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Fluctuations and Instabilities in Multifragmentation*

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Invited talk at the Fifth International Conference on Nucleus-Nucleus Collisions, Taormina, Italy, 30 May - 4 June 1994.

SCAN-9409022

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In this article we review the foundations of dynamical description of unstable systems. We describe the concept of stochastic one-body theories. In particular, spinodal instabilities are accounted for: the most unstable modes which depend mostly on the range of the nuclear force are shown to determine the early unstable dynamics and therefore the initial multifragmentation; the characteristic time scales for the growth of these modes and the typical time scales for cluster formation, as a function of the amplitude of fluctuations, are discussed. The differences between such a fragmentation pattern, essentially determined by the instabilities, and fragmentation structures obtained from the hypothesis of statistical equilibrium are stressed. Finally, we propose that the occurrence of spinodal instabilities can be signed by a lack of small size fragments associated to a tendency to break the system in close-mass pieces.

* This work was supported in part by the Commission of the European Communities, under Contract No.ERBCHBICT930619.

1. Introduction

In the recent years the genesis of multifragmentation of nuclear systems has been extensively studied[1]. It has been suggested that the fragment production can be related to the occurrence of instabilities during the dynamical evolution of the nuclear system produced by a heavy ion reaction. In particular possible sources of instabilities - shape instabilities, some time related with the Coulomb interaction[2], or volume (also called spinodal) instabilities[3-6] - have been identified.

However, the direct study of the dynamics of unstable nuclear systems has only been made possible with the development of stochastic mean field approaches[7-9]. Indeed, since quantum mechanics is at work in the nucleus because of both the Heisenberg uncertainty principle and the Pauli exclusion principle, no exact many-body simulations are possible. However, it has been argued that, alike many static or dynamical properties of the nuclei, instabilities can be well described within the mean-field approximation[10, 11] and that stochastic approaches may provide a well-founded description of the onset of fragmentation.

In this article we will stress the fact that the occurrence of instabilities generally introduces: **typical size scales** and **typical time scales**. These features may provide a general signature of instabilities. We will show that the dynamical evolution (such as the fragmentation time) is governed by the magnitude of the stochastic terms in the equation of motion. We will discuss the fact that the final state interaction may partly affect the considered signal of instabilities because it provides a route toward statistical equilibration. Finally, we will show that experimental findings about the fragmentation times do not contradict the instability scenario and that some experimental results about the fragmentation pattern plead in favour of the occurrence of instabilities.

Figure 1. Cluster formation obtained by randomly distributing small drops of water using an atomizer (a) or formed by the clusterization of a thin unstable layer of water, obtained using a brush (b).

However, before getting into the details of the nuclear multifragmentation let us first illustrate the concepts of statistical distribution, equilibrium, time scales, fragment size, instabilities and the idea of the signatures of instabilities on a simple every-day-life experiment, the so called window-cleaner experiment.

2. Window-cleaner Experiment

If small drops of water, of approximately equal size, are randomly distributed on a surface, one obtains a statistical distribution of clusters. This can be realized in practice by using an atomizer to spray water on a pane (see Fig. 1). Depending upon the density of the water sprayed on the screen, one may get different cluster distributions (see Fig. 1): at low density the mass spectrum is strongly peaked on the sprayed-drop mass; as the density is increased larger clusters are formed; at a given density the system passes through a percolative phase transition above which at least one very large fragment (infinite) is observed.

The described system, at a given density of water on the pane, may easily be placed out of equilibrium by a stroke of the brush. In this way one may create a thin layer of water which is unstable. Therefore, the system rapidly clusterizes in rather large drops characteristic of the instabilities. After the fast formation of the initial clusters the system does not really evolve. This example clearly illustrates the fact that instabilities are

Figure 2. Phase diagram of (3D) nuclear matter: the liquid region and the gas region appear respectively on the right and on the left of the figure. The coexistence region is decomposed into two zones : a metastable zone , where a uniform liquid (or gas) phase may survive during a long time and a spinodal zone, where uniform nuclear matter is unstable against density variations. The critical point is located on the top of the spinodal region. The curves inside the spinodal region join the points characterized by the same value of τ , being τ the characteristic growing time of perturbations with $\lambda = 10 \text{ fm}$, the most unstable mode well inside the spinodal. Typical values, around 30-50 fm/c, are obtained.

associated to characteristic time scales much shorter than equilibration time so that the fingerprints of the instabilities may be kept over in the clusterization pattern very different from a statistical distribution, in particular through the manifestation of particular size scales.

