Coherent interactions of axions with microwave photons in a resonant cavity to search for cosmic axions

S. Matsuki and I. Ogawa

*Nuclear Science Division, Institute for Chemical Research, Kyoto University
Uji, Kyoto 611, Japan*

and

K. Yamamoto

Department of Nuclear Engineering, Kyoto University, Kyoto 606-01, Japan

Abstract

A new scheme to search for cosmic axions is proposed, which utilizes coherent nature of the axions: The cosmic or galactic halo axions having a long de Broglie wavelength (over 100m) due to its small momentum coherently interact with artificially driven microwave photons in a resonant cavity, producing an interference term with low beat-frequency. By calculating the autocorrelation of the low frequency components of the output power picked up from the cavity, this coherent interference term is selectively enhanced while the incoherent background noise arising from the amplifier system is strongly suppressed. This would provide a much more efficient scheme to search for the cosmic axions than the conventional amplification-heterodyne-methods without driven photons in a cavity.

Search for the non-baryonic dark matter particles is one of the most challenging issues related to recent particle physics and cosmology [1]. The invisible axion is now one of the most attractive candidates for such non-baryonic dark matter particles among other candidates like the neutralinos and massive neutrinos. It is, however, quite difficult to detect the cosmic axions due to their extremely weak interactions with the ordinary matters, although a few tries were reported already [2, 3]. A promising way to detect the cosmic axions is to convert the axions into microwave photons in a resonant cavity under a strong magnetic field, as originally proposed by Sikivie [4]. In its original scheme, the microwave power in a cooled high- Q cavity (as high as 10^5) under a magnetic field (7T or so) is measured with a superheterodyne system after amplified by a cryogenic amplifier. The resonant frequency of the cavity is varied so as to cover the expected range of the axion mass.

Mostly a severe problem in this scheme is the background; one unavoidable source of the background is due to the blackbody radiations from the cavity wall. This background, however, can be reduced by cooling down the cavity; if the cavity is cooled down to say, 10mK with a dilution refrigerator, for example, then the mean number of photons at 2.4 GHz (10^{-5} eV) in the cavity becomes as small as 10^{-5} . This number is comparable or smaller than the expected mean number of the axion-converted photons in the cavity with conventional magnetic field strength and cavity volume. Another important source of the background is due to the inherent noise from the amplifier system; presently the best cryogenic amplifier has a noise temperature of about 4K, which is achieved by cooling the amplifier down to 2K – 4.2K [5]. Contrary to the thermal blackbody photon noise from the cavity mentioned above, this inherent noise of the amplifier, however, can not be reduced any more, even if the system is cooled down further. It is rather determined by a factor independent on the temperature of the system, so that the minimum noise limit arises below some temperature [5]. Thus, the ultimate background of this scheme will be placed by the noise of the cryogenic amplifier. Unfortunately, since the number of axion-converted photons in conventional circumstances is quite low (less than 1 photon per second), rather long measurement time and large cavity volume are required in order to achieve an appreciable signal to noise ratio (S/N).

In the present paper, we propose a new scheme to search for the cosmic axions, where the noise is separated very efficiently from the signal by utilizing the coherent nature of the cosmic axions. Then, the theoretical limit on the axion density comprising our

galactic dark halo could be checked out within a reasonable measuring time.

The principle of the method proposed here depends on the coherent nature of the cosmic axions: The axions are coherently produced in the early Universe when the global Peccei-Quinn symmetry $U(1)_{PQ}$ is spontaneously broken at a temperature $\sim f_a$ (axion decay constant), and then they acquire a mass $m_a \sim f_\pi m_\pi / f_a$ after the QCD phase transition takes place [6]. The initial coherency of the axions is quite high enough so that the axions are regarded truly as Bose condensate [6]. Although the coherently oscillating axion field is disturbed by the gravitational interactions, it would still maintain coherence to some extent in the present age. In our galactic halo, the axions are expected to be rotating around the center of our galaxy, and the velocity dispersion is estimated from the virial theorem to be about 300 km/sec, i.e. $\beta_a \sim 10^{-3}$, near the earth. Such axions have quite a long de Broglie wavelength due to their small momentum:

$$\lambda_a = h/k_a \simeq 120\text{m} \left(\frac{10^{-5}\text{eV}}{m_a} \right) \left(\frac{10^{-3}}{\beta_a} \right). \quad (1)$$

