the gluon). With production only by γ , at low M , our K -factor is not dissimilar to that corresponding to Fig. 5 of [15]. On the other hand, the structure of our HOC looks rather different. E.g. our final result, Eq. (18), contains a piece $\sim 2\pi^2/3$ typical of Drell-Yan from unpolarized or longitudinally polarized hadrons [16]; such a piece is absent in the final result of [15].

In conclusion, we have shown that one-loop HOC to $A_\uparrow + B_\uparrow \rightarrow \ell^- \ell^+ + X$ enhance the contribution of the Born term; and in particular near the Z peak fairly sizeable rates may be anticipated. Our work lends further support to proposed experiments at RHIC.

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FIGURE CAPTIONS

- Fig. 1. K-factors: (a) $K_\eta = (\sigma_{\eta,\text{Born}} + \sigma_{\eta,\text{HOC}})/\sigma_{\eta,\text{Born}}$, where $\sigma_\eta \equiv \Delta_T d\sigma/dM^2 d\eta_3 d\phi_3$ for lepton rapidities $\eta_3 = 0$ and 1. (b) $K = (\sigma_{\text{Born}} + \sigma_{\text{HOC}})/\sigma_{\text{Born}}$, where $\sigma \equiv \Delta_T d\sigma/dM^2 d\phi_3$.
- Fig. 2. Inclusive cross sections: (a) $\sigma \equiv \Delta_T d\sigma/dM^2 d\phi_3$ for M near M_Z . (b) The quantity $-M^4 \sigma_\eta / F_u(M)$, where $\sigma_\eta \equiv \Delta_T d\sigma/dM^2 d\eta_3 d\phi_3$ for $\eta_3 = 0$ and 1.
- Fig. 3. Dominance of soft, collinear and virtual gluons: (a) The ratio $L \equiv \sigma_H / \sigma_S$. (b) The ratio $L_\eta \equiv \sigma_{\eta,H} / \sigma_{\eta,S}$ for $\eta_3 = 0$ and 1.

Fig. 1

Fig. 2

Fig. 3