3. Instabilities and Mean-Field Approximations

Let us now turn back to the nuclear physics case. During a heavy ion collision the nuclear system is strongly stressed and therefore might get out of equilibrium. During this fast evolution the system might face instabilities. For example if the system is rapidly expanded it might be pushed in a forbidden region of the phase space such as the spinodal zone. The phase diagram of nuclear matter is supposed to present a liquid-gas phase transition². Within the coexistence area it exists a zone of mechanical instabilities called the spinodal zone. eventually will lead to the partition of the system in clusters.

Instabilities as well as phase diagram can be qualitatively studied within a mean field approximation. From the static point of view, this approximation often provides a good description of the properties of many-body system, except sometimes in the vicinity of the critical point where it is well known that mean field approximation may give a poor description of critical properties such as critical exponents.

From the dynamical point of view mean field approximations are often a good building block for more elaborate theories. In particular constrained Hartree-Fock methods or other one-body approaches such as liquid-drop calculations are often used to describe complex landscape of potential energy (and masses) as a function of deformation or shape parameters (or even partitions of the system). These calculations present barriers and more complex instabilities, bifurcations, etc. and have shown that for many nuclear physics problems mean-field approximation is a good starting point even when the problem

involves some instabilities.

4. Fluctuation and Langevin Dynamics.

It is clear from the intuitive point of view that a given unstable system may lead to different final states. However, in a deterministic approach there will be no possibilities to decide which way to go and a unique final state will be predicted. This is for example the problem encountered by time dependent mean field descriptions of systems in the spinodal region: if the initial system corresponds to a uniform density, a mean field dynamics will never break the translational symmetry and the uniform density system will appear as stationary. This is obviously an unphysical solution because any fluctuation (or correlation) may trigger the instabilities which will lead the system to break apart into a mixing of the two phases: the liquid and the gas.

The missing piece of the puzzle is that any theory which describes only a part of the system (collective variables, one-body densities ...) must be considered as statistical in the sense that the lack of information associated with the projection onto the retained variables corresponds to some entropy and thus to some disorder. This disorder, which corresponds to the unknown correlations, will introduce some stochasticity in the motion of the collective variables.

In the past many stochastic approaches have been developed: from master (or rate) equation to Fokker-Planck and Langevin approaches[12]. Number of them, designed to describe for example induced fission, heavy-ion deep-inelastic reactions, compound-nucleus statistical shape-average, etc are related to mean field approaches. The success of these studies has demonstrated the validity of the concepts of stochastic approaches based on mean-field treatments of the nuclear dynamics.

Very recently fully microscopic stochastic mean-field theories have been developed [7]. The basic idea is to mimic the microscopic description of the brownian motion by considering that the most important source of fluctuations in a one body dynamics is coming from the stochastic aspect of the collisions between particles. This leads to the so-called Boltzmann-Langevin (B.L.) dynamics, in which the mean-field evolution is complemented by a fluctuating collision term. It has been shown that these approaches correctly describe the thermalization of a Fermi gas[8, 9, 13] (see fig.3). Therefore it can be considered as a microscopic dynamical extension of the various stochastic (or statistical) macroscopic approaches. However, until now, it has been solved only in 2D.

In accordance with the usual Langevin treatment of the Brownian motion, it has been proposed to replace the complex description of the stochastic collisions by a simple random force[14, 6]. It has been shown that, when the strength of the random term is correctly tuned, these simple Langevin approaches provide a good approximation to the complete treatment, even when the system presents instabilities.

As far as unstable systems are concerned, it is of paramount importance to notice that the strength of the fluctuating terms determines the time scale of the whole dynamics. Indeed, the larger the noise the sooner the system will reach a given fluctuation amplitude. In fact the instability time introduces a time unit, but the magnitude of the Langevin term defines how many time units are needed to get to a particular stage of the dynamics (such as for example the fragmentation of the system). Therefore, without a correct description

Figure 3. The results of a B.L. treatment, implemented on a lattice, (histogram) are compared with the prediction of statistical mechanics (full line) for the variance $\sigma^2(p_x, 0)$ (top, left) [8] and covariance σ_{cov}^2 (top, center) [13] of a Fermi gas. The agreement shows that the B.L. correctly takes into account thermal fluctuations. A non-equilibrium case is also considered (top, right) [15]. For a system which lies in the spinodal region we show the exponential increase of the variance σ_k^2 associated to the most unstable modes, in the case of the full B.L. calculation (stars) and using a simpler stochastic approach where the fluctuations are coming from the use of a finite number of test particles in ordinary *BUU* calculations. We obtain a good agreement as soon as a suitable number of test particles is used. This agreement is observed also looking at the temporal evolution of the spatial density (bottom, left: B.L. lattice treatment, right: test particle treatment) and of the Fourier transform of the (ensemble averaged) correlation function σ^2 , which is displayed on the right of the density plots [15].

of the stochastic part of the dynamics, only the onset of the instabilities can be inferred but no conclusion about the real dynamics can be drawn.

