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The phase of coherent K_S regeneration and dispersion relations

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Abstract

The phase of nuclear regeneration of K_S mesons in a K_L beam is related to the momentum dependence of the regeneration amplitude $(f(0) - \bar{f}(0))/k$, via a dispersion relation. Using experimental data on the momentum dependence of the modulus of the amplitude, values for the regeneration phase are derived for the regenerator materials copper, lead, carbon and carborundum (B_4C). The precision of these phases is about $\pm 3^\circ$ in the momentum region from 40 to 150 GeV/c. This uncertainty limits the precision of measurements on the CP-violation phase ϕ_{+-} in regenerator experiments.

Introduction

The measurement of the phase differences ϕ_{+-} and ϕ_{00} between decay amplitudes of the CP-violating decays $K_L \rightarrow \pi^+\pi^-$ and $K_L \rightarrow \pi^0\pi^0$ and the corresponding CP-conserving decays, $K_S \rightarrow \pi^+\pi^-$ and $K_L \rightarrow \pi^0\pi^0$, is important for the elucidation of the mechanism of CP violation. This measurement [1] can be done by two different experimental methods: i) the study of the time dependent interference of pure strangeness states, or of an incoherent mixture of K^0 and $\bar{K}^0$, ii) the study of the time-dependent interference in the coherent state of K_L and K_S mesons produced by nuclear regeneration in a K_L beam.

In thirty years since the discovery of CP violation, initial experiments used the regeneration method, until the technique of slow extracted proton beams made experiments near to the proton target possible. This new technique gave the most precise results on ϕ_{+-} in 1974 [2]. Experiments using the regeneration method for the production of a coherent K_S state from a K_L beam measured, by interference, the phase difference between the CP violation phase ϕ_{+-} and the nuclear regeneration phase ϕ_f , $\phi_{+-} - \phi_f$. The phase ϕ_f was determined in an independent experiment by measuring the time-dependent charge asymmetry in K_{l3} decays behind a regenerator [3].

In recent years, again the interference near a proton target [4] and the regeneration technique [5] were taken up, and a new level of precision in the measurement of the phase difference $\phi_{+-} - \phi_f$ was achieved. At the same time, a theoretical argument is used to calculate the nuclear regeneration phase ϕ_f from the measured momentum dependence of the regeneration amplitude in a limited momentum region (from 40 to 150 GeV/c). The purpose of this note is to point out that this argument has theoretical uncertainties that are much larger than the 0.5° systematic error attributed to the regeneration phase by the authors in ref.[5].

Regeneration

The transmission regeneration of K_S mesons from a beam of K_L mesons incident on a slab of nuclear matter is a coherent process where the phase of the outgoing K_S wave is shifted by a fixed amount relative to the incident K_L wave. This coherence is due to the fact that for scattering in the forward direction at angles less than 10^{-7} radians, the recoil momentum in the forward scattering process is transmitted to the regenerator block as a whole and the resulting recoil energy is negligibly small [6,7].

Coherent regeneration on nuclei is used as a tool for preparing a K_S beam from a K_L beam far from the original target in which the K^0 and $\bar{K}^0$ mesons are produced. For the measurement of the phases ϕ_{+-} and ϕ_{00} , the time-dependent interference in the decays of a coherent mixture of K_L and K_S

mesons behind a regenerator block is measured. This time dependence for the $\pi^+\pi^-$ decay mode is given [1] by:

$$I(\pi^+\pi^-, t) \propto |\rho|^2 e^{-\Gamma_S t} + 2|\rho| |\eta| e^{-(\Gamma_S + \Gamma_L)t/2} \cdot \cos(\Delta m t - (\phi_{+-} - \phi_\rho)) + |\eta|^2 e^{-\Gamma_L t}$$

Here, ρ , the amplitude of coherent nuclear regeneration behind a block of length L , is described as

$$\rho = \pi i \frac{f(0) - \bar{f}(0)}{k} \Lambda_S N G(L) \quad (1)$$

and $\Lambda_S = \beta \gamma \tau_S$ is the K_S decay length, N the nuclear density, $G(L)$ a geometrical function depending on the length L of the regenerator, Γ_S and Γ_L the decay rates of K_S and K_L mesons, $\Delta m = m_L - m_S$ the mass difference of K_L and K_S mesons, k the wave number of the K_L mesons, and η the ratio of the decay amplitudes for $K_L \rightarrow \pi^+\pi^-$ and $K_S \rightarrow \pi^+\pi^-$.

