Open Charm as a Probe of Pre-Equilibrium Dynamics in Nuclear Collisions

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August 1994

Abstract

The pre-equilibrium contribution to open charm production in nuclear collisions at $\sqrt{s} = 200$ AGeV is calculated and shown to be sensitive to the early time evolution of the initial mini-jet gluon plasma. The role of charm as a probe of the formation zone physics and correlations between momentum and space-time coordinates is emphasized. While ideal (Bjorken) correlation between the rapidity $y$ and space-time rapidity $\eta$ of mini-jet gluons suppresses greatly the pre-equilibrium yield, the minimal correlation with finite geometrical spread enhances the initial fusion rate by a factor $\sim 2$ in the moderate $p_\perp \sim 2 - 6$ GeV range. That is dominated by fusion of “semi-hard” $p_\perp > 2$ GeV mini-jet gluons and secondary “soft” $p_\perp < 2$ GeV gluons from initial and final state radiation. The “intrinsic” charm flavor excitation process is negligible in the mid-rapidity domain.
I. INTRODUCTION

Open charm production, direct photon, and dilepton production are among the most direct probes [1–4] of the early time evolution of the quark-gluon plasma produced in ultra-relativistic nuclear reactions. At collider energies $\sqrt{s} > 200$ AGeV the initial mini-jet plasma is mostly gluonic [6,7] with a quark content far below its chemical equilibrium value. Furthermore, the initial transverse momentum distribution of those gluons is very broad [3] resembling hot thermal gas of gluons with an effective temperature $T \sim 500$ MeV [6]. Because charm is produced mainly through gluon fusion, open charm production is an ideal probe of that initial gluonic state. In contrast, hidden charm [5] is mostly sensitive to final state interactions in the later stages of evolution. Photons and dileptons are complementary probes of the evolution of the suppressed quark component of the plasma.

The present study is motivated by two recent studies [3,4] of open charm which predict widely different rates in nuclear collisions. In ref. [3] the pre-equilibrium contribution was found to be almost equal to initial gluon fusion rate. In ref. [4], a more provocative claim was made that open charm would be enhanced by over an order of magnitude above the initial pQCD rate! The main result of the present study is that pre-equilibrium open charm production is indeed comparable to the initial fusion rate as in [3]. However, we find an interesting sensitivity to the space-time and momentum space correlations neglected in ref. [3] and to details of the mini-jet formation physics. The copious charm production in ref. [4] is found to be due to an overestimation of the intrinsic charm contribution and the use of an unrealistic $A^n$ scaling from $pp$ reactions. Our calculations show furthermore that the dominant source of pre-equilibrium charm is the fusion of semi-hard mini-jets ( with $p_\perp > 2$ GeV ) with soft gluons ( with $p_\perp < 2$ GeV) originating from initial and final state radiation. Thus, open charm probes the interesting interplay between soft and hard components of the plasma [7] and is thus an important diagnostic of the initial non-equilibrium partonic state in $A + A$ collisions.

The paper is organized as follows: In section 2 the dependence of the direct pQCD
rates for charm production on structure functions, $Q^2$ scale, and $K$ factor is reviewed and compared to existing data. The beam energy dependence and the $A$ dependence of the initial charm production are compared to results in ref. [4]. In section 3, the pre-equilibrium charm production is calculated. The mini-jet rapidity and transverse momentum distribution are fit to results of the Monte Carlo HIJING model [8] including initial and final state radiation. Then three different models for the space-time and momentum correlations are studied and the influence on the charm yield is shown to be significant. We also study the sensitivity of the results to different models of the formation physics comparing the usual $\theta(\tau - \tau_0)$ parameterization with the form $(1 + (\tau_0/\tau)^2)^{-1}$ that more accurately represents pQCD interference [9]. In section 4, we give some discussions and the summary.

II. INITIAL CHARM PRODUCTION

Heavy quark production in $pp$ reactions was studied long ago in pQCD [12] including both fusion and heavy flavor excitation processes in the leading order. It was found that the flavor excitation processes were dominant at high energies because a small $Q^2$ exchange can easily liberate any intrinsic charm component while gluon fusion requires $Q^2 \sim 4M_f^2$. In the Partron Cascade Model [4], Geiger incorporated both mechanisms to calculate s, c, b quark production in nuclear collisions. Geiger's results suggested flavor excitation of the intrinsic charm part of nuclear structure functions would be by far the dominant source of charm production in nuclear collisions as well. However, it is pointed out [13] that the original flavor excitation rates in ref. [12] were a factor of 100 too high in the $x_f \sim 0$ region due to neglected interference with other pQCD amplitudes to the same order. On the other hand, the contribution of intrinsic charm was shown in [14,15] to be important at large $x_f$. In the midrapidity region, where most of the charm is made, the contribution of the intrinsic charm component is only about 10%, and is well within the uncertainties from other sources.