In conclusion, the various studies about B.L. approaches and its simplified versions have shown that these theories provide a well-founded frame-work for the study of fluctuations, even in the presence of instabilities: they allow the system to undergo spontaneous symmetry breaking and to explore instabilities, bifurcations and chaos; moreover they introduce the concept of statistical ensemble of trajectories which restore the predictive power of the theory in terms of probability distribution (or Monte-Carlo event generator).

5. Small Time Behaviour of Spinodal Decomposition.

Using the previously introduced stochastic one-body approach it is now possible to describe the dynamics of the onset of instabilities, relying on the mean field to describe the characteristics of the instabilities and on the stochastic terms to incorporate the correct fluctuations, including eventually the thermal and statistical ones.

Let us now illustrate these concepts on the example of the spinodal instabilities. In this section, we will gather several results concerning this spinodal decomposition for finite and infinite (with and without expansion) systems in 2D and 3D. The predicted scale of the instabilities dominating the dynamics appears as a very robust quantity since the range of the effective interaction in a semi-classical calculation is coming from both the range of the bare nuclear force from quantum and surface effects. Since these properties are rather independent of the physical conditions of the system such as the density and the temperature, they can be constrained by the value of the surface energy of normal nuclei at $T=0$.

In this section, we will discuss the memory lost of these initial characteristics and we will propose a possible signature of spinodal decomposition, in short-lived finite-size systems as a lack of small cluster formation.

5.1. Formation of pre-clusters.

As a first illustration, ref.[15] presents the results of a schematic calculation of a piece of a two-dimensional nuclear matter at half normal density and at a temperature $T=3$ MeV. The range of the nuclear force is simulated by a gaussian with a full width at half maximum of about 2 fm. In the first results the mean field is averaged on the y direction in order to simplify the discussion of the clusterization process.

Fig 3a presents a lattice simulation [9] of the B.L. evolution in which the one-body distribution function f experiences a stochastic evolution and therefore fluctuates, in response to the random effect of the Langevin force. Since the mean field is unstable, these fluctuations are amplified and eventually lead to the formation of fragments. In Fig.3b, we show a similar time evolution computed in a simplified Langevin approach using a test particle method. Now fluctuations are present since the beginning due to the finite sampling of the phase space distribution but, starting from a given time ($t \approx 50$ fm/c), the evolution becomes comparable to that of the lattice calculation (part a), reaching finally a similar structure. This amazing equivalence between two independent sets of events computed with two totally different computer codes and two different Langevin forces is also apparent on the ensemble average of the Fourier transform of the density fluctuations (see fig.3). These correlation functions are all clearly peaked at a value which

Figure 4. Clusterization of a periodic box (a), a finite charged box (b) and an expanding box (c). The mean field has been averaged on the y direction. The Coulomb repulsion and the expansion velocity help the system to keep the memory of the initial instabilities.

corresponds to $\lambda \approx 10$. fm .

The same conclusions can be drawn from the study of uniform nuclear matter in expanding geometries [16] and of finite size systems [16]. An example of the latter results is presented in figure 4. It can be seen that the most unstable wave length is approximately independent of the considered system.

The figure 5b presents the results of actual 2D calculations and one sees that indeed the fragments formed are approximately 10 fm apart one from the each other as the presented event-averaged Fourier transform is confirming.

This behaviour is also clearly visible in the 3D Langevin evolution of a hot expanded system presented in Figure 6. Rather large fragments are formed in the first stage of the evolution.

5.2. Interpretation of the Results in the Linear Response Theory.

In refs [14, 6] the occurrence of a given size scale has been related to the range of the forces (and surface terms) through the use of linear response to stochastic mean field theory. For example Figure 5 presents the dispersion relation associated with the studied 2D systems. One can see that it presents a maximum of instability for a given wave number which corresponds to $\lambda = 10$ fm . This can be easily understood. Indeed, for a scale-free system this dispersion relation would be linear in k : i.e. the smaller the wave length the more unstable the mode. However, as soon as we introduce a scale such as a range for the forces, the mean field will not be sensitive to perturbation over a small size. Therefore the small wave length (the high k) cannot be unstable and the dispersion relation has to present a peak for a given size.