Then, the axion field may be considered to be coherent at least within a time scale $\sim \lambda_a/\beta_a$, during which the axions pass through the de Broglie wavelength; the coherence time will be about 10^{-4}sec if the axion mass lies in the range of 10^{-5}eV . The axion field is thus of quite different characteristics from those of the thermal blackbody photons and the amplifier noise which have no inherent nature of coherence. (Recently Sikivie and Ipser discussed the extent to which the phase space distribution of cold dark matter particles is thermalized in a galactic halo, and showed that there are large deviations from a thermal distribution in that the highest energy particles have discrete values of velocity [7]. If this is the case actually in our galactic halo, then the dark matter axions may have a much longer coherence time at these discrete velocities than estimated above.)

The present method utilizes the coherent interactions of the cosmic axions with the microwave photons in the cavity, which could produce a significant interference effect with low beat-frequency between the axion-converted photons and the microwave photons driven from an outer source. Such an interference effect, however, does not appear for the thermal blackbody photons from the cavity wall without coherent nature. Thus, it is sufficient in the present method to cool down the cavity to a temperature $\sim 4\text{K}$. This coherent interference term is quite large compared to the axion-converted photon intensity itself when the driven microwave field is appreciable. Then, by calculating the autocorrelation of the low frequency components of the output power picked up from the

cavity, we can selectively enhance the interference term while suppressing strongly the background arising from the incoherent amplifier noise. This is due to the fact that the incoherent amplifier noise has a correlation time much shorter than the coherence time of the axion field. Thus it would be possible to search for the cosmic axions with much better efficiency than the conventional amplification-heterodyne-methods without driven microwave photons [2, 3], as will be explained in detail in the followings.

The radiation field in a cavity with a resonant frequency ω_c is described quantum-mechanically in terms of creation and annihilation operators ($c^\dagger$ and c). The electric field operator is then given in the Schrödinger picture by

$$\mathbf{E}(\mathbf{x}) = \sqrt{\frac{\hbar\omega_c}{2\epsilon_0 V}} \left[\boldsymbol{\epsilon} f(\mathbf{x})c + \boldsymbol{\epsilon}^* f(\mathbf{x})^* c^\dagger \right], \quad (2)$$

where $\boldsymbol{\epsilon}$ is the polarization vector, and the wave function $f(\mathbf{x})$ is normalized by the condition $\int_V |f(\mathbf{x})|^2 d^3x = V$ (volume of the cavity). The time evolution of the system in the cavity at a finite temperature T_c is governed by the Schrödinger equation for the density operator:

$$\frac{d\rho}{dt} = \frac{1}{i\hbar} [H, \rho] + \Lambda\rho, \quad H = H_0 + \Delta H(t), \quad (3)$$

where

$$H_0 = \hbar\omega_c \left[c^\dagger c + \frac{1}{2} \right], \quad (4)$$

$$\Lambda\rho = -\frac{\omega_c}{2Q} [c^\dagger c\rho + \rho c^\dagger c - 2c\rho c^\dagger] - \frac{\beta(T_c)\omega_c}{2Q} [cc^\dagger\rho + \rho cc^\dagger - 2c^\dagger\rho c]. \quad (5)$$

The Liouvillian relaxation in the cavity is represented by the operator $\Lambda\rho$ in terms of the quality factor Q of the cavity and the Boltzmann factor

$$\beta(T_c) = e^{-\hbar\omega_c/k_B T_c} = \frac{\bar{n}(T_c)}{\bar{n}(T_c) + 1}, \quad (6)$$

where $\bar{n}(T_c)$ is the average number of thermal photons in the cavity [8].

The perturbation Hamiltonian $\Delta H(t)$, which may in general be time-dependent, is provided by the interactions with the external sources, the driven microwave power and the coherent axion field in the present situation. It is expressed as

$$\Delta H(t) = i\hbar [\zeta(t)c - \zeta^*(t)c^\dagger], \quad (7)$$

where $\zeta(t)$ is a c -number function of time. The perturbation term $\Delta_p H(t)$ caused by driving microwave power of frequency ω_p into the cavity is then given by a function

$$\zeta_p(t) = C e^{-i(\omega_p t + \delta_p)} . \quad (8)$$

The magnitude C is determined with the solution of eq.(3) for $T_c \rightarrow 0$, which will be derived later, so as to give $\text{Tr}(\rho c^\dagger c) = N_p$ for the number of photons stored in the cavity at the resonant frequency $\omega_p = \omega_c$:

$$C = \frac{\omega_c N_p^{1/2}}{2Q} \simeq 1\text{eV} \left(\frac{P}{10^2\text{W}} \right)^{1/2} \left(\frac{10^5}{Q} \right)^{1/2} , \quad (9)$$

where the relation $N_p = PQ/\omega_c^2$ is used in terms of the driven power P and the quality factor Q .