In this paper we only include fusion processes for the parton level cross sections as in ref. [3]. For the production in $p - p$ collisions, we use the light quark and gluon structure
functions from Glück et al [16] and Duke-Owens [17] for comparison. The pQCD differential cross sections for $a + b \to c\bar{c} + X$, are taken from ref. [12]. For example,

$$\sigma_{q\bar{q} \to c\bar{c}} = \frac{8\pi \alpha_s^2(Q^2)}{27\hat{s}^2} (\hat{s} + 2M_c^2) \chi$$

(1)

$$\sigma_{gg \to c\bar{c}} = \frac{\pi \alpha_s^2(Q^2)}{3\hat{s}} \left[ -\left( 7 + \frac{31M_c^2}{\hat{s}^2} \right) \frac{1}{4} \chi + \left( 1 + \frac{4M_c^2}{\hat{s}^2} + \frac{M_c^4}{\hat{s}^2} \right) \log \frac{1 + \chi}{1 - \chi} \right]$$

(2)

where $\chi = \sqrt{1 - 4M_c^2/\hat{s}}$ and we consider the following two choices for the scale $Q^2$ in the coupling constant $\alpha_s(Q^2) = 12\pi/((33 - 2n_f) \log(Q^2/\Lambda^2))$ from ref. [12]:

1. for $gg \to c\bar{c}, Q^2 = \hat{s}/2$; for $q\bar{q} \to c\bar{c}, Q^2 = \hat{s}$. ($Q^2$ choice-1)

2. for both $gg \to c\bar{c}$ & $q\bar{q} \to c\bar{c}, Q^2 = \hat{s}$. ($Q^2$ choice-2)

We take $n_f = 4$ for charm quark production and $n_f = 5$ for bottom quark production. The QCD scale $\Lambda$ depends on the choice of parton distribution functions and is given in the table below.

<table>
<thead>
<tr>
<th>Parton distribution functions</th>
<th>$\Lambda$(GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRV-LO set</td>
<td>0.25</td>
</tr>
<tr>
<td>GRV-HO set</td>
<td>0.20</td>
</tr>
<tr>
<td>Duke-Owens set 1(DO1)</td>
<td>0.20</td>
</tr>
<tr>
<td>Duke-Owens set 2(DO2)</td>
<td>0.40</td>
</tr>
</tbody>
</table>

To incorporate approximately the next-to-leading-order corrections to the above rates we multiply the leading order results by a K-factor. In general, K-factor depends on the choice of parton distribution functions, the center of mass energy of the collision, and the type of the projectile and target particles. Calculations to order $O(\alpha_s^3)$ for the subprocesses were carried out [18,19], and afterwards the calculations to order $O(\alpha_s^3)$ for $p + p$ and Au + Au collisions were made [20,21]. For DO1, $M_c = 1.5$ GeV, $Q_2 = 4M_c^2$, $P_{lab} = 100 - 1000$ GeV, the K-factor for p-p collisions [20] was found to range from 2.85 to 4.1.

In Fig 1.1 we compare the so calculated charm cross section to the limited data on inclusive $c\bar{c}$ production in $p - p$ collisions. The two data points are $\sigma = 18.5 \pm 4.5 \mu b$ at lab-frame energy $P = 400 GeV/c$, and $\sigma = 42 \pm 13 \mu b$ at $P = 800 GeV/c$ [20,22]. As a consistency check, we also plot the long-dashed curve using the same parameters as in Fig 1.1 of ref. [20]
(i.e. DO1, $M_c = 1.5 GeV$, $Q^2 = 4M^2_c$) using constant $K = 3$ for simplicity. Comparing the solid and dot-dashed curves shows the strong dependence on the assumed charm quark mass for the GRV-HO set. Comparing the solid and dashed curve we see that different choices for the $Q^2$ scale can be compensated for by shifts in the $K$ factor. These results together with the paucity of data emphasize the need to measure $pp$ and $pA$ to fix uncertainties in the initial charm production rate in order that the final state contribution in $AA$ can be properly identified.