The amplitude ρ contains the difference of the nuclear forward scattering amplitudes for $K^0 N$ and $\bar{K}^0 N$ scattering, $f(0) - \bar{f}(0)$, and geometrical factors. Correspondingly the phase of ρ can be separated into the nuclear part $\phi_f = \arg \{ i [f(0) - \bar{f}(0)] \}$ and the geometrical part ϕ_G :

$$\phi_\rho = \phi_f + \phi_G \quad (2)$$

The phase ϕ_G is known from the length of the regenerator, the mass difference $\Delta m = m_L - m_S$, and the K_S lifetime. The nuclear regeneration phase ϕ_f can be measured from the time-dependent charge asymmetry in semileptonic K_L decays behind a regenerator. This has been done with a precision of 2.6 degrees for carbon [3]. In addition, there is a theoretical argument [8,9] on the basis of the Regge model, connecting the momentum dependence of the modulus of $f(0) - \bar{f}(0)$ to the regeneration phase ϕ_f . The argument is valid if the scattering amplitudes are described by the Regge model, and if the difference of $f(0)$ and $\bar{f}(0)$ is given by only one Regge exchange diagram involving ω meson exchange. In this ideal case, the difference $f(0) - \bar{f}(0)$ would have a power-law momentum dependence

$$f(0) - \bar{f}(0) \propto p^{\alpha(0)-1} = p^\beta \quad (3)$$

where $\alpha(0)$ is the intercept of the Regge trajectory of the ω meson at zero momentum transfer. In this case, the phase would be

$$\phi_f = -\pi(1 + \beta)/2 \quad (4)$$

and the value of ϕ_f could be inferred from a measurement of the single exponent β .

Measurements of the modulus of the regeneration amplitude have been done for several nuclear materials [10-15]. The momentum dependences have been measured in the range from 40 GeV/c to 150 GeV/c, for some materials also below 40 GeV/c. Some exponents of the power law behaviour are listed in table 1. The exponents appear to be similar for several nuclear materials, but not equal. The exponent for lead differs from the one of B₄C by 0.058 ± 0.020 , but also the exponent for carbon differs from the one for B₄C.

Most important for the consideration of the regeneration phase is the observation of the Chicago-Fermilab-collaboration [11], that there is a clear break in the momentum dependence, the data above 20 GeV/c falling off more steeply. The difference in exponents is 0.17 ± 0.05 for lead and 0.20 ± 0.03 for copper. Such a momentum dependence is incompatible with Regge behaviour because a second Regge trajectory contributing at low momenta necessarily would have a steeper slope while a second trajectory with a less steep fall-off would be seen at high momenta above 150 GeV/c. Since this peculiar momentum dependence cannot be reconciled with an idealized single trajectory Regge behaviour, the regeneration phase cannot be obtained simply from the Regge model.

Dispersion relations and analyticity

There is, however, a more reliable theoretical argument relating the regeneration phase to the modulus of the regeneration amplitude: for analytic amplitudes, causality allows the derivation of dispersion relations between these quantities. If A is the modulus of an analytic amplitude and ϕ its phase and if A has no zeroes in the complex plane, then the dispersion relation yields

$$\phi(p_0) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{d \ln A}{du} \ln \coth \frac{|u|}{2} du \quad (5)$$

where $u = \ln(p/p_0)$ is the integration variable. In order to obtain the phase $\phi(p_0)$ at a certain momentum p_0 , the momentum dependence of the modulus $A(p)$ must be known over the entire range of values of p from zero to infinity.