The effective coupling between the electric field $\mathbf{E}$ and the axion field ϕ under a time-independent strong magnetic field $\mathbf{B}_0$ is given by

$$\mathcal{L}_a = \epsilon_0 g_{a\gamma\gamma} \phi \mathbf{E} \cdot \mathbf{B}_0 , \quad (10)$$

where the permittivity of free space is included explicitly so that $g_{a\gamma\gamma}$ is given independently of ϵ_0 ; $g_{a\gamma\gamma} \sim (\alpha/\pi) f_a^{-1}$ with $m_a f_a \sim m_\pi f_\pi$ in the axion models [6]. Then, this axion-photon coupling provides a perturbation Hamiltonian to describe the emission and absorption of photons mediated by the coherent state of axion $|\eta_a(t)\rangle$:

$$\Delta_a H(t) = \langle \eta_a(t) | \{ - \int d^3x \mathcal{L}_a(t=0) \} | \eta_a(t) \rangle . \quad (11)$$

By using eqs.(2) and (10), it is calculated as

$$\Delta_a H(t) = i\hbar [\kappa z(t)c - \kappa z^*(t)c^\dagger] \quad (12)$$

with

$$\kappa = \epsilon_0 g_{a\gamma\gamma} B_0 \bar{\phi} \sqrt{\frac{\omega_c V}{2\hbar\epsilon_0}} , \quad (13)$$

$$z(t) = \frac{1}{iV} \int_V \left[\frac{\bar{\phi}(\mathbf{x}, t)}{\bar{\phi}} \right] \left[\frac{\boldsymbol{\epsilon} \cdot \mathbf{B}_0(\mathbf{x})}{B_0} \right] f(\mathbf{x}) d^3x , \quad (14)$$

$$\bar{\phi}(\mathbf{x}, t) = \langle \eta_a(t) | \phi(\mathbf{x}, 0) | \eta_a(t) \rangle , \quad (15)$$

where B_0 is the mean value of the applied magnetic field.

The mean value of the coherent axion field $\bar{\phi}(\mathbf{x}, t)$ is fixed by the axion energy density ρ_a as

$$\bar{\phi} = \rho_a^{1/2} / m_a . \quad (16)$$

Then, the small effective coupling κ is estimated for $\omega_c \simeq m_a$ from eq.(13) as

$$\kappa \simeq 2 \times 10^{-13} \text{eV} \left(\frac{g_{a\gamma\gamma}}{10^{-15} \text{GeV}^{-1}} \right) \left(\frac{10^{-5} \text{eV}}{m_a} \right)^{1/2} \left(\frac{\rho_a}{\rho_{\text{halo}}} \right)^{1/2} \left(\frac{B_0}{7T} \right) \left(\frac{V}{2000 \text{cm}^3} \right)^{1/2}, \quad (17)$$

where $\epsilon_0^{1/2} T = 1.95 \text{eV}^2$ is given by the experimental value of the Bohr magneton with $c = \hbar = 1$, and $\rho_{\text{halo}} \simeq 0.3 \text{GeVcm}^{-3}$ is the galactic dark halo density. This coupling should be compared with C for the driven power in eq.(9). The disturbance on the axion field due to the axion-photon conversion can be neglected here in a good approximation; such an effect contributes only to the next-to-leading order corrections $\sim \kappa^2$ in determining the radiation field in the cavity.