Next we compare our results for the rapidity density of produced $c\bar{c}$ pairs at $y = 0$ with results of ref. [4]. In Fig 1.2 energy dependence in the range between RHIC and LHC ($\sqrt{s} = 200 - 6300$ AGeV ) for $Au + Au$ collisions are shown. The solid and thick dashed curves are our $pp$ results scaled to $AA$ via

$$\left(\frac{dN}{dY}\right)^{AA}_{Y=0} = A^{0+1/3} \left(\frac{d\sigma}{dY}\right)^{pp}_{Y=0} / \sigma^{pp}_{inelastic}$$

(3)

where $\sigma^{pp}_{inelastic}$ is taken from ref. [23]. Glauber geometry for central high $A + A$ collisions gives $\alpha = 1$. In Fig 1.2 The top dotted curve labeled by triangle is the parton cascade model result [4] for the so-called QGP formation case. That curve is higher than our curves by about an order of magnitude because it includes the bogus intrinsic charm contribution. The dotted curve with filled squares show that the contribution to the previous curve from fusion processes only ( processes (1) and (2) in the notation of ref. [4] ) is indeed very close to our results. Shown also in Fig 1.2 by lower long dashed curve with open circles is a curve in ref. [4] from the estimate without QGP formation. It is lower than the solid curve by a factor of 6 to 2.5. The main source of this difference is the use of an unphysical $A^\alpha$ scaling with $\alpha = 0.76$ for the $A$-dependence of $p - A$ cross sections instead of the value $\alpha = 1$ we use from Glauber geometry. The long-dashed curve with filled circles is the lower curve multiplied by a factor of $A^{1.0}/A^{0.76} = 3.55$ to demonstrate this. In summary, the factor $\sim 50$ enhancement of charm production suggested in ref. [4] comparing curves G1 with G4 for charm production at RHIC is exaggerated by the intrinsic charm and $A^\alpha$ scaling.

As a further check on the parameters we compare charmed hadron $x_f$ results in Fig 1.3
with 400 GeV $p-p$ data. We use the $\delta$-function fragmentation. A more realistic fragmentation function used in ref. [14] lowers the curves slightly and reveals the true high-$x_f$ intrinsic charm component. As a further check we compare $b\bar{b}$ production in Fig. 1.4. Here we take $M_c = 4.75$ GeV as in ref. [21], with $K = 3, n_f = 5$. The data point at $\sqrt{S} = 630$ GeV is from ref. [27]: $\sigma(p\bar{p} \rightarrow b + X) = 19.3 \pm 7(\exp.\) $\pm 9(\th.)\mu b$, and only the experimental error is indicated in Fig 1.4. At $\sqrt{S} = 1.8 TeV$, our value is $41.8\mu b \times K = 125\mu b$. This is significantly larger than found in ref. [21].

III. PRE-EQUILIBRIUM CHARM PRODUCTION

We consider next the pre-equilibrium contribution to the charm yield in $A + A$. This is the charm produced through final state interactions between partons in the dense mini-jet plasma.

A. Spectrum of Mini-Jets

The spectrum of mini-jet quarks and gluons in leading-order follows from ref. [28]

$$\frac{d\hat{\sigma}}{d\hat{t}}_{gg-gg} = \frac{9}{2} \frac{\pi \alpha_s^2}{\hat{s}^2} \left[ 3 - \frac{\hat{u}\hat{t}}{\hat{s}^2} - \frac{\hat{u}\hat{s}}{\hat{t}^2} - \frac{\hat{s}^2}{\hat{u}^2} \right]$$

(4)

$$\frac{d\hat{\sigma}}{d\hat{t}}_{gq-gq} = \frac{\pi \alpha_s^2}{\hat{s}^2} \left[ -4 \frac{\hat{u}^2 + \hat{s}^2}{\hat{u}\hat{s}} + \frac{\hat{t}^2}{\hat{t}^2} + \frac{\hat{s}^2}{\hat{u}^2} \right].$$

(5)

The term mini-jets refers to unresolved jets at a scale $p_\perp > p_\perp \text{cut} = 2 GeV$. The inclusive cross section to produce mini-jets is given by

$$\frac{d\sigma}{dy dp_\perp^2} = \int dy_3 x_1 f_1 x_2 f_2 \frac{d\hat{\sigma}}{d\hat{t}}(1 + 2 \rightarrow 3 + 4)$$

(6)

where $f_1$ is the incident parton distribution evaluated at $x_1 = p_\perp (e^y + e^{-y})/\sqrt{s}$ at a scale $Q^2 = \hat{s}$, and $y_3$ is the rapidity of the unobserved final parton. The subprocess Mandelstam variables are $\hat{s} = sx_1 x_2$ etc. and the light-cone coordinates of the initial and final partons are $p_1 = [2x_1 p_0, 0, 0]$, $p_2 = [0, 2x_2 p_0, 0]$, $p_3 = [m_\perp e^{y_3}, m_\perp e^{-y_3}, -p_\perp]$ and the observed
parton has \( p = [m_1 e^\eta, m_1 e^{-\eta}, p_\perp] \). The resulting transverse momentum distribution of mid-rapidity mini-jet gluons at \( \sqrt{s} = 200 \) AGeV is shown by the open circles in Fig. 2.1. We call this distribution with \( p_\perp \text{cut} = 2 \) GeV the hard distribution. It is compared to the output of the Monte Carlo calculation via the HIJING model [8] that includes initial and final state radiation.

