Chiral Symmetry Breaking and the Polarized Photon Structure Function in QED

A.Freund and L.M.Sehgal

III. Physikalisches Institut, RWTH Aachen

D-52074 Aachen, Germany

Abstract

The structure function of a polarized photon $g_1^\gamma(x)$ satisfies a remarkable sum rule: $\int g_1^\gamma(x) dx = 0$. The zero arises as a cancellation between explicit (mass-induced) and anomalous chiral symmetry breaking. We analyse this cancellation by studying the helicity structure of the cross section $\gamma^*(\lambda_1) + \gamma(\lambda_2) \rightarrow l^+(\lambda_3) + l^-(\lambda_4)$. The structure function $g_1^\gamma(x)$ is found to consist of two pieces, $g_{1,\text{like}}$ and $g_{1,\text{unlike}}$, corresponding to final l^+l^- states with like and unlike helicities. The function $g_{1,\text{like}}$ is constant in x , and integrates to $+1/2$, while $g_{1,\text{unlike}}$ is x -dependent and integrates to $-1/2$. When the target photon is virtual ($|k^2| \gg m^2$), the integral $\int g_{1,\text{like}} dx$ drops to zero, while $\int g_{1,\text{unlike}} dx$ stays constant at the value $-1/2$. We conclude that the "explicit" and "anomalous" parts of $g_1^\gamma(x)$ are uniquely associated with final l^+l^- states with equal or opposite helicities. The survival of the like-helicity component is a "quasi-paradox" analogous to the survival of helicity-flip bremsstrahlung at high energies. Some implications for the QCD process $\gamma^*g \rightarrow Q\bar{Q}$ are noted.

In attempting to understand the role of the gluon in the structure function of a polarized proton, attention has turned to a simpler question in a more familiar context: the structure function of a polarized photon in QED [1]. This quantity may be defined as [2,3]

$$g_1^\gamma(x, Q^2) = \frac{1}{2}(\sigma_{++}(x, Q^2) - \sigma_{+-}(x, Q^2))/\sigma_0 \quad (1)$$

where σ_{++} and σ_{+-} are the cross sections for the annihilation of a virtual photon of mass q^2 ($= -Q^2$) on a real photon (mass $k^2 = 0$) in helicity states $++$ and $+-$ respectively. Here x is defined as $x = \frac{Q^2}{s + Q^2 - k^2}$, s being the invariant mass squared of the two photon system, and σ_0 is a convenient scale factor $\sigma_0 = e^4/(2\pi(s + Q^2 - k^2))$. In the lowest order of QED, σ_{++} and σ_{+-} are simply the cross sections for pair production $\gamma^*\gamma_{+(-)} \rightarrow l^+l^-$, and have the explicit expressions [4]

$$\begin{aligned} \sigma_{+-} &= \sigma_0(1-x)^2 \left(\frac{1}{2} \ln\left(\frac{1+v}{1-v}\right)(5-v^4) + v^4 - 5v \right) \\ \sigma_{++} &= \sigma_{+-} + \sigma_0 \left(\ln\left(\frac{1+v}{1-v}\right)(2x-1) + v(3-4x) \right) \end{aligned} \quad (2)$$

Thus the structure function g_1^γ is

$$g_1^\gamma(x, Q^2) = \frac{1}{2} \left(\ln\left(\frac{1+v}{1-v}\right)(2x-1) + v(3-4x) \right) \quad (3)$$

with $v = \sqrt{1 - \frac{4m^2}{Q^2}}$, m being the mass of the lepton l . In the deep inelastic domain,