The time variation of the perturbation Hamiltonian $\Delta_a H(t)$ is represented by the function $z(t)$, whose magnitude is generally $|z(t)| \sim 1$ by definition for $|\bar{\phi}(\mathbf{x}, t)| \sim \bar{\phi}$, $|\mathbf{B}_0(\mathbf{x})| \sim B_0$ and $|f(\mathbf{x})| \sim 1$. It may be expressed in terms of the Fourier transform as

$$z(t) = \frac{T^{1/2}}{2\pi} \int_{-\infty}^{\infty} \tilde{z}(\omega) e^{-i\omega t} d\omega \quad (-T/2 \leq t \leq T/2), \quad (18)$$

$$\tilde{z}(\omega) = \frac{1}{T^{1/2}} \int_{-T/2}^{T/2} z(t) e^{i\omega t} dt, \quad (19)$$

where T is a large reference time. Since $z(t)$ is determined essentially by the coherent axion field in eq.(14), its power spectrum has the following form with peaks around $\omega \simeq \pm m_a$:

$$|\tilde{z}(\omega)| \sim (\pi/\Delta\omega_a)^{1/2} \quad (0 \leq |\omega| - m_a \lesssim 2\Delta\omega_a),$$

$$|\tilde{z}(\omega)| \approx 0 \quad (|\omega| - m_a > 2\Delta\omega_a), \quad |\tilde{z}(\omega)| = 0 \quad (|\omega| < m_a), \quad (20)$$

where the dispersion of the axion energy is given by

$$\Delta\omega_a \sim \frac{1}{2} \beta_a^2 m_a \quad (21)$$

for the small mean velocity $\beta_a \sim 10^{-3}$, and the peak value of $|\tilde{z}(\omega)|$ is estimated by using the Parseval formula for $|z(t)| \sim 1$.

The Schrödinger equation (3) can be solved by introducing a new set of creation and annihilation operators ($\tilde{c}^\dagger$ and $\tilde{c}$) which describe a Fock space constructed on a time-varying coherent state $|\eta(t)\rangle$ specified by a certain function $\eta(t)$:

$$\tilde{c}(\eta) = c - \eta(t), \quad [\tilde{c}(\eta), \tilde{c}^\dagger(\eta)] = [c, c^\dagger] = 1,$$

$$|\eta\rangle = \exp\left(-\frac{|\eta|^2}{2} + \eta c^\dagger\right) |0\rangle, \quad \tilde{c}(\eta)|\eta\rangle = (c - \eta)|\eta\rangle = 0,$$

$$|n, \eta\rangle = \frac{1}{\sqrt{n!}} (\tilde{c}^\dagger(\eta))^n |\eta\rangle . \quad (22)$$

The thermal ensemble may be taken for the density operator,

$$\rho = (1 - \beta) \sum_{n=0}^{\infty} \beta^n |n, \eta\rangle \langle n, \eta| , \quad (23)$$

so that

$$\langle \tilde{c}^\dagger \tilde{c} \rangle = \text{Tr}(\rho \tilde{c}^\dagger \tilde{c}) = \bar{n}(T_c) . \quad (24)$$

For this form of the density operator, the Liouvillian relaxation term is, in particular, expressed effectively as $\Lambda\rho = \frac{1}{i\hbar} [\Delta_\eta H, \rho]$ with $\Delta_\eta H = \frac{i\hbar\omega_c}{2(\bar{n}+1)Q} [\eta^* \tilde{c}(\eta) - \eta \tilde{c}^\dagger(\eta)]$. Then, the Schrödinger equation (3) is reduced to a master equation to determine the coherent state of radiation realized in the cavity:

$$\frac{d\eta(t)}{dt} = -K\eta(t) + \zeta_p(t) + \kappa z(t) \quad [K = i\omega_c + \frac{\omega_c}{2Q}] , \quad (25)$$

where $\zeta(t) = \zeta_p(t) + \kappa z(t)$ is given for $\Delta H(t)$.

The linear differential equation (25) with inhomogeneous terms is readily solved by the usual method. Then, by using eqs.(8) and (18) the coherent state of photons in the cavity is determined as

$$\eta(t) \approx \frac{C e^{-i(\omega_p t + \delta_p)}}{i(\omega_c - \omega_p) + (\omega_c/2Q)} + \frac{\kappa T^{1/2}}{2\pi} \int_{-\infty}^{\infty} \frac{\tilde{z}(\omega) e^{-i\omega t}}{i(\omega_c - \omega) + (\omega_c/2Q)} d\omega . \quad (26)$$