For convenience we have parameterized the Monte Carlo results in the range \( |y| < 3.7 \) with

\[
\frac{dN}{dy dp_\perp} = 0.06 e^{-1.25 p_\perp} \cos \left[ \frac{\pi (\frac{y}{3.7})^{1.8}}{2} \right]
\]

(7)

In the following, we call this parameterized distribution the soft+hard distribution. In Fig. 2.1, the soft+hard, hard, and Monte Carlo distributions are very close to each other in the semi-hard \( p_\perp > 2 \) GeV region at \( y = 0 \). However the parametrized distribution falls underneath the Monte Carlo result for \( p_\perp < 1 \) GeV. We emphasize that the soft component is strongly model dependent as it requires the furthest extrapolation from the pQCD hard domain. The Hijing yield in that region is due to initial and final state radiation. Other contributions in this soft domain from coherent string are possible [7]. While most of the following results are obtained with the simple parametrization above, we will check the sensitivity to variations of the soft component as well.

B. \( \eta - y \) correlations

1. Bjorken correlations

In ideal Bjorken dynamics, the space-time rapidity \( \eta = 1/2 \log((t + z)/(t - z)) \) and the true momentum rapidity \( y = 1/2 \log((E + p_z)/(E - p_z)) \) are assumed to be perfectly correlated. This is referred to as the inside-outside picture and the phase-space distribution function in this case has the form

\[
F(\vec{x}, \vec{p}, t)_{Bj} = \frac{(2\pi)^2}{p_\perp^2} \frac{dN}{dy dp_\perp} \frac{\delta(y - \eta)}{\tau \pi R_A^2} \Theta(\tau - \tau_i) \Theta(\tau_f - \tau)
\]

(8)
where $\tau_i = 0.1 \text{fm}/c$ is the mini-jet formation time and $\tau_f \approx 1 \text{fm}/c$ is an estimate of the proper time interval of the pre-equilibrium phase during which the energy density falls by an order of magnitude due to rapid longitudinal expansion.

The phase space distribution is normalized such that

$$
\int \frac{F(\vec{x}, \vec{p}, t) \delta^D x}{(2\pi)^3} = \frac{d^3 N}{d^3 p} = \frac{1}{2\pi E_p} \frac{dN}{dy dp},
$$

(9)

We are mainly interested in the charm production at $y = 0$ where $p_{T1} = (\cos \phi_1, \sin \phi_1, 0)p_{\perp 1}$. The pre-equilibrium contribution is then from ref. [3] as

$$
\left( E \frac{d^3 N}{d^3 p} \right)_{y=0} = \int d^4 x \int \frac{1}{32(2\pi)^8} \frac{d^6 k_1 d^6 k_2 d^6 p_2}{\omega_1 \omega_2 E_2} F(\vec{x}, \vec{k}_1, t) F(\vec{x}, \vec{k}_2, t) |M|^2 \delta^D \left( \sum P^0 \right)
$$

(10)

The above expression show clearly why charm is sensitive to the space and momentum correlations of the evolving gluon distributions. Denoting $dN/dy dp_\perp \equiv g(y, p_\perp)$, the ideal $\eta - y$ correlations lead to

$$
\left( E \frac{d^3 N}{d^3 p} \right)_{y=0} = \int_{\tau_i}^{\tau_f} \frac{d\tau}{32(2\pi)^6 R_A^2} \int d\eta dp_{\perp 1} dp_{\perp 2} d\phi_1 d\phi_2 \frac{g(\eta, p_{\perp 1})g(\eta, p_{\perp 2}) \delta(\sum E) |M|^2}{p_{\perp 1} p_{\perp 2} E_2}
$$

$$
= \frac{\ln(\tau_f/\tau_i)}{32(2\pi)^6 R_A^2} \int d\eta dp_{\perp 2} d\phi_1 d\phi_2 \frac{g(\eta, p_{\perp 1,0})g(\eta, p_{\perp 2}) |M|^2}{p_{\perp 2} \gamma (E \cosh \eta - p \cos \phi_2)}
$$

(11)

