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Baryogenesis Constraints on Two Higgs Doublet Models¹

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Abstract

The requirement that sphaleron transitions be sufficiently suppressed after the electroweak phase transition to preserve any then existing baryon asymmetry is studied perturbatively in a general two Higgs doublet model. We find that about 5% of otherwise acceptable Higgs potentials evade baryon number washout and that the presence of a soft CP violating term makes it easier to do so. The spectrum of Higgs particles for these acceptable cases is discussed. The Minimal Supersymmetric Standard Model is treated as a special case; here we only find acceptable cases in an otherwise disfavoured region of parameter space, $\tan \beta < 1$.

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1 Introduction

It is well known that there is difficulty in sustaining the hypothesis [1] of baryogenesis at the electroweak phase transition in the minimal standard model. In this standard model one Higgs doublet case the source of CP violation is the CKM quark mixing matrix which is too small to explain the observed baryon to entropy ratio. Also, requirements on the phase transition (in a perturbative treatment) seem to lead to a Higgs mass smaller than the experimental lower limit [2,3]. To overcome these difficulties, attention has been given to extensions of the minimal standard model, involving the addition of extra scalars [3, 4, 5, 6, 7]. Prominent among these is the extension to two Higgs doublets, and this is what we discuss here. Our treatment is perturbative; we do not treat the non-perturbative ideas which it has been suggested recently, might rehabilitate baryogenesis in the minimal, one Higgs doublet, standard model [8].

One of the advantages of the two Higgs doublet model is that it can come with an extra mechanism for CP breaking, in addition to the usual breaking via the CKM matrix. The two Higgs doublet potential can contain a term which softly breaks CP invariance without violating the necessary suppression of flavour changing neutral currents. Mechanisms have been proposed [4, 7] where precisely this term generates the necessary change in the Chern-Simons winding number to mediate baryogenesis at the electroweak phase transition. Whether or not one of these particular proposed mechanisms be the means by which baryons are produced, the two Higgs doublet potential should be such that the phase transition is first order, with the sphaleron mass after the transition large enough to suppress the baryon-number violating processes which would otherwise wash out any net baryon number previously produced. It is precisely this requirement which leads, in the one Higgs doublet case, to a Higgs mass which is too small to agree with the experimental bound [2, 3].

In the two doublet potential of Eq.(1) below there are many parameters. These have to be such that the potential is bounded below, that the $SU(2) \times U(1)$ symmetry breaking minimum of the potential conserves electromagnetic charge, and to satisfy the experimental bounds on the masses of the 5 consequent physical scalar particles. The resultant parameter space is our basis space. We want to assess how limiting the electroweak baryogenesis hypothesis is for the parameters of the potential, and so we shall estimate

the proportion of the basis parameter space compatible with the hypothesis. An important free parameter is μ_3^2 of Eq.(1), the coefficient of the term which softly breaks CP invariance when μ_3^2 is complex. This term could provide the most important source for as yet unobserved electric dipole moments in quarks and leptons through CP violating neutral scalar exchange [9, 10]. Thus we shall make our estimate for a range of particular values of complex μ_3^2 ; another parameter of our theory is the mass of the top quark which we take to be 175 GeV.

We also make some investigations with our methods of the Minimal Supersymmetric Standard Model. The similarity to the theory outlined above is that there are just two Higgs doublets and the quarks (and leptons) couple to the Higgs doublets in the usual way; the dissimilarity is that the supersymmetry prescription strongly limits the number of free parameters in the potential, one of which can be taken to be μ_3^2 which now must be real. There are however other possible sources of CP violation involving the interactions of the supersymmetric partners of the Standard Model particles [11].