Here the term $\sim e^{-Kt} = e^{-\omega_c t/2Q} e^{-i\omega_c t}$ to specify the initial condition is neglected, since it diminishes rapidly for the time scale $(\omega_c/Q)^{-1} \sim 10^{-5} \text{sec} (10^{-5} \text{eV}/\omega_c)(Q/10^5)$. The classical electric field is given with this solution of $\eta(t)$ as

$$\bar{\mathbf{E}}(\mathbf{x}, t) = \text{Tr}[\rho(t) \mathbf{E}(\mathbf{x})] = \epsilon \sqrt{\frac{\hbar\omega_c}{2\epsilon_0 V}} [f(\mathbf{x})\eta(t) + f(\mathbf{x})^* \eta^*(t)] , \quad (27)$$

where the real polarization vector ϵ with $\epsilon \cdot \epsilon = 1$ is taken for definiteness. Then, the square of the electric field at a certain point $\mathbf{x}_0$ ($f(\mathbf{x}_0) \neq 0$) is observed as

$$\begin{aligned} G(t) &= \text{Tr}[\rho(t) : \mathbf{E}(\mathbf{x}_0) \cdot \mathbf{E}(\mathbf{x}_0) :] \\ &= \bar{E}_0^2 [\bar{\eta}(t)^2 + \bar{n}(T_c)] \end{aligned} \quad (28)$$

with a real function

$$\bar{\eta}(t) = \frac{1}{\sqrt{2}} [\eta(t) e^{-i\delta_0} + \eta^*(t) e^{i\delta_0}] , \quad (29)$$

where

$$\bar{E}_0 \equiv \sqrt{\frac{\hbar\omega_c}{\epsilon_0 V}} |f(\mathbf{x}_0)|, \quad f(\mathbf{x}_0)^2 \equiv |f(\mathbf{x}_0)|^2 e^{-2i\delta_0}. \quad (30)$$

In eq.(28), the symbol $::$ denotes the normal-order product, and the relations $\langle \tilde{c} \rangle = \langle \tilde{c}\tilde{c} \rangle = 0$ and $\langle c^\dagger c \rangle = \langle \tilde{c}^\dagger \tilde{c} \rangle + |\eta(t)|^2$ with eq.(24) are taken into account.

It is practically convenient in the followings to tune the driven microwave frequency as $\omega_p = \omega_c$. Then, by using eqs.(26) and (29) with C given already in eq.(9), $G(t)$ is calculated as

$$\begin{aligned} G(t)/\bar{E}_0^2 &\simeq 2N_p \cos^2(\omega_c t + \delta_p + \delta_0) + \bar{n}(T_c) \\ &+ \frac{\kappa N_p^{1/2} T^{1/2}}{2\pi} \int_{-\infty}^{\infty} [\tilde{g}_1(\omega) + \tilde{g}_2(\omega)] e^{-i\omega t} d\omega, \end{aligned} \quad (31)$$

where

$$\begin{aligned} \tilde{g}_1(\omega) &= \frac{e^{i\delta_p} \tilde{z}(\omega + \omega_c) + e^{-i\delta_p} \tilde{z}^*(-\omega + \omega_c)}{-i\omega + (\omega_c/2Q)}, \\ \tilde{g}_2(\omega) &= \frac{e^{-i(\delta_p + 2\delta_0)} \tilde{z}(\omega - \omega_c)}{-i(\omega - 2\omega_c) + (\omega_c/2Q)} + \frac{e^{i(\delta_p + 2\delta_0)} \tilde{z}^*(-\omega - \omega_c)}{-i(\omega + 2\omega_c) + (\omega_c/2Q)}, \end{aligned} \quad (32)$$

and $|\tilde{g}_{1,2}(\omega)| = |\tilde{g}_{1,2}(-\omega)|$ due to the reality of $G(t)$. The tiny contribution $\sim \kappa^2$ can be neglected in eq.(31). The contributions given by $\tilde{g}_1(\omega)$ and $\tilde{g}_2(\omega)$ stem from the interference between the driven microwave photons and the axion-converted photons. Contrarily, no such an interference term appears for the thermal blackbody photons without coherent nature. By noting the behavior of $\tilde{z}(\omega)$ in eq.(20), it is found that $|\tilde{g}_1(\omega)|$ and $|\tilde{g}_2(\omega)|$ have peaks around

$$|\omega| \simeq \omega_{\pm} \equiv |m_a + \Delta\omega_a \pm \omega_c|. \quad (33)$$