$Q^2 \gg m^2$, the variable x has the limits 0 and 1, and the integral of g_1^γ is

$$\int_0^1 g_1^\gamma(x, Q^2) dx = 0 \quad (4)$$

The sum rule (4) has an interesting interpretation in terms of chiral symmetry breaking in QED. The operator product expansion applied to the product of two electromagnetic currents $J_\mu(x)J_\nu(0)$ (with $J_\mu = \bar{l} \gamma_\mu l$) allows one to relate $\int_0^1 g_1^\gamma dx$ to the expectation value of the axial charge operator $Q_5 = \int d^3x \bar{l} \gamma_0 \gamma_5 l$ in a one-photon state [5,2,3]. The calculation of $\langle \gamma(k) | J_{\mu 5} | \gamma(k) \rangle$ involves the well known triangle diagram and the result (for non-zero m)

depends on the method of regularization. Taking the mass of the photon k^2 as a regulator, one obtains the sum rule [5,6]

$$\int_0^1 g_1^\gamma(x) dx = -\frac{1}{2} + \frac{1}{4} \frac{r}{\sqrt{r+1}} \ln\left(\frac{\sqrt{r+1}+1}{\sqrt{r+1}-1}\right) \quad (5)$$

where $r = \frac{4m^2}{|k^2|}$. For $m \neq 0$, the real photon limit $k^2 \rightarrow 0$ corresponds to $r \rightarrow \infty$, and the two terms in (5) cancel, yielding the sum rule (4). On the other hand, if $m \rightarrow 0$ in such a way that $r \rightarrow 0$, the second term in (5) vanishes and one obtains the "anomaly" result $\int g_1^\gamma dx = -\frac{1}{2}$ [7]. Clearly, the two terms in (5) correspond to the two pieces in the divergence of the axial vector current:

$$\partial_\mu J_5^\mu = 2im\bar{l}\gamma_5 l + \frac{\alpha}{4\pi} F_{\mu\nu} \hat{F}^{\mu\nu} \quad (6)$$

The explicit (mass-dependent) term is responsible for the second term in Eq. (5), while the anomaly produces the first term, the two together yielding the sum rule $\int g_1^\gamma dx = 0$ in the limit $r \rightarrow 0$.

The question we wish to raise is, whether one can identify and separate these two pieces of the structure function at the level of the $\gamma^* \gamma \rightarrow l^+ l^-$ cross section. To answer this, we examine the lowest order expressions for the helicity cross sections in QED:

$$\sigma_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = \sigma(\gamma^*(\lambda_1) \gamma(\lambda_2) \rightarrow l^+(\lambda_3) l^-(\lambda_4)) \quad ,$$

specifying both the photon and lepton helicities. These are

$$\sigma_{+-+-} = \sigma_{+--+} = \sigma_0 \frac{1}{2} (1-x)^2 \left(\ln\left(\frac{1+v}{1-v}\right) \left(v^2 + \frac{3}{v^2}\right) - v - \frac{3}{v} \right)$$

$$\sigma_{+-++} = \sigma_{+---} = \sigma_0 \frac{1}{2} (1-x)^2 \left(\ln\left(\frac{1+v}{1-v}\right) \left(5 - v^4 - v^2 - \frac{3}{v^2}\right) + v^3 - 4v + \frac{3}{v} \right)$$

$$\begin{aligned} \sigma_{+++-(-+)} = \sigma_{+-+-(+)} + \sigma_0 \frac{1}{4} \left((1-x) \left(\ln\left(\frac{1+v}{1-v}\right) \left(-2v^2 - \frac{6}{v^2}\right) + 4v + \frac{12}{v} \right) + \ln\left(\frac{1+v}{1-v}\right) \left(v^2 + \frac{3}{v^2}\right) \right. \\ \left. - 2v - \frac{6}{v} \right) \end{aligned}$$

$$\begin{aligned}\sigma_{++++}(-) = \sigma_{+-++}(-) + \sigma_0 \frac{1}{4} ((1-x) \ln(\frac{1+v}{1-v}) (2v^2 - (+)2v^3 + (-)\frac{2}{v} + \frac{6}{v^2} - 8) + 12v - \frac{12}{v} \\ + (-)4v^2 - (+)4) + \ln(\frac{1+v}{1-v}) (-v^2 + (-)2v - (+)\frac{2}{v} - \frac{3}{v^2} + 4) - 2v + \frac{6}{v} + (-)4) \quad (7)\end{aligned}$$