In deriving the above, we have used kinematic relations $E_2 = (p_{\perp 1} + p_{\perp 2}) \cosh \eta - E$,

$$
\frac{\delta(\sum E)}{p_{\perp 1} E_2} = \frac{\delta(p_{\perp 1} - p_{\perp 1,0})}{p_{\perp 2} (E \cosh \eta - p \cos \phi_2)}
$$

and

$$
p_{\perp 1,0} = p_{\perp 2} \frac{E \cosh \eta - p \cos \phi_2}{p_{\perp 2} (1 - \cos(\phi_1 - \phi_2)) - (E \cosh \eta - p \cos \phi_1)}.
$$

Numerical integration of the above integral in equation (11) leads to the results shown in Fig. 2.2. The solid line is the $p_{\perp}$-distribution for the initial charm production, from section 2. We see that the pre-equilibrium contribution in this strongly correlated case is totally negligible. This result is similar to the thermal charm production contribution calculated in ref. [3] except in our case the curve extends to higher $p_{\perp}$ because of the broader initial mini-jet distribution in $p_{\perp}$. 7
2. Uncorrelated $\eta - y$

In ref. [3], another extreme case opposite to the ideal Bjorken picture was considered. In that case the gluon distribution is assumed to be completely uncorrelated as in an ideal thermal fireball. This assumption leads to

$$ F(x, \bar{p}, t)_{MW} = \frac{(2\pi)^2}{p^2_{\perp}} \frac{1}{V} \frac{dN}{dy dp_{\perp}} $$

(12)

From equation (10), we have

$$ \left( E \frac{d^3N}{d^3p} \right)_{y=0} = \frac{I(p_{\perp})}{32(2\pi)^4} \int \frac{d^4x}{V^2} $$

(13)

where

$$ I(p_{\perp}) = \int dy_1 dy_2 dp_{\perp 1} dp_{\perp 2} d\phi_1 d\phi_2 \frac{g(y_1, p_{\perp 1}, 0)g(y_2, p_{\perp 2}, 0) |M|^2}{\cosh y_1 \cosh y_2 p_{\perp 2}^2 (E \cosh y_2 - p \cos \phi_2)} $$

(14)

$$ p_{\perp 1,0} = p_{\perp 2}^2 \frac{E \cosh y_2 - p \cos \phi_2}{\cosh(y_1 - y_2) - \cos(\phi_1 - \phi_2)} - (E \cosh y_1 - p \cos \phi_1) $$

(15)

If one assumes a fixed volume $V = \tau_i \pi R_A^2$, then $\int dt \sim \tau_f - \tau_i$, and

$$ \int \frac{d^4x}{V^2} \sim \frac{1}{\pi R_A^2} \frac{\tau_f}{\tau_i} \quad \text{as in ref. [3].} $$

(16)

For uncorrelated case, the pre-equilibrium charm production is much larger than the Bjorken-correlation case. In ref. [3], the pre-equilibrium charm production has almost the same magnitude and $p_{\perp}$-shape as the initial charm.

3. Minimally-correlated $\eta - y$

We consider here the simplest source $\eta - y$ correlations resulting from the minimal geometrical spread in initial production points required by the uncertainty principle. This type of correlations are included in the parton cascade model and discussed in ref. [10]. The phase space distribution function including such minimal correlations has the form

$$ F(x, \bar{p}, t)_{\text{min}} = N \int \frac{dN}{dy dp_{\perp}^2} \frac{\theta(\tau_{\text{max}} - \frac{t}{\cosh y_0})}{1 + \left(\frac{\tau_{\text{max}}}{\Delta t}\right)^2} \rho_0(x_0, t_0) \delta(x - x_0 - \bar{v} \Delta t) d^3x_0 dt_0. $$

(17)
The integration is over the space-time coordinates \((\vec{x}_0, t_0)\) of the production points of the gluons. These points are distributed according to a normalized density \(\rho_0(\vec{x}_0, t_0)\). The delta function arises to take into account free streaming of the partons from the production point, with velocity \(\vec{v} = \vec{p}/E\), where \(E = p_\perp \cosh y\) and \(p_z = p_\perp \sinh y\). The theta function defines what we mean by pre-equilibrium. The proper time when the pre-equilibrium fusion is terminated is \(\tau_{\text{max}}\), which is determined below in Fig 2.3. The theta function insures that only those gluons with proper time less than \(\tau_{\text{max}}\) contribute.