In particular, when the frequency of the cavity is tuned to be close enough to the axion mass, the peak of $|\tilde{g}_1(\omega)|$ around the low beat-frequency ω_- is resonantly enhanced, as seen from eq.(32):

$$|\tilde{g}_1(\omega)| \sim \frac{Q(\pi/\Delta\omega_a)^{1/2}}{\omega_c} \quad (|\omega| \simeq \omega_- \lesssim \omega_c/2Q). \quad (34)$$

The experimental procedure to observe this interference effect with the low beat-frequency is shown schematically in fig.1: The microwave from a synthesized signal generator is fed to a high- Q cavity cooled down to the liquid He temperature in a strong magnetic field. The electric (magnetic) field in the cavity picked up by a coupling capacitor (loop coil) is squared by a diode or a mixer after amplified. The output from the

diode (mixer) is passed through a low-frequency band-pass filter amplifier to select the components in a range $\omega_{\min} \leq |\omega| \leq \omega_{\max}$ with

$$0 < \omega_{\min} < \omega_c/2Q \ll \omega_{\max} \ll \omega_c . \quad (35)$$

This range of the filter is chosen so as to eliminate the irrelevant frequency components of the output coming from the mixer. Then, if the frequency of the cavity is chosen to meet the condition $\omega_{\min} < \omega_- < \omega_c/2Q$, only the resonantly enhanced $\tilde{g}_1$ -term among the contributions in eq.(31) is passed through the filter. The low-frequency components passed through the filter are measured for a certain time interval with a fast analog-to-digital converter (ADC). The resultant time spectra stored in the memory are used to calculate the autocorrelation function by computer. In this autocorrelation function, the contribution of the incoherent amplifier noise is localized in a small region around the time lag $\tau = 0$, as will be explained later in detail. Thus, the autocorrelation function can be Fourier analyzed by cutting the vicinity of $\tau = 0$, from which the power spectrum of the axion field could be observed. Since the coherence time $\sim \Delta\omega_a^{-1}$ of the axion field is much longer than the correlation time of the incoherent amplifier noise, this cut-off reduces substantially the noise contribution while almost unaffected the signal of the axion-converted photons.

The above procedure is repeated by varying the frequency ω_c with a tuning rod in the cavity while matching the driven microwave frequency ω_p to ω_c . The frequency step to be varied for a given octave in frequency from ω_{c0} to $2\omega_{c0}$ should be taken as

$$\Delta\omega_{c0} \equiv r \frac{\omega_{c0}}{2Q} < \frac{\omega_{c0}}{2Q} - \omega_{\min} \quad (r < 1) , \quad (36)$$

so that if the axion mass m_a lies in this octave the following resonant condition is satisfied at some scanning step:

$$\omega_{\min} < \omega_- < \omega_{\min} + \Delta\omega_{c0} < \omega_{c0}/2Q . \quad (37)$$

The measurement time T with the ADC should be sufficiently longer than the axion coherence time, e.g. $\Delta\omega_a^{-1} \sim 10^{-4}\text{sec}$ for $m_a \sim 10^{-5}\text{eV}$ and $\beta_a \sim 10^{-3}$. Then, the time required to scan an octave in frequency may be estimated as

$$T_{\text{oct}} = \frac{\omega_{c0}}{\Delta\omega_{c0}} T = 10^5 \text{sec} \left(\frac{Q}{10^5} \right) \left(\frac{T}{10^{-1}\text{sec}} \right) \left(\frac{0.2}{r} \right) . \quad (38)$$

It should also be noted that the time step available in the current ADC is $\Delta t \sim 10^{-6}$ sec. This time step will be small enough to measure the time spectrum of the low beat-frequency interference effect, since its intrinsic time scale $\omega_{\min}^{-1}$ becomes as long as 10^{-5} sec by taking $\omega_{\min} = 10$ kHz for the filter. (This value of $\omega_{\min}$ will be required, since the noise is not considered to be “white” for the frequency below about 10kHz.)