Of particular note are the following features: (I) In the limit $Q^2 \rightarrow 0$ (or $x \rightarrow 0$) the above cross sections reduce to the known helicity cross sections for real $\gamma\gamma$ annihilation [8]. (II) In the deep inelastic limit $Q^2, s \gg m^2$, which amounts to the limit $v \rightarrow 1$, the helicity cross sections $\sigma_{+-++}, \sigma_{+---}$ and σ_{++--} vanish. The only independent cross sections in this limit are $\sigma_{+-+-} (= \sigma_{+--+}), \sigma_{+++-} (= \sigma_{+-+})$ and σ_{++++} . These are plotted as functions of x in Fig.1. (III) Especially interesting is the feature that the cross section σ_{++++} (with l^+l^- possessing the same helicities) is non-vanishing and constant in x , even in the chiral limit $4m^2/Q^2 \rightarrow 0$.

We now define two new structure functions, analogous to g_1^γ , which are specific to final l^+l^- states with like ($++$, $--$) and unlike ($+-$, $-+$) helicities:

$$\begin{aligned}g_{1,\text{like}} &= \frac{1}{2} ((\sigma_{++++} + \sigma_{++--}) - (\sigma_{+-++} + \sigma_{+---})) \\ g_{1,\text{unlike}} &= \frac{1}{2} ((\sigma_{+++-} + \sigma_{+-+}) - (\sigma_{+-+-} + \sigma_{+--+}))\end{aligned} \quad (8)$$

Using the formulas (7), we have:

$$\begin{aligned}g_{1,\text{like}} &= \frac{1}{4} ((1-x) \ln(\frac{1+v}{1-v}) (2v^2 + \frac{6}{v^2} - 8) + 12v - \frac{12}{v}) + \ln(\frac{1+v}{1-v}) (-v^2 - \frac{3}{v^2} + 4) \\ &\quad - 2v + \frac{6}{v}) \\ g_{1,\text{unlike}} &= \frac{1}{4} ((1-x) \ln(\frac{1+v}{1-v}) (-2v^2 - \frac{6}{v^2}) + 4v + \frac{12}{v}) + \ln(\frac{1+v}{1-v}) (v^2 + \frac{3}{v^2}) - 2v - \frac{6}{v})\end{aligned} \quad (9)$$

which add up to give the structure function in Eq.(3). In the deep inelastic limit, the function $g_{1,\text{like}}$ approaches a constant ($= -1/2$), while $g_{1,\text{unlike}}$ is x -dependent (Fig.2). The integrals of these functions are:

$$\int g_{1,\text{like}} dx = \frac{1}{2} \text{ and } \int g_{1,\text{unlike}} dx = -\frac{1}{2} \quad (10)$$

These are exactly the contributions to $\int g_1 dx$ associated with explicit and anomalous chiral symmetry breaking.

Our analysis has implications for the QCD process $\gamma^* g \rightarrow Q\bar{Q}$, where $Q\bar{Q}$ is a heavy quark-anti-quark pair. To the extent that $Q^2 \gg m_Q^2 \gg \Lambda_{\text{QCD}}^2$, the corresponding polarized gluon structure function should satisfy $\int g_1^{Q\bar{Q}} dx = 0$. This sum rule is valid even if the gluon is not on its mass-shell, as long as its virtuality $|k^2| \sim \Lambda_{\text{QCD}}^2$ does not exceed m_Q^2 . We predict that $Q\bar{Q}$ pairs of like helicity (especially the cross section σ_{++++}) will make a contribution to $g_1^{Q\bar{Q}}$. By contrast, in $\gamma^* g$ annihilation into light quark-antiquark pairs, the virtuality of (confined) gluons unavoidably exceeds the light quark mass (i.e. $|k^2| \sim \Lambda_{\text{QCD}}^2 \gg m_{u,d}^2$) and one expects the integral of $\int g_1 dx$ to have a nonzero value, dictated by the U(1) chiral anomaly of QCD [5,7].

