Cosmology with a master coupling in flipped SU(5) × U(1): The λ₆ universe

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**A B S T R A C T**

We propose a complete cosmological scenario based on a flipped SU(5) × U(1) GUT model that incorporates Starobinsky-like inflation, taking the subsequent cosmological evolution carefully into account. A single master coupling, λ₆, connects the singlet, GUT Higgs and matter fields, controlling 1) inflaton decays and reheating, 2) the gravitino production rate and therefore the non-thermal abundance of the supersymmetric cold dark matter particle, 3) neutrino masses and 4) the baryon asymmetry of the Universe.

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It is common lore that the Universe may have been in a symmetric state soon after the Big Bang, but its subsequent evolution to the present-day universe with its content of matter, dark matter and neutrinos remains problematic. Typical grand unified theory (GUT) models require many seemingly unrelated couplings to explain various physical observables. In this Letter we develop a complete cosmological scenario based on a detailed flipped SU(5) × U(1) GUT model [1,2] incorporating Starobinsky-like inflation [3], and relate a host of cosmological observables through a single master coupling, denoted by λ₆.

In addition to quark, lepton and Higgs fields, the model contains four gauge singlets that drive inflation, provide a µ-term for the mixing of the electroweak Higgs doublets, and a seesaw mechanism [4,5] for neutrino masses. Among the superpotential couplings of the singlet fields there is one that couples the singlet, GUT Higgs fields and matter, denoted by λ₆. Remarkably this one coupling controls 1) inflaton decays and therefore the reheating temperature, 2) the gravitino production rate and therefore the non-thermal abundance of the lightest supersymmetric particle (LSP) that is a candidate for cold dark matter, 3) neutrino masses, and 4) the baryon asymmetry of the Universe through leptogenesis [6]. This Letter explores the deep correlations between these apparently disparate quantities that are all related by the master coupling λ₆—the λ₆ Universe.

In the flipped SU(5) × U(1) GUT [7–9] motivated by string theory [10], all of the Standard Model (SM) matter fields, as well as right-handed neutrinos, are embedded in three generations of ¹⁰ (1), ⁵ (−3), and ¹ (5) representations of SU(5), which are denoted by F₁, F₂, and ℓ₁, respectively, where the numbers in the parentheses show the U(1) charges in units of 1/√40 and i = 1, 2, 3 are generation indices. The representation assignments of the right-handed quarks and leptons are “flipped” with respect to those in standard SU(5). The minimal supersymmetric standard model (MSSM) Higgs fields H₂ and H₁ are in ⁵ (−2) and ⁵ (2) representations, denoted by h and H, respectively. The GUT gauge group is broken into the SM gauge group by ¹⁰ (1) and ¹⁰ (−1) Higgs representations of SU(5), which are denoted by H and H, respectively. The four singlet chiral multiplets are denoted φ₀ (a = 0, . . . , 3), and we assume that the inflaton can be identified with one of these, which we denote by φ₀.

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The superpotential of this model [1] is given by
\[
W = \lambda_1 F_i F_j h + \lambda_2 F_i \tilde{F}_j \tilde{h} + \lambda_3 F_i \tilde{F}_j \phi_u + \lambda_4 h_1 \phi_u \phi_d + \lambda_5 \phi_u \phi_d \phi_e + \mu^a b \phi_u \phi_d + \mu^a b \phi_u \phi_d,
\]
where we impose a $Z_2$ symmetry: $H \leftrightarrow -H$, which forbids the mixing between the SM matter fields, Higgs color triplets, and the Higgs doublets, and suppresses the supersymmetric mass term for $H$ and $\tilde{H}$. Owing to the absence of these terms, rapid proton decay due to colored Higgs exchange is avoided. In addition, the doublet-triplet splitting problem is solved by the missing-partner mechanism [9,11]. Without loss of generality, we take $\lambda_{12} \equiv \lambda_{12} \equiv 0$ and $\mu_{ab} \equiv \mu_{ab} \equiv 0$ to be real and diagonal in what follows. Down-quark, up-quark and charged-lepton masses are related to the $\lambda_{1,2,3}$ couplings, respectively, and (neglecting renormalization group effects for simplicity) $\lambda_2 \equiv \text{diag}(m_u, m_c, m_t)/(|h|^2)$, where $M_P \equiv (8\pi G_N)^{-1/2}$ is the reduced Planck mass, the asymptotically-flat Starobinsky-like potential is realized for the inflation field $\phi_0$ [14]. The value $m_t \simeq 3 \times 10^{13}$ GeV reproduces the measured value of the primordial power spectrum amplitude [11].