The formation physics is included via the Lorentzian formation factor [9]

\[
(1 + (t_f(p) / \Delta t)^2)^{-1} ,
\]

where \(\Delta t = t - t_0\) is the elapsed time, and the formation time is given by

\[
t_f(p) \approx \cosh y \frac{0.2 GeV}{p_\perp} (fm) .
\]

We note that the above formation factor more accurately describes the interference phenomena suppressing production at small times than the conventionally assumed factor

\[
\theta(\Delta t - t_f(p)) .
\]

In the following we consider both formation functions for comparison to check for the sensitivity to this formation physics.

For \(\rho_0(\vec{x}_0, t_0)\) we assume that \(\int \rho_0(\vec{x}_0, t_0) d^3x_0 dt_0 = 1\). In this case the normalization factor is \(\mathcal{N} = (2\pi)^3/(E\pi)\).

\[
\lim_{t \to \infty} \int F(\vec{x}, \vec{p}, t)_{\text{min}} d^3x / (2\pi)^3 = d^3 N / d^3 p .
\]

As discussed in ref. [10], the production points are spread along the beam axis according to the uncertainty principle by an amount \(\delta z \equiv d \sim \hbar / p_\perp\) since the dominant parton interaction leading to a \(y = 0\) parton with final \(p_\perp\) has an initial longitudinal momentum \(x P_0 \sim p_\perp\). We take as a particular model

\[
d = \frac{1}{\frac{p_\perp}{6\pi} + \Delta z^{-1} (fm)}
\]
where $\Delta z = 1$ fm. Clearly this is only a rough guess, but it allows us at least to investigate the sensitivity of the results to a particular $\eta - y$ correlation that results from this spatial spreading of the production points. We emphasize that it is precisely the uncertainty of the initial space-time formation physics that leads us to study the role of open charm production as an experimental probe of that physics.

Given the above assumption we take

$$
\rho_0(\vec{x}, t_0) = \frac{1}{\pi R_A^2} \delta(t_0) e^{-\frac{\vec{x}^2}{\langle 2d^2 \rangle}} \frac{1}{\sqrt{2\pi d}}, \quad d = \frac{1}{\rho_{/z}} + \frac{1}{\Delta z} (fm)
$$

(23)

where $d$ is the mean spread for gluons depending on $p_\perp$ from above. This distribution only spreads out the production points along the beam axis. A more realistic treatment would also smear out in the time coordinate.

Neglecting transverse expansion, we obtain finally

$$
F(\vec{x}, \vec{p}, t)_{\min} = \left( \frac{2\pi}{\Delta z} \right)^3 \frac{\rho_0(\vec{x}, t_0)}{\sqrt{2\pi R_A^2}} (\Delta z^{-1} + \frac{p_\perp}{0.2}) e^{-\left(\frac{x-y_1}{\rho} \right)^2 (\Delta z^{-1} + \frac{p_{+/z}}{0.2})} e^{-\left(\frac{y_1}{\rho} \right)^2 (\Delta z^{-1} + \frac{p_{+/y}}{0.2})} \frac{dN}{1 + \left( \frac{0.2 \cosh y_1}{p_{/\perp}} \right)^2} \frac{dN}{1 + \left( \frac{0.2 \cosh y_2}{p_{/\perp}} \right)^2} \frac{dN}{1 + \left( \frac{0.2 \cosh y_2}{p_{/\perp}} \right)^2}
$$

(24)

let $a_1 = \tanh y_1 \ t$, $a_2 = \tanh y_2 \ t$, $b_1 = \left( \Delta z^{-1} + \frac{p_{+/z}}{0.2} \right)^2 / 2$, $b_2 = \left( \Delta z^{-1} + \frac{p_{+/y}}{0.2} \right)^2 / 2$, then after integration over $z$, we have the final expression as the following:

$$
\left( \left( \frac{d^3N}{d^3p} \right)_{\eta=0} \right)_{\min} = \left( \frac{\sqrt{2\pi}}{128\pi^3(2\pi)^3 R_A^2} \right) \int dy_2 dy_1 dp_{+/z} dp_{+/y} \frac{\left( \Delta z^{-1} + \frac{p_{+/z}}{0.2} \right)^2 \left( \Delta z^{-1} + \frac{p_{+/y}}{0.2} \right)^2 \left( M \right)^2}{\cosh y_1 \cosh y_2 \rho_{/\perp} \rho_{/\parallel}}
$$

$$
= \int_{0}^{t_f} \left( \frac{\left( a_1 - a_2 \right)^2}{e^{b_1 - b_2}} \right) \left[ 1 + \left( \frac{0.2 \cosh y_1}{p_{/\perp}} \right)^2 \right] \left[ 1 + \left( \frac{0.2 \cosh y_2}{p_{/\perp}} \right)^2 \right]
$$

(25)

where $t_f = \tau_{max} \min(\cosh y_1, \cosh y_2)$, and $p_{+/z}$ is the same as in equation (15).