Appendix

$$\sigma_{+-+-} = \sigma_{+--+} = \sigma_0 \frac{x^2(Q^2(1-x)/x + k^2)^2}{8Q^4u} \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{v^2}{u} + \frac{3}{v^2u^5} \right) - \frac{2v}{u^2} - \frac{6}{vu^4} \right)$$

$$\sigma_{++-+} = \sigma_{+---} = \sigma_0 \frac{x^2(Q^2(1-x)/x + k^2)^2}{8Q^4u} \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{v^2}{u} - \frac{v^4}{u} - \frac{2v^2}{u^3} + \frac{2}{u^3} + \frac{3}{u^5} - \frac{3}{v^2u^5} \right) + \frac{2v^3}{u^2} - \frac{2v}{u^2} - \frac{6v}{u^4} + \frac{6}{vu^4} \right)$$

$$\begin{aligned} \sigma_{++-}(++) &= \sigma_{+--}(++) + \sigma_0 \frac{1}{8u} \left((1-x + \frac{xk^2}{Q^2}) \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{2v^2}{u} - \frac{2}{u} + (-)2v^3 - (+)2v + (-)\frac{2v}{u^2} - (+)\frac{2}{vu^2} - \frac{6}{u^3} \right. \right. \right. \\ &\quad \left. \left. + \frac{6}{v^2u^3} \right) + (-)\frac{4}{u} - (+)\frac{4v^2}{u} - \frac{12}{vu^2} + \frac{12v}{u^2} \right) + (\frac{x^2k^2}{Q^2}) \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(-(+)\frac{4v}{u^2} + (-)\frac{4}{vu^2} - \frac{4}{u^3} + \frac{4}{v^2u^3} + \frac{4}{u} - \frac{4v^2}{u} \right) \right. \\ &\quad \left. - \frac{8(1-v^2)}{vu^2} - (+)\frac{8(1-v^2)}{u(1-v^2u^2)} \right) + \ln\left(\frac{1+uv}{1-uv}\right) \left(-\frac{v^2}{u} + \frac{5}{u} - \frac{4}{v^2u} - (+)\frac{v}{u^2} + (-)\frac{1}{vu^2} - \frac{1}{u^3} + \frac{1}{v^2u^3} - (+)v + (-)\frac{1}{v} \right) \\ &\quad \left. + \frac{4(1-v^2)}{v(1-v^2u^2)} - \frac{2(1-v^2)}{vu^2} - 4v + \frac{4}{v} - (+)\frac{2(1-v^2)(1+u^2)}{u(1-v^2u^2)} \right) \end{aligned}$$

$$\begin{aligned} \sigma_{++-}(+-) &= \sigma_{+--}(+-) + \sigma_0 \frac{1}{8u} \left((1-x + \frac{xk^2}{Q^2}) \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(-\frac{2v^2}{u} + \frac{2}{u} - \frac{6}{v^2u^3} - \frac{2}{u^3} \right) + \frac{4v}{u^2} + \frac{12}{vu^2} \right) + (\frac{x^2k^2}{Q^2}) * \right. \\ &\quad \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{4}{u^3} - \frac{4}{v^2u^3} + (-)\frac{4v}{u^2} + (-)\frac{4}{vu^2} + \frac{4v^2}{u} - \frac{4}{u} \right) + \frac{8(1-v^2)}{vu^2(1-v^2u^2)} + \frac{8v(1+v^2)}{1-v^2u^2} - (+)\frac{8(1+v^2)}{u(1-v^2u^2)} \right) \\ &\quad \left. + \ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{1}{u^3} - \frac{1}{v^2u^3} + (-)\frac{v}{u^2} + (-)\frac{1}{vu^2} + \frac{v^2}{u} - \frac{1}{u} + \frac{4}{v^2u} - (+)v - (+)\frac{1}{v} \right) - \frac{4}{v} + \frac{2(1-v^2)}{vu^2} - \frac{4(1-v^2)}{v(1-v^2u^2)} \right. \\ &\quad \left. + (-)\frac{2u(1+v^2)}{1-v^2u^2} - (+)\frac{2(1+v^2)}{u(1-v^2u^2)} \right) \end{aligned}$$

with $u = \sqrt{1 + \frac{4x^2k^2}{Q^2}}$. We have checked that the above results coincide, when summed over the lepton helicities, with those given in [4].