Figure 5. The 2D dispersion relation (left, top): the most unstable mode lies around $\lambda = 10 \text{ fm}$. The clusters obtained in an exact simulation of a classical gas, prepared inside the spinodal region, using a two-body force with the same gross properties of the nuclear force, top right figure, for a 2D system. One can observe that the clusterization process is dominated by a typical wave length. This result is similar to the B.L. prediction for nuclear matter (center, right). The comparison of the dispersion relation with the Fourier transform of the correlation function (on the right of each cluster distribution) shows that the clusterization is mainly driven by the most important unstable modes. The size distribution at the time when the fragments are formed and the fragment multiplicity as function of time are displayed on the left. The fragments which are formed may later on interact (center, right): in this case the memory of the initial instabilities is lost (see also the Fourier spectrum). The cluster formation obtained distributing randomly the particles in space is shown (right, bottom). In this case the Fourier spectrum is white and we do not observe any peculiar structure in the fragmentation.

Figure 6. Fragmentation of a 3D nuclear system of 100 nucleons, at $\rho = 0.4\rho_0$, $T = 3 \text{ MeV}$ and with an expansion velocity of $v = 0.1 c$ (left) and $v = 0.2 c$ (right). A simplified Langevin approach has been used to introduce fluctuations in the dynamics. A number of fragments strictly related to the most unstable modes in the considered situation is obtained. Because of some final state interaction, in the case of the lower expansion velocity, we finally get three fragments. No small fragments are observed. (Images: Groupe VISIons)

It should be noticed that the linear response approach has been shown to correctly explain the average behaviours such as the average fragment multiplicity and the correlation function. However, the linear response approach does not give information on the fluctuations of such ensemble average quantities which are the result of the interplay between the complex non-linear dynamics and the source of stochasticity. Therefore, it is clear that a detailed study of the predictions of stochastic mean field approach necessitate the use of direct simulation of many events.

5.3. Spinodal Decomposition in Molecular Dynamics.

One may worry if the stochastic mean-field approximation provides a valuable description of dynamics of unstable systems.

Since the prediction of the occurrence of a well-defined scale in the dynamics of the growth of clusters in the spinodal region appears as a very robust prediction it can be also tested on physically different situation. In this section we report on the study of the spinodal decomposition of a classical system[17]. The advantage of a classical system is that direct simulations of the exact dynamics can be performed using a molecular dynamics approach.

Figure 5 presents the results of one exact simulation of the evolution of an initially uniform classical system located in its spinodal zone. The simulation has been performed using a force with a range comparable with the one used in the nuclear case. The right part of the figure shows the Fourier analysis of the correlation function. These results show that a classical dynamics is analogous to the one predicted by the stochastic approaches. In particular the short time behaviour of the clusterization process appears to be dominated by the most unstable modes, the size of which is clearly related to the actual range of the force. Whatever these results need to be completed[17] one can conclude that the predictions we made within the B.L. approach are confirmed by the exact solution of a classical gas dynamics.

5.4. Signatures in Fragmentation pattern.

As expected the spinodal instabilities produce few small fragments as it can be seen from fragment size distribution (see fig. 5). However, if the gas of clusters remains in interaction as in the closed box case the system will try to reach statistical equilibrium. The clusters therefore evolve through a kind of coalescence mechanism as can be seen on fig. 5. However, in an open geometry (or in an expanding system) this final state interaction is blocked after a given freeze-out time. In this case the system keeps memory of clusterization pattern produced by the instabilities and a lack of small fragment production is observed [16]. This is shown on fig. 6 which presents results in open geometry.

6. Discussion and Conclusion.

In conclusion, the spinodal instabilities produce typical fragmentation pattern characterised by the presence of approximately equal size fragments ($Z \approx 10 - 20$). This signal can be partly altered by the final state interaction, depending upon the freeze-out time and by the subsequent decay of these hot fragments. However, spinodal instabilities can probably be evidenced by a reduction of the production of small intermediate mass fragments. From the experimental point of view some observation might already plead in

favour of the occurrence of spinodal instabilities. First of all fragmentation time have now been measured[18] and it seems that the time to produce fragments from a hot system is about 100-200 fm/c. This time is compatible with the theoretical predictions discussed in this article: i.e. the time needed to get deep enough in the spinodal zone, to produce fragments through instabilities and to reach a freeze-out configuration. Indeed, as can be seen from fig. 2 the instability time in 3D is at maximum around 30 fm/c so that the time to produce fragments which is 2 or 3 times this time is about 100 fm/c.

As far as the fragmentation pattern is concerned, several experiments are now reporting on the presence of equal size fragments in central collisions[19, 18]. Moreover, it seems that this observation cannot be reproduced by neither sequential decay simulation nor by statistical multifragmentation calculations. In particular it seems that the latter are predicting too many small size fragments as compared with experiments[19]. Therefore it might be that some experimental results are already presenting some signal of the spinodal decomposition in nuclear reactions. However more systematic studies are called for.

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