The inflaton $\phi_0$ couples directly to the fields $F_i$ via the couplings $\lambda_{10}^i$, which play a central role in our analysis. Two other singlet fields, $\phi_1$ and $\phi_2$, also couple to $F_i$. The remaining singlet field does not couple to $F_i$, and develops a supersymmetry breaking scale VEV which violates a $\mu$ mixing term for the MSSM Higgs doublets. We assume that $\lambda_{10}^i = 0$ \((a = 0, 1, 2)\), so as to suppress $R$-parity violation. This setup was introduced in Ref. [1], where it was called “Scenario B”. In this case, $R$-parity is violated in the singlet sector, which is sufficiently sequestered from the observable sector that the LSP has a lifetime much longer than the age of the Universe [2].

A general challenge in supersymmetric GUTs is the presence of multiple degenerate vacua [15,16]. While inflation might have left the Universe in the correct vacuum state, one should follow the dynamic evolution of the universe, showing that the GUT phase transition occurred. Finite-temperature effects break the vacuum degeneracy through differences in the numbers of degrees of freedom associated with the different phases [15–17]. Although the global minimum generally lies in the symmetric state at temperatures of order the GUT scale, a GUT like SU(5) confines at lower temperatures $T \sim 10^{10}$ GeV. This raises the GUT-symmetric vacuum energy, and opens the way towards successful cosmological evolution.

GUT symmetry breaking in our model occurs along one of the $F$- and $D$-flat directions in the scalar potential: a linear combination of $v_1$ and $v_2$, which are the SM singlet components of $H$ and $\tilde{H}$, respectively. We denote this combination by $\Phi$, and call it the flaton. Once $\Phi$ acquires a VEV, the SU(5) × U(1) GUT gauge group is broken into the SM gauge group. The thirteen Nambu-Goldstone chiral multiplets in $H$ and $\tilde{H}$ are absorbed by gauge multiplets, and the other six physical components are combined with the triplet components in $h$ and $\tilde{h}$ to make them massive. The flat direction can be lifted by non-renormalizable superpotential terms, e.g., of the form $W_{\text{NR}} \simeq (H \tilde{H})^2/M_P^2$. The flaton and flatino then obtain masses of order the supersymmetry-breaking scale.

We focus on the portion of parameter space where the strong reheating scenario discussed in Ref. [2] is realized. As shown in Ref. [2], in this case the GUT symmetry is unbroken at the end of inflation. We further assume that the system remains in the unbroken phase during reheating, as is confirmed in the following analysis. The phase transition is triggered by the difference in the number of light degrees of freedom, $g$, between the broken and unbroken phases [1,2,15–17]. Massless superfields provide a thermal correction to the effective potential of $-g T^2 \phi^4/90$, where $T$ denotes the temperature of the Universe. Since the number of light degrees of freedom in the unbroken phase ($g = 103$) is larger than that in the Higgs phase ($g = 62$), $\Phi$ is kept at the origin at high temperatures. However, once the temperature drops below the confinement scale of the SU(5) gauge theory, $\Lambda_c$, the number of light degrees of freedom significantly decreases ($g \leq 25$), and thus the Higgs phase becomes energetically favored [1]. We have found that in this strong reheating scenario the incoherent component of the flaton drives the phase transition if $\Lambda_c \gtrsim 20 (m_{\phi} M_{\text{GUT}})^{1/2}$ [2], where $m_{\phi}$ and $M_{\text{GUT}}$ are the flaton mass and the GUT scale, respectively. For $m_{\phi} = 10^4$ GeV and $M_{\text{GUT}} = 10^{16}$ GeV, the above condition leads to $\Lambda_c \gtrsim 2.3 \times 10^{16}$ GeV.