Figure Captions:

Fig.1 Helicity cross sections for $\gamma^*(\lambda_1) + \gamma(\lambda_2) \rightarrow l^+(\lambda_3) + l^-(\lambda_4)$ in units of $\sigma_0 = e^4/(2\pi(s + Q^2 - k^2))$ for $\frac{4m^2}{Q^2} = 10^{-6}$.

Fig.2 Plots of the functions $g_{1,\text{like}}$ and $g_{1,\text{unlike}}$ for $\frac{4m^2}{Q^2} = 10^{-6}$. The full structure function is $g_1 = g_{1,\text{like}} + g_{1,\text{unlike}}$.

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In attempting to understand the role of the gluon in the structure function of a polarized proton, attention has turned to a simpler question in a more familiar context: the structure function of a polarized photon in QED [1]. This quantity may be defined as [2,3]

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where σ_{++} and σ_{+-} are the cross sections for the annihilation of a virtual photon of mass q^2 ($= -Q^2$) on a real photon (mass $k^2 = 0$) in helicity states $++$ and $+-$ respectively. Here x is defined as $x = \frac{Q^2}{s + Q^2 - k^2}$, s being the invariant mass squared of the two photon system, and σ_0 is a convenient scale factor $\sigma_0 = e^4/(2\pi(s + Q^2 - k^2))$. In the lowest order of QED, σ_{++} and σ_{+-} are simply the cross sections for pair production $\gamma^*\gamma_{+(-)} \rightarrow l^+l^-$, and have the explicit expressions [4]

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Thus the structure function g_1^γ is

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depends on the method of regularization. Taking the mass of the photon k^2 as a regulator, one obtains the sum rule [5,6]

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where $r = \frac{4m^2}{|k^2|}$. For $m \neq 0$, the real photon limit $k^2 \rightarrow 0$ corresponds to $r \rightarrow \infty$, and the two terms in (5) cancel, yielding the sum rule (4). On the other hand, if $m \rightarrow 0$ in such a way that $r \rightarrow 0$, the second term in (5) vanishes and one obtains the "anomaly" result $\int g_1^\gamma dx = -\frac{1}{2}$ [7]. Clearly, the two terms in (5) correspond to the two pieces in the divergence of the axial vector current:

$$\partial_\mu J_5^\mu = 2im\bar{\psi}\gamma_5\psi + \frac{\alpha}{4\pi} F_{\mu\nu} \hat{F}^{\mu\nu} \quad (6)$$

The explicit (mass-dependent) term is responsible for the second term in Eq. (5), while the anomaly produces the first term, the two together yielding the sum rule $\int g_1^\gamma dx = 0$ in the limit $r \rightarrow 0$.

The question we wish to raise is, whether one can identify and separate these two pieces of the structure function at the level of the $\gamma^*\gamma \rightarrow l^+l^-$ cross section. To answer this, we examine the lowest order expressions for the helicity cross sections in QED:

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specifying both the photon and lepton helicities. These are

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$$\sigma_{-++-} = \sigma_{-+--} = \sigma_0 \frac{1}{2} (1-x)^2 \left(\ln\left(\frac{1+v}{1-v}\right) \left(5 - v^4 - v^2 - \frac{3}{v^2}\right) + v^3 - 4v + \frac{3}{v} \right)$$

$$\begin{aligned} \sigma_{++++}(-+) = \sigma_{+-+-}(-+) + \sigma_0 \frac{1}{4} \left((1-x) \left(\ln\left(\frac{1+v}{1-v}\right) \left(-2v^2 - \frac{6}{v^2}\right) + 4v + \frac{12}{v} \right) + \ln\left(\frac{1+v}{1-v}\right) \left(v^2 + \frac{3}{v^2}\right) \right. \\ \left. - 2v - \frac{6}{v} \right) \end{aligned}$$