As mentioned in the above, the incoherent amplifier noise can be separated efficiently from the signal of the axion-converted photons by calculating the autocorrelation function for the output $u(t)$ passed through the band-pass amplifier:

$$\psi(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} u(t)u(t+\tau)dt . \quad (39)$$

The electric field in the cavity is picked up as voltage $d\epsilon \cdot \mathbf{E}(\mathbf{x}_0)$ by the coupling capacitor of size d (1cm typically). Thus, the output through the mixer is given by $d^2G(t)$, if the noise contribution is not considered. It is then passed through the filter so that for the desired case of eq.(37) only the resonantly enhanced interference term is left as the signal:

$$u_s(t) \simeq \kappa N_p^{1/2} \bar{V}_0^2 \frac{T^{1/2}}{2\pi} \int_{-\infty}^{\infty} \tilde{g}_1(\omega) e^{-i\omega t} d\omega , \quad (40)$$

where $\bar{V}_0 = d\bar{E}_0$. By using the Wiener-Khintchine relation, the autocorrelation of this signal can be calculated in terms of the power spectrum as

$$\psi_s(\tau) \simeq \frac{\kappa^2 N_p \bar{V}_0^4}{2\pi} \int_{-\infty}^{\infty} |\tilde{g}_1(\omega)|^2 e^{-i\omega\tau} d\omega \sim \psi_s(0) e^{-\Delta\omega_a |\tau|} \cos(\omega_- \tau) , \quad (41)$$

where for $m_a \lesssim 10^{-4}$ eV and $\beta_a \sim 10^{-3}$ the coherence time $\Delta\omega_a^{-1}$ of the axion field is longer than the beat preperiod $\omega_{\min}^{-1} \sim \omega_{\min}^{-1} \sim 10^{-5}$ sec. By using eqs.(13), (20), (30), (32) and (34), the magnitude is estimated as

$$\psi_s(0) \sim \left(\frac{N_p \bar{V}_0^2}{\epsilon_0} \right) \cdot \frac{1}{2} d^2 Q^2 (\epsilon_0^{1/2} B_0)^2 \rho_a \left(\frac{g_a \gamma \gamma}{m_a} \right)^2 . \quad (42)$$

On the other hand, the incoherent white noise, which mainly comes from the first stage amplifier at the cavity output, causes additional voltage,

$$v_n(t) = \frac{T^{1/2}}{2\pi} \int_{-\infty}^{\infty} \tilde{v}_n(\omega) e^{-i\omega t} d\omega , \quad |\tilde{v}_n(\omega)| \simeq (w_n/\epsilon_0)^{1/2} , \quad (43)$$

where w_n is the noise power per Hz. After passed through the mixer, this noise contribution is multiplied by the voltage, $dG(t)^{1/2} \simeq (2N_p)^{1/2} \cos(\omega_c + \delta_p + \delta_0)$, provided by the microwave in the cavity:

$$u_n(t) \simeq (2N_p)^{1/2} \bar{V}_0 \mathcal{W}_{\text{filter}} [2v_n(t) \cos(\omega_c t + \delta_p + \delta_0)] , \quad (44)$$

where $\mathcal{W}_{\text{filter}}$ represents the deformation made by the filter of eq.(35). (The contribution of $v_n(t)^2$ may be neglected here in a good approximation, since the noise power w_n is small enough.) Then, the autocorrelation of this output due to the amplifier noise will damp in a short time scale reciprocal to the band width $\simeq \omega_{\text{max}}$ of the filter [9] as

$$\psi_n(\tau) \sim \psi_n(0)e^{-\omega_{\text{max}}|\tau|} \cos(\omega_{\text{max}}\tau/2). \quad (45)$$

Thus, if the band width of the filter is taken as $\omega_{\text{max}} \sim 10^{-9}\text{eV}$ ($\sim 100\text{kHz}$), this noise component $\psi_n(\tau)$ becomes negligible within the coherence time of the axion field $\Delta\omega_a^{-1} \gtrsim 10^{-5}\text{sec}$ for $m_a \lesssim 10^{-4}\text{eV}$, so that the noise is separated very efficiently from the signal in the autocorrelation function. The magnitude $\psi_n(0)$ is estimated in terms of the power spectrum of $u_n(t)$ given by $|\tilde{u}_n(\omega)|^2 \sim 8N_p \bar{V}_0^2 w_n / \epsilon_0$ for the frequency range $\omega_{\text{min}} \lesssim |\omega| \lesssim \omega_{\text{max}}$:

$$\psi_n(0) \sim \frac{1}{\pi} \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} |\tilde{u}_n(\omega)|^2 d\omega \sim \left(\frac{N_p \bar{V}_0^2}{\epsilon_0} \right) \cdot \frac{8w_n \omega_{\text{max}}}{\pi}. \quad (46)$$

In practice, this behavior of the noise component can be monitored experimentally. The correlation between the signal and noise will also be vanishing, since these processes are considered to be independent. These features of the signal and noise components in the autocorrelation function are schematically shown in fig.2.