We also plot the energy density curve at $z = 0$ as a function of time in Fig. 2.3. We see that it increases first, and reaches maximum at the time about $0.1 fm/c$, then the energy density decreases linearly to $\sim 2 GeV/ fm^3$ at $\sim 0.9 fm/c$ for hard (soft+hard) distribution. We choose the above time as the cutoff $\tau_{max}$. The numerical results are shown in Fig. 2.4. We can see that in the region $p_{+/z} \sim 4 GeV$, the pre-equilibrium contribution is comparable with the initial one. This window is similar to the window in the dilepton spectrum calculated by Shuryak and Xiong [2].
The previous uncorrelated case neglects the finite formation times of the mini-jets. In order to see the formation-time effect, we also use \( \theta(t/ \cosh y - 0.2/p_\perp) \) instead of the Lorentzian\( (1 + (0.2 \cosh y/(p_\perp t))^2)^{-1} \) for the formation-time effect. The result from this \( \theta \)-function is about 10\% lower at \( p_\perp = 0 \text{GeV} \), and 30\% lower at \( p_\perp = 8 \text{GeV} \), as shown in Fig. 2.5.

We also see that for the soft+hard distribution the soft gluons significantly increase the pre-equilibrium charm production in both low-\( p_\perp \) and high-\( p_\perp \) region, with the largest increase in low-\( p_\perp \) region. It’s interesting to see where the enhancement comes from. In Fig. 2.4, the curve with diamonds shows the contribution from the fusion of soft gluons both with \( p_\perp < 2 \text{GeV} \), and the curve with unfilled squares shows the contribution from the fusion of hard gluons both with \( p_\perp > 2 \text{GeV} \). These two curves are both very low compared with the curve calculated from the soft+hard distribution. So most of the enhancement going from hard distribution to soft+hard distribution comes from the fusion of hard and soft mini-jet gluons.

We have noted before that our fit for the mini-jet gluon spectrum falls below the Monte Carlo result from HIJING calculation. We can fit the soft gluons from HIJING better by using \( 0.265 e^{-2.6 p_\perp} \) for \( p_\perp \in (0, 1.1) \text{GeV} \), and use the old fit \( 0.06 e^{-1.25 p_\perp} \) for higher-\( p_\perp \) gluons. This new fit gives us more very soft gluons. We have done the calculation for minimally-correlated case using the new fit, and the result is different only by less than 10\%, which means the super-soft gluons are not very important for the pre-equilibrium charm production.

The pre-equilibrium charm production at non-zero rapidity is also interesting. The preliminary results show an increase of \( dN/dy \) (about a factor of 2) at mid-rapidity for Bjorken-correlation case and minimally-correlated case, but for uncorrelated case \( dN/dy \) seems to decrease monotonously as rapidity increases. Also, when we change the parameter \( \Delta_z \) in the mean beam-axis spread (see equation (22)) by a factor of 2, the results don’t change much (less than 10\%).
IV. DISCUSSION AND SUMMARY

In this paper, we calculated initial and pre-equilibrium charm production in nuclear collisions to test for the sensitivity of this probe to the unknown initial conditions in such reactions. For the initial charm production, the dependence on the choice of structure functions, the $Q^2$ scale, and the K-factor was studied. The parameters were fixed by fitting the limited available experimental data at lower energies. We pointed out that the copious charm production predicted in ref. [4] was due to the improper inclusion of the intrinsic charm flavor excitation processes and also to the use of unrealistic $A^{1.09}$ scaling from $pp$ to $AA$. Our calculated charm yields are close to those computed in ref. [3].

For the contribution from pre-equilibrium charm production, we studied the effect of correlations between the rapidity $y$ and space-time rapidity $\eta$ of mini-jet gluons. For the ideal Bjorken-correlated case, where $\eta = y$, the pre-equilibrium charm production is negligible compared with the yield due to initial gluon fusion. For the opposite extreme fireball case, corresponding to uncorrelated $y$ and $\eta$, the pre-equilibrium charm production is several orders of magnitude larger than in the Bjorken-correlated case, in agreement with the findings of ref. [3]. Therefore, the pre-equilibrium charm production is very sensitive to the $(\eta - y)$-correlations in the initial state. By loosening the correlation, gluons with different rapidities and transverse momenta can fuse and enhance greatly the charm yield.