$$\begin{aligned}\sigma_{++++}(-) = \sigma_{+-+-}(-) + \sigma_0 \frac{1}{4} ((1-x) \ln(\frac{1+v}{1-v}) (2v^2 - (+)2v^3 + (-)\frac{2}{v} + \frac{6}{v^2} - 8) + 12v - \frac{12}{v} \\ + (-)4v^2 - (+)4) + \ln(\frac{1+v}{1-v}) (-v^2 + (-)2v - (+)\frac{2}{v} - \frac{3}{v^2} + 4) - 2v + \frac{6}{v} + (-)4) \quad (7)\end{aligned}$$

Of particular note are the following features: (I) In the limit $Q^2 \rightarrow 0$ (or $x \rightarrow 0$) the above cross sections reduce to the known helicity cross sections for real $\gamma\gamma$ annihilation [8]. (II) In the deep inelastic limit $Q^2, s \gg m^2$, which amounts to the limit $v \rightarrow 1$, the helicity cross sections $\sigma_{+-+-}, \sigma_{+---}$ and σ_{++--} vanish. The only independent cross sections in this limit are $\sigma_{+-+-} (= \sigma_{+--+}), \sigma_{++++} (= \sigma_{+++-})$ and σ_{++++} . These are plotted as functions of x in Fig. 1. (III) Especially interesting is the feature that the cross section σ_{++++} (with l^+l^- possessing the same helicities) is non-vanishing and constant in x , even in the chiral limit $4m^2/Q^2 \rightarrow 0$.

We now define two new structure functions, analogous to g_1^γ , which are specific to final l^+l^- states with like (+ + , - -) and unlike (+ -, - +) helicities:

$$\begin{aligned}g_{1,\text{like}} &= \frac{1}{2} ((\sigma_{++++} + \sigma_{++--}) - (\sigma_{+-+-} + \sigma_{+---})) \\ g_{1,\text{unlike}} &= \frac{1}{2} ((\sigma_{+++-} + \sigma_{+-+-}) - (\sigma_{+-+-} + \sigma_{+---}))\end{aligned} \quad (8)$$

Using the formulas (7), we have:

$$\begin{aligned}g_{1,\text{like}} &= \frac{1}{4} ((1-x) \ln(\frac{1+v}{1-v}) (2v^2 + \frac{6}{v^2} - 8) + 12v - \frac{12}{v}) + \ln(\frac{1+v}{1-v}) (-v^2 - \frac{3}{v^2} + 4) \\ &\quad - 2v + \frac{6}{v}) \\ g_{1,\text{unlike}} &= \frac{1}{4} ((1-x) \ln(\frac{1+v}{1-v}) (-2v^2 - \frac{6}{v^2}) + 4v + \frac{12}{v}) + \ln(\frac{1+v}{1-v}) (v^2 + \frac{3}{v^2}) - 2v - \frac{6}{v})\end{aligned} \quad (9)$$

which add up to give the structure function in Eq.(3). In the deep inelastic limit, the function $g_{1,\text{like}}$ approaches a constant (= -1/2), while $g_{1,\text{unlike}}$ is x -dependent (Fig.2). The integrals of these functions are:

$$\int g_{1,\text{like}} dx = \frac{1}{2} \text{ and } \int g_{1,\text{unlike}} dx = -\frac{1}{2} \quad (10)$$

These are exactly the contributions to $\int g_1 dx$ associated with explicit and anomalous chiral symmetry breaking.