The S/N ratio in the autocorrelation is estimated from eqs.(42) and (46) for the case of Peccei-Quinn models with the relations $g_{a\gamma\gamma} \sim (\alpha/\pi)f_a^{-1}$ and $m_a f_a \sim m_\pi f_\pi$:

$$\frac{\psi_s(0)}{\psi_n(0)} \sim 2 \times 10^{-6} \left(\frac{d}{1\text{cm}} \right)^2 \left(\frac{Q}{10^5} \right)^2 \left(\frac{10^{-9}\text{eV}}{\omega_{\text{max}}} \right) \left(\frac{4\text{K}}{w_n} \right) \left(\frac{B_0}{7\text{T}} \right)^2 \left(\frac{\rho_a}{\rho_{\text{halo}}} \right). \quad (47)$$

(The enhancement factors provided by the amplifiers cancel in this ratio, since the signal and noise are amplified in the same way.) It is here interesting to see that this S/N ratio does not depend on the cavity volume which is typically $\sim 10^3\text{cm}^3$. This will be favorable to search for the higher axion mass region of $m_a \sim 10^{-4}$. The corresponding S/N ratio for the mean magnitudes $\bar{u}_s$ and $\bar{u}_n$ of the signal and noise contributions, respectively, in the output amplitude $u(t)$ is thus given for the typical values of the parameters taken in eq.(47) by

$$\frac{\bar{u}_s}{\bar{u}_n} = \left[\frac{\psi_s(0)}{\psi_n(0)} \right]^{1/2} \sim 10^{-3}. \quad (48)$$

Since the maximum number of the scale units of the ADC to measure the time spectra is typically 12 bits = 4096, this S/N ratio is large enough for the signal not to be rounded off

by the computer calculation. In fact, the intrinsic noise $\sim (2^{12})^{-2} \sim 10^{-7}$ in the relative magnitude is observed in calculating the autocorrelation, which would originate from the computational errors.

In conclusion, we have shown that the coherent interactions of the cosmic axions with driven microwave photons in a resonant cavity could produce a significant interference term with low beat-frequency, while no such interference appears for the thermal black-body photons from the cavity wall. This interference term is much more stronger in proportion to the square root of the intensity of the driven microwave than the original intensity of the axion-converted photons. By calculating the autocorrelation of the low frequency components of the output power picked up from the cavity, the noise component is separated very efficiently from the signal component of the interference term; the noise component with very short correlation time is localized near the zero time lag, while the signal component lasts much longer during the coherence time of the axion field. Then, by cutting out the part near zero time lag, where the noise component mainly contributes, the autocorrelation function is Fourier analyzed, resulting the frequency power spectrum of the interference peak with much better signal-to-noise ratio. This would provide quite an efficient scheme to search for the cosmic axions, with which the theoretical limit on the axion density comprising our galactic dark halo could be checked for reasonable measuring time (~ 1 day to scan an octave in frequency), cavity volume ($\sim 10^3 \text{cm}^3$) and temperature of the system ($\sim 4\text{K}$). This method is applicable up to a higher axion mass region $\sim 10^{-4} \text{eV}$, as long as the coherence time of the axions is longer than the time step $\sim 1 \mu\text{sec}$ of the ADC. Furthermore, this method would clarify the intrinsic nature of coherence of the axions in the present age, if real signals of the axions are found experimentally, thus providing an important way to investigate the characteristics of the axions in the early Universe.

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Figure Captions

Figure 1. Schematic diagram of the present experimental system. The time spectra of the low-frequency interference peak once stored in a computer memory are analyzed by calculating the autocorrelation function. Then, by cutting out the part near zero time lag, where the noise component mainly contributes, the autocorrelation function is Fourier analyzed, resulting the frequency power spectrum of the interference peak with much better signal-to-noise ratio.

Figure 2. Schematic features of the signal and white noise components in the autocorrelation function. The noise component with very short correlation time is localized near the zero time lag, while the signal component lasts much longer during the coherence time of the axion field.

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Fig. 2.
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