In order to investigate the effect of more realistic correlations that may exist in the initial mini-jet plasma, we introduced a minimal correlation model taking into account the uncertainty principle and finite formation times. Our main result is that this minimal correlation also leads to a very large increase (about two orders of magnitude) of the pre-equilibrium charm production relative to the ideal Bjorken-correlated case. We find a window in transverse momentum between 3–5 GeV where the pre-equilibrium enhancement is comparable to the initial charm yield. We showed furthermore that reasonable variations of the formation probability distribution led to measurable effects on the order of 50%.

Given the large uncertainty about the mini-jet initial conditions, as apparent for exam-
ple by comparing the widely different predictions for a variety of other observables from HIJING [8] and the Parton Cascade [4] models for nuclear collisions, it is essential to find experimental probes that provide direct information about those initial conditions. Our calculations confirm that open charm production provides a powerful independent probe that is especially sensitive to the phase space correlations at early times in nuclear collisions.

Acknowledgements: We thank K. Geiger, L. Xiong, B. Müller, X.N. Wang for useful discussions and A. Muller for bringing refs. [13,15] to our attention.
REFERENCES


FIGURE CAPTIONS

Fig1.1 The cross section for $pp \rightarrow c\bar{c}X$ is plotted as a function of $P_{lab}$. The data are from [22]. The solid line is our result with $M_c = 1.3$ GeV, $K = 3$, $Q^2$ choice-1 and GRV-HO set. The long-dashed curve is the result with the same parameters as in Fig 1 of [20], but using a K-factor of 3 instead of doing $O(\alpha_s^2)$ calculation.

Fig1.2 $dN_{c\bar{c}}/dY$ at $Y = 0$ for $Au - Au$ collisions vs $\sqrt{S}/A$. The top curve is Geiger’s total charm production with QGP formation using Parton Cascade Model [4]. The solid (dashed) curve is our result using first (second) parameterization. The dotted curve with filled squares is Geiger’s charm production from fusion processes only in the case of QGP formation. The bottom curve is Geiger’s parton model result without QGP formation using $\alpha = 0.76$. We use $\alpha = 1$, for the $A$-dependence of $p - A$ cross sections. The long-dashed curve with filled circles is the bottom curve multiplied by 3.55, which is the factor between the two choices of $\alpha$ for $Au - Au$ collisions.

Fig1.3 The production of charmed hadrons as a function of $x_f = x_1 - x_2$ for $p - p$ collisions at $P_{lab} = 400$ GeV/c [26]. The solid curve is our result for $d\sigma_{pp \rightarrow c\bar{c}+X}/dx_f$ using the first parameterization. The dashed curve is our result using the second parameterization. These curves assume a delta function charm fragmentation function.

Fig1.4 The cross section for $p\bar{p} \rightarrow b\bar{b} + X$ vs $\sqrt{S}/A$. The data point at $\sqrt{S} = 630$ GeV is from [27]. The dashed cross at $\sqrt{S} = 1.8$ Tev is obtained indirectly from [21], and the error bar is only illustrative.

Fig2.1 The gluon distribution $dN/dy d^2p_\perp$ at $y = 0$ is plotted. The solid curve is taken from the HIJING calculation with radiation effects included, and the circles are our result from the initial production. The dashed curve is the fit $0.06 e^{-1.25p_\perp}$.

Fig2.2 The distribution $Ed^2N/d^2p$ at $y = 0$ of charm quark production using $\delta(\eta - y)$-correlation is plotted as a function of $p_\perp$. The solid curve is the initial charm pro-
duction. The curve labeled with filled diamonds is the pre-equilibrium contribution including both the soft \((p_\perp < 2 \text{ GeV})\) and hard \((p_\perp > 2 \text{ GeV})\) components. The curve labeled with unfilled diamonds is the pre-equilibrium contribution including only the hard component.

**Fig. 2.3** The energy density at \(z = 0\) is plotted as a function of proper time assuming minimal correlations and Lorentzian formation probability. The solid curve includes both soft and hard components while the dashed curve includes only the hard component.

**Fig. 2.4** The distribution \(E d^3 N/d^3 p\) at \(y = 0\) of charm quark production using minimal \(\eta - y\) correlations is plotted as a function of \(p_\perp\). The curve labeled with filled squares include both components while that labeled with unfilled squares include only the fusion of hard gluons. The curve labeled with diamonds shows the contribution from fusion of only soft gluons with \(p_\perp < 2 \text{ GeV}\). This shows that the dominant pre-equilibrium contribution comes from the fusion of soft and hard gluons.

**Fig. 2.5** The distribution \(E d^3 N/d^3 p\) at \(y = 0\) of charm quark production using different formation-time probability distributions as a function of \(p_\perp\). The curve with filled squares is obtained using the Lorentzian form (18), and the dashed curve using the theta function form (20).