In the case of such strong reheating, the flaton decouples from the thermal bath, and when $T \lesssim m_{\phi}$, it becomes non-relativistic and eventually dominates the energy density of the Universe until it decays. The decay of the flaton generates a second period of reheating. The amount of entropy released by the flaton decay is estimated to be
\[
\Delta \simeq 1.6 \times 10^4 \lambda_{2,3,7}^{-2} \left( \frac{M_{\text{GUT}}}{10^{16} \text{ GeV}} \right) \left( \frac{10 \text{ TeV}}{m_{\text{soft}}/m_{\phi}} \right)^{1/2},
\]
where $m_{\text{soft}}$ stands for the typical value of fermion masses. It was shown in Ref. [2] that $|\lambda_{0}^a| \gtrsim O(10^{-4})$, reheating is completed in the symmetric phase via the dominant inflaton decay channel $\phi_0 \rightarrow F_i H$. The reheating temperature in this case is given by
\[
T_{\text{reh}} \simeq 5.4 \times 10^{14} \text{GeV} \sqrt{10 \sum_i |\lambda_{0}^i|^2},
\]
indicating a direct relation between $T_{\text{reh}}$ and $\lambda_{6}$. During reheating, gravitinos are produced via the scattering/decay of particles in the thermal bath [18–38]. For the calculation of the gravitino production rate, we use the formalism outlined in [36], but using the group theoretical factors and couplings appropriate to flipped SU(5)×U(1).

These gravitinos eventually decay into LSPs, and the resultant “non-thermal” contribution to the LSP abundance is given by
\[
\Omega_{\text{DM}} h^2 \simeq 0.12 \left( \frac{1.6 \times 10^4}{\Delta} \right) \left( \frac{m_{\text{LSP}}}{1 \text{ TeV}} \right) \left( \frac{\sqrt{\sum_i |\lambda_{0}^i|^2}}{0.0097} \right)
= 0.12 \left( \frac{1.6 \times 10^4}{\Delta} \right) \left( \frac{m_{\text{LSP}}}{1 \text{ TeV}} \right) \left( \frac{T_{\text{reh}}}{1.6 \times 10^{13} \text{GeV}} \right).\]

The total dark matter abundance is obtained by adding this non-thermal component to the thermal relic density of the LSP, which is reduced by a dilution factor $\Delta$. Thus the LSP relic density is also directly related to $\lambda_{6}$.

The neutrino mass structure in this model was studied in Refs. [1,2]. As we noted above, only three singlet fields, including the inflaton, couple to the neutrino sector. The masses of the heavy states are approximately $(m_{\mu}, m_{\tau}, m_{\mu}/2)$, and the mass matrix of the left-handed neutrinos is obtained from a first seesaw mechanism:
(m_v)_{ij} = \sum_{a=0,1,2} \frac{\lambda_{\alpha}^a \lambda_{\beta}^a \langle \bar{\nu}_a | \bar{U}_v | \nu_i \rangle^2}{\mu_a^2}, \quad (5)

where we take \((\bar{\nu}_a^\nu) = 10^{16}\) GeV in this paper. We diagonalize the mass matrix \((5)\) by a unitary matrix \(U_{v}: m_v^D_{ij} = U_v^\dagger m_v U_v\). The light neutrino mass matrix is then obtained through a second seesaw mechanism \(4,5\):

\[(m_v)_{ij} = \sum_k \frac{\lambda_{\alpha}^a \lambda_{\beta}^a (U_v)_{ik}(U_v^\dagger)_{jk}(\bar{\nu}_a^\nu)^2}{(m_v^D)_{jk}}. \quad (6)\]

This mass matrix is diagonalized by a unitary matrix \(U_v\) as \(m_v^D = U_v^* m_v U_v^\dagger\). We note that, given a matrix \(\lambda_{\alpha}^a\), the mass eigenvalues of \(m_v\) are uniquely determined as functions of \(\mu_1\) and \(\mu_2\) via Eqs. \((5)\) and \((6)\).