To confirm this connection, we have calculated the helicity cross sections for the process $\gamma^*(Q^2)\gamma^*(k^2) \rightarrow l^+l^-$, when the target photon is given a (space-like) mass equal to k^2 . These results are listed in the Appendix. The resulting expressions for $g_{1,\text{like}}$ and $g_{1,\text{unlike}}$, in the limit $Q^2 \gg |k^2|, m^2$ are:

$$\begin{aligned} g_{1,\text{like}} &= \frac{r}{8x(1-x) + 2r} \\ g_{1,\text{unlike}} &= \frac{1}{2}((2x-1)(\ln(\frac{(1-x)}{x^2(1-x) + xr/4}) - 1) + (1-x)(\frac{\frac{r}{2} - x(2x-1)}{x(1-x) + \frac{r}{4}}) - \frac{r}{4x(1-x) + r}) \end{aligned} \quad (11)$$

which satisfy the sum rules:

$$\begin{aligned} \int g_{1,\text{like}} dx &= \frac{1}{4} \frac{r}{\sqrt{r+1}} \ln\left(\frac{\sqrt{r+1} + 1}{\sqrt{r+1} - 1}\right) \\ \int g_{1,\text{unlike}} dx &= -\frac{1}{2} \end{aligned} \quad (12)$$

thus reproducing the two terms in Eq.(5).

Conclusions

We conclude that the chiral anomaly contribution to the polarized photon structure function is associated with l^+l^- final states in the "allowed" helicity combinations $+-$ or $-+$, normally expected for QED reactions at high energies. There is, however, an additional contribution to g_1^γ , associated with the "forbidden" helicity configurations $++$, $--$ for the l^+l^- system, originating in the explicit (mass-dependent) chiral symmetry breaking term in the divergence of the axial vector current. The integrals of these two components of g_1^γ are equal and opposite, leading to the sum rule $\int_0^1 g_1^\gamma dx = 0$. The fact that a "chirality-forbidden" cross section such as $\gamma^*(+)\gamma^*(+)\rightarrow l^+(+)l^-(+)$ makes a finite contribution to g_1^γ , even in the high energy limit $Q^2, s \gg m^2$, is one more example of a "quasi-paradox" [9], analogous to the survival of helicity-flip bremsstrahlung in the high energy limit of QED [10].

Our analysis has implications for the QCD process $\gamma^* g \rightarrow Q\bar{Q}$, where $Q\bar{Q}$ is a heavy quark-anti-quark pair. To the extent that $Q^2 \gg m_Q^2 \gg \Lambda_{\text{QCD}}^2$, the corresponding polarized gluon structure function should satisfy $\int g_1^{Q\bar{Q}} dx = 0$. This sum rule is valid even if the gluon is not on its mass-shell, as long as its virtuality $|k^2| \sim \Lambda_{\text{QCD}}^2$ does not exceed m_Q^2 . We predict that $Q\bar{Q}$ pairs of like helicity (especially the cross section σ_{++++}) will make a contribution to $g_1^{Q\bar{Q}}$. By contrast, in $\gamma^* g$ annihilation into light quark-antiquark pairs, the virtuality of (confined) gluons unavoidably exceeds the light quark mass (i.e. $|k^2| \sim \Lambda_{\text{QCD}}^2 \gg m_{u,d}^2$) and one expects the integral of $\int g_1 dx$ to have a nonzero value, dictated by the U(1) chiral anomaly of QCD [5,7].

Appendix

$$\sigma_{+-+-} = \sigma_{+--+} = \sigma_0 \frac{x^2(Q^2(1-x)/x + k^2)^2}{8Q^4u} \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{v^2}{u} + \frac{3}{v^2u^5} \right) - \frac{2v}{u^2} - \frac{6}{vu^4} \right)$$

$$\sigma_{+--+} = \sigma_{+---} = \sigma_0 \frac{x^2(Q^2(1-x)/x + k^2)^2}{8Q^4u} \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{v^2}{u} - \frac{v^4}{u} - \frac{2v^2}{u^3} + \frac{2}{u^3} + \frac{3}{u^5} - \frac{3}{v^2u^5} \right) + \frac{2v^3}{u^2} - \frac{2v}{u^2} - \frac{6v}{u^4} + \frac{6}{vu^4} \right)$$