For the nuclear regeneration phase ϕ_f , we obtain:

$$\phi_f(p_0) = -(\pi/2 + \phi(p_0)) \quad (6)$$

Applying this relation to the regeneration problem, we use experimental data for the momentum dependences of the moduli of the amplitude. For Cu, we use the measured exponents $\beta = -0.43 \pm 0.03$ in the momentum region below 20 GeV/c and $\beta = -0.63 \pm 0.01$ from 20 to 140 GeV/c, and estimate $\beta = -0.63 \pm 0.05$ as a guess for the unknown high-energy behaviour. The estimated uncertainty on this unknown exponent is one quarter of the break in exponents of 0.20 ± 0.03 observed at 20 GeV/c in ref. [11]. For Pb, we use $\beta = -0.45 \pm 0.05$ below 20 GeV/c, $\beta = -0.654 \pm 0.018$ from 20 to 140 GeV/c, and $\beta = -0.654 \pm 0.05$ above 140 GeV/c. For Carbon using experimental data without theoretical input, these measurements give $\beta = -0.632 \pm 0.025$ for

momenta from 4 to 10 GeV/c [14], and $\beta = -0.607 \pm 0.020$ from 30 to 130 GeV [10]. Above 130 GeV, we assume a continuous extrapolation $\beta = -0.607 \pm 0.05$ with the error as a reflection of a possible additional contribution at high energy. For carborundum (B_4C), the exponent is only measured in the range from 40 to 150 GeV for the neutral decay mode and from 30 to 160 GeV/c for the charged decay mode, to be $\beta = -0.596 \pm 0.009$. Outside this range we assume an uncertainty of ± 0.05 in the exponent.

The results for the regeneration phases from the integration of the dispersion relation eq. (5) are shown in figs. 1a to 1d, as a function of Kaon momentum. It appears that using experimental data in the measured range and assuming an extrapolated continuous behaviour in the unexplored momentum domain, dispersion relations can give regeneration phases with uncertainties of ± 2.5 degrees for Cu, ± 3 degrees for Pb, ± 3 degrees for Carbon, and ± 2.5 degrees for B_4C , in the relevant momentum region from 40 to 150 GeV/c. The uncertainties in the upper half of this momentum region are related to the assumed uncertainty of ± 0.05 in the exponent of the power-law behaviour of the regeneration amplitude. Of course, assuming a single Regge trajectory with constant exponent in the power-law momentum dependence above 150 GeV/c would decrease this uncertainty, but then the resulting regeneration phase and the CP-phase ϕ_{+-} deduced from it cannot be used as a test of CPT invariance.

These errors on the regeneration phase are about of the same magnitude as the ones from a direct measurement of ϕ_f from the time-dependent charge asymmetry of semileptonic decays behind a regenerator [3].

Since the theoretical uncertainties from the integration over unexplored momentum domains in the dispersion method cannot easily be reduced, the most promising route for improving on the precision of the knowledge of the regeneration phase is probably a new measurement of this charge asymmetry with improved statistical precision.

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Table 1

Exponents of power-law behaviour of nuclear regeneration amplitudes, $(f(0) - \bar{f}(0))/k$, with kaon momentum

Material	exponent	momentum range	Ref.
B ₄ C	-0.596 ± 0.009	30-160 GeV/c	[12]
C, Al, Cu, Sn, Pb	-0.614 ± 0.009	20-140 GeV/c, average over different nuclei	[11]
Pb	$\Delta = 0.17 \pm 0.05$	difference of exponents above and below 20 GeV/c	[11]
	-0.654 ± 0.018	35-95 GeV/c	[13]
	-0.45 ± 0.05	2-10 GeV/c	[15]
Cu	$\Delta = 0.20 \pm 0.03$	difference of exponents above and below 20 GeV/c	[11]
C	-0.607 ± 0.020	30-130 GeV/c	[10]
	-0.632 ± 0.025	4-10 GeV/c	[14]

Figure captions

Fig. 1: Nuclear regeneration phase ϕ_f calculated from the measured and extrapolated exponents of the momentum dependence of the modulus of the regeneration amplitude, via the dispersion relation. Central value (diamonds) and error interval (crosses, squares) are shown: (a) for copper, (b) for lead, (c) for carbon, (d) for carborundum B_4C .

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 1d