On the other hand, as discussed in Ref. \([39]\), the PMNS matrix differs from \(U_v\) by an additional factor of a unitary matrix \(U_{PMNS} = U U_v^\dagger\). This prevents us from predicting the PMNS matrix in this framework. We note, however, that we can instead use this equation to determine \(U_{PMNS} (\text{given } U_v)\). It was found in \([39]\) that the matrix \(U_{PMNS}\) affects the ratios between proton decay channels; for instance, \(\Gamma(p \rightarrow \mu \pi^0 \pi^0) / \Gamma(p \rightarrow e^+ \pi^0 \pi^0) \equiv |(U_{PMNS}^\dagger)(U)|^2/\langle |(U)|^2 \rangle\), which is in general different from the ratio predicted in an ordinary SU(5) GUT. A more detailed discussion of proton decay will be given in a forthcoming paper \([40]\).

As can be seen from Eq. \((5)\), right-handed neutrinos become massive after \(H\) develops a VEV. In the strong reheating scenario, therefore, right-handed neutrinos are massless and in thermal equilibrium right after the reheating is completed. They become massive and drop out of equilibrium almost instantaneously at the time of the GUT phase transition and eventually decay non-thermally \([2,16]\) to generate a lepton asymmetry \([6]\). The lepton asymmetry is then converted to a baryon asymmetry via the sphaleron process \([41]\). The resultant baryon number density is given by

\[\frac{n_B}{s} = \frac{28}{79} \frac{135 \zeta(3)}{4 \pi^4 k_{\text{reh}} \Delta} \sum_{i=1,2,3} \epsilon_i, \quad (7)\]

where \([2,39]\)

\[\epsilon_i = \frac{1}{2\pi} \sum_{\mu \neq i} \text{Im} \left[ \frac{\langle U_v^\dagger (\lambda_{\alpha}^\mu)^2 \lambda_{\beta}^\mu \rangle^2}{\langle U_v^\dagger (\lambda_{\alpha}^\nu)^2 \lambda_{\beta}^\nu \rangle^2} \right] g \left( \frac{m_v^2}{m_v^2} \right). \quad (8)\]

with \([42]\)

\[g(x) = -\sqrt{x} \left[ \frac{2}{x - 1} + \ln \left( \frac{1 + x}{x} \right) \right]. \quad (9)\]

It is important to note that the sign in \((7)\) is fixed: in order to obtain \(n_B/s > 0\), we must require \(\epsilon_i < 0\).

As we see in Eqs. \((3)\) and \((4)\), the coupling \(\lambda_{\alpha}\) determines the reheating temperature and the non-thermal component of the dark matter abundance. This coupling also controls the neutrino mass and baryon asymmetry through the right-handed neutrino mass matrix in Eq. \((5)\).

We now investigate numerically the effect of the \(\lambda_{\alpha}\) coupling on these physical observables. To this end, we perform a parameter scan of \(\lambda_{\alpha}\). We first write it in the form \(\lambda_{\alpha} = r_{\alpha} M_\alpha\), where \(r_{\alpha}\) is a real constant and \(M_\alpha\) is a complex \(3 \times 3\) matrix. We then scan \(r_{\alpha}\) logarithmically over the range \((0.1, 1)\) choosing a total of 2000 values. For each value of \(r_{\alpha}\), we generate 2000 random complex \(3 \times 3\) matrices \(M_\alpha\) with each component taking a value of \(O(1)\).

For each value of \(\lambda_{\alpha}\), we obtain the mass eigenvalues of light neutrinos as functions of \(\mu_1\) and \(\mu_2\) as described above. We then determine these two \(\mu\) parameters by requiring that the observed values of the squared mass differences are reproduced; namely, for the normal ordering (NO) case, \(m_2 - m_1 = \Delta m^2_{21} = 7.39 \times 10^{-5} \text{ eV}^2\) and \(m_2 - m_3 = \Delta m^2_{31} = 2.525 \times 10^{-3} \text{ eV}^2\), and for the inverted ordering (IO) case, \(m_3 - m_1 = 7.39 \times 10^{-5} \text{ eV}^2\) and \(m_3 - m_2 = \Delta m^2_{32} = -2.512 \times 10^{-3} \text{ eV}^2\) \([43]\).