$$\begin{aligned} \sigma_{+-+-(++)} &= \sigma_{+---(++)} + \sigma_0 \frac{1}{8u} \left((1-x + \frac{xk^2}{Q^2}) \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{2v^2}{u} - \frac{2}{u} + (-)2v^3 - (+)2v + (-)\frac{2v}{u^2} - (+)\frac{2}{vu^2} - \frac{6}{u^3} \right. \right. \right. \\ &\quad \left. \left. + \frac{6}{v^2u^3} \right) + (-)\frac{4}{u} - (+)\frac{4v^2}{u} - \frac{12}{vu^2} + \frac{12v}{u^2} \right) + \left(\frac{x^2k^2}{Q^2} \right) \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(- (+)\frac{4v}{u^2} + (-)\frac{4}{vu^2} - \frac{4}{u^3} + \frac{4}{v^2u^3} + \frac{4}{u} - \frac{4v^2}{u} \right) \right. \\ &\quad \left. - \frac{8(1-v^2)}{vu^2} - (+)\frac{8(1-v^2)}{u(1-v^2u^2)} \right) + \ln\left(\frac{1+uv}{1-uv}\right) \left(- \frac{v^2}{u} + \frac{5}{u} - \frac{4}{v^2u} - (+)\frac{v}{u^2} + (-)\frac{1}{vu^2} - \frac{1}{u^3} + \frac{1}{v^2u^3} - (+)v + (-)\frac{1}{v} \right) \\ &\quad \left. + \frac{4(1-v^2)}{v(1-v^2u^2)} - \frac{2(1-v^2)}{vu^2} - 4v + \frac{4}{v} - (+)\frac{2(1-v^2)(1+u^2)}{u(1-v^2u^2)} \right) \end{aligned}$$

$$\begin{aligned} \sigma_{+-+-(+-)} &= \sigma_{+---(+-)} + \sigma_0 \frac{1}{8u} \left((1-x + \frac{xk^2}{Q^2}) \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(- \frac{2v^2}{u} + \frac{2}{u} - \frac{6}{v^2u^3} - \frac{2}{u^3} \right) + \frac{4v}{u^2} + \frac{12}{vu^2} \right) + \left(\frac{x^2k^2}{Q^2} \right) * \right. \\ &\quad \left(\ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{4}{u^3} - \frac{4}{v^2u^3} + (-)\frac{4v}{u^2} + (-)\frac{4}{vu^2} + \frac{4v^2}{u} - \frac{4}{u} \right) + \frac{8(1-v^2)}{vu^2(1-v^2u^2)} + \frac{8v(1+v^2)}{1-v^2u^2} - (+)\frac{8(1+v^2)}{u(1-v^2u^2)} \right) \\ &\quad \left. + \ln\left(\frac{1+uv}{1-uv}\right) \left(\frac{1}{u^3} - \frac{1}{v^2u^3} + (-)\frac{v}{u^2} + (-)\frac{1}{vu^2} + \frac{v^2}{u} - \frac{1}{u} + \frac{4}{v^2u} - (+)v - (+)\frac{1}{v} \right) - \frac{4}{v} + \frac{2(1-v^2)}{vu^2} - \frac{4(1-v^2)}{v(1-v^2u^2)} \right. \\ &\quad \left. + (-)\frac{2u(1+v^2)}{1-v^2u^2} - (+)\frac{2(1+v^2)}{u(1-v^2u^2)} \right) \end{aligned}$$

with $u = \sqrt{1 + \frac{4x^2k^2}{Q^2}}$. We have checked that the above results coincide, when summed over the lepton helicities, with those given in [4].

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Figure Captions:

Fig.1 Helicity cross sections for $\gamma^*(\lambda_1) + \gamma(\lambda_2) \rightarrow l^+(\lambda_3) + l^-(\lambda_4)$ in units
of $\sigma_0 = e^4/(2\pi(s + Q^2 - k^2))$ for $\frac{4m^2}{Q^2} = 10^{-6}$.

Fig.2 Plots of the functions $g_{1,\text{like}}$ and $g_{1,\text{unlike}}$ for $\frac{4m^2}{Q^2} = 10^{-6}$. The full
structure function is $g_1 = g_{1,\text{like}} + g_{1,\text{unlike}}$.

Fig.1

Fig. 2