We generate the same number of \(\lambda_{\alpha}\) matrices for each mass ordering, and find solutions for 9839 and 730 matrix choices for the NO and IO cases, respectively, out of a total of \(4 \times 10^6\) models sampled. This difference indicates that the NO case is favored in our model. We find that the lightest neutrino mass is \(\lesssim 10^{-5}\) eV in both cases. In the case of NO, the heavier neutrinos have masses \(\simeq \sqrt{\Delta m^2_{21}} = 8.6 \times 10^{-3}\) eV and \(\simeq \sqrt{\Delta m^2_{31}} = 5.0 \times 10^{-2}\) eV. In the IO case, on the other hand, both of the heavier states have masses \(\simeq \sqrt{\Delta m^2_{32}} = 5.0 \times 10^{-2}\) eV. The sum of the neutrino masses is then given by \(\sum m_{\nu_i} = 0.06\) eV and 0.1 eV for NO and IO, respectively. These predicted values are below the current limit imposed by Planck 2018 \([44]\), \(\sum m_{\nu_i} < 0.12\) eV, but can be probed in future CMB experiments such as CMB-S4 \([45]\). Moreover, the IO case can be probed in future neutrino-less double beta decay experiments, whereas testing the NO case in these experiments is quite challenging \([46]\).

We show in Fig. 1 the distribution of the non-thermal dark matter density produced by gravitino decays in these solutions for \(\lambda_{\alpha}\). We find that many parameter solutions predict \(\Omega_{\text{DM}} h^2 \lesssim 10^{-2}\) for \(m_{\text{LSP}} = 1\) TeV, corresponding to \(T_{\text{reh}} \simeq 10^{12}\) GeV (see Eq. \((4)\)), while some solutions yield \(\Omega_{\text{DM}} h^2 \lesssim 10^{-1}\) corresponding to a reheating temperature as high as \(T_{\text{reh}} \simeq 10^{13}\) GeV. In both cases, the reheating temperature is much higher than the SU(5) confinement scale \(\Lambda_5\), satisfying the strong reheating condition \([2]\).

In Fig. 2 we show the distribution of \(n_B/s\) for \(\Delta = 10^4\), where we see that both positive and negative baryon asymmetries can be obtained. In particular, the observed value (in both magnitude and sign) of the baryon asymmetry \(n_B/s = 0.87 \times 10^{-10}\) \([44]\), which is shown as the vertical solid line, can easily be explained in our scenario.

In Fig. 3, we plot the non-thermal contribution to the LSP abundance from gravitino decay against the baryon asymmetry predicted at the same parameter point, assuming \(\Delta = 10^4\). The vertical black and horizontal green lines show, respectively, the observed values of baryon asymmetry and dark matter abundance.
In summary, we have examined the correlations between inflationary reheating, the non-thermal dark matter abundance produced by gravitino decays, neutrino masses, and the baryon asymmetry in a simple model based on a single master superpotential coupling $\lambda_6$ involving a gauge singlet, a heavy Higgs breaking the GUT gauge symmetry and the (flipped) top matter representation. Using the known neutrino mass-squared differences as a constraint, we find that the typical reheating temperature is $10^{12}$ GeV and the typical baryon-to-entropy ratio lies between $n_B/s \sim (10^{-13} - 10^{-7})$, embracing the observed value near $10^{-10}$. For the preferred value of the baryon asymmetry, we find that, for NO neutrino masses, the non-thermal LSP abundance may saturate the measured relic density of dark matter, but may be significantly lower, leaving open the possibility of a dominant thermal contribution. With IO masses, the non-thermal component is typically subdominant. In this case, because of late entropy production, regions of parameter space that would yield $\Omega_{DM} h^2 \sim 1000$ in standard cosmology are preferred, opening new regions of supersymmetric parameter space for experimental searches.

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