2 The Temperature Dependent Effective Potential

We adopt the usual theory where the down quarks (d_i) only couple to each other through the Higgs doublet Φ_1 , and the up quarks (u_i) only couple to each other through Φ_2 , thus ensuring tree level flavour conservation of scalar mediated neutral currents. In order to guarantee the structure against radiatively induced conservation breaking terms from the quartic couplings of the scalar fields, our quartic and Yukawa terms obey symmetry under the usual discrete transformation $\Phi_2 \rightarrow -\Phi_2, (u_i)_R \rightarrow -(u_i)_R$. This symmetry is softly and safely broken (that is, without inducing tree level flavour changing neutral currents) by the CP violating quadratic coupling of Φ_1 to Φ_2 (discussed in §(1)) which appears in Eq.(1). Subject to the restriction of the above symmetry, the most general, renormalisable scalar interaction consistent with gauge invariance is [12, 13]

$$\begin{aligned}
V_0 = & -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 - \mu_3^2 \Phi_1^\dagger \Phi_2 - \mu_3^{2*} \Phi_2^\dagger \Phi_1 + \\
& \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \\
& \lambda_4 \left| \Phi_1^\dagger \Phi_2 \right|^2 + \frac{1}{2} \left[\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_5^* (\Phi_2^\dagger \Phi_1)^2 \right]
\end{aligned} \tag{1}$$

where μ_1^2, μ_2^2 and the dimensionless coefficients $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are real. Without loss of generality we take $\lambda_5 = \lambda_5^*$ to be real, and then Eq.(1) is CP invariant if and only if μ_3^2 is real or pure imaginary; as stated above we shall consider the general case where μ_3^2 is complex.

The two Higgs doublets are standardly expressed in terms of real fields ϕ_i , $i = 1, 2, \dots, 8$, through

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix} \quad (2)$$

From Eqs.(1) and (2) we obtain the zero temperature potential $V_0(\phi)$ as a function of $\phi = \phi_1, \phi_2, \dots, \phi_8$ with the parameters λ_i assumed renormalised. The vacuum expectation values of the scalars are to be found at the minimum of V_0 and thus satisfy

$$\partial_i V_0 \big|_{\phi=\langle\phi\rangle} = 0, \quad \partial_i \equiv \frac{\partial}{\partial \phi_i}, i = 1, 2, \dots, 8 \quad (3)$$

and also the requirement that $\langle\phi\rangle$ be a minimum.

We take as part of the definition of our basis parameter space that Φ_1 and Φ_2 are parallel at the minimum to ensure charge conservation. Without any loss of generality, we take the usual convention that the lower components of both doublets are the neutral fields and that the vacuum expectation values are then

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 e^{i\theta} \end{pmatrix}, \quad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \quad (4)$$

where v_1 and v_2 are real and $v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV}$. The scalar mass-squared matrix is given by

$$M_{ij}^2 = \partial_i \partial_j V_0 \big|_{\phi=\langle\phi\rangle} \quad (5)$$

and gives rise to 5 massive physical particles and 3 zero mass would-be Goldstone bosons. For given λ_i , Eq.(3) can be taken to determine μ_1^2, μ_2^2 and θ , which leaves λ_i , $\tan \beta = v_2/v_1$ and μ_3^2 as the free parameters of the theory [14].

At non-zero temperature, T , we develop the loop expansion by the usual methods [15], and make the assumption, which will be discussed later, that the temperature is sufficiently high. The result is, in the approximation used,

$$V(\phi, T) = V_0(\phi) - \frac{N_{eff} \pi^2 T^4}{90} + V_2(\phi, T) + V_3(\phi, T) \quad (6)$$

$$V_2(\phi, T) = \frac{T^2}{24} \left\{ \begin{aligned} &-4(\mu_1^2 + \mu_2^2) + (6\lambda_1 + 2\lambda_3 + \lambda_4 + 3g_b^2 + g_\tau^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2) \sum_{i=1}^4 \phi_i^2 \\ &+ (6\lambda_2 + 2\lambda_3 + \lambda_4 + 3g_t^2 + \frac{9}{4}g^2 + \frac{3}{4}g'^2) \sum_{i=5}^8 \phi_i^2 \end{aligned} \right\} \quad (7)$$

$$V_3(\phi, T) = -\frac{T}{12\pi} \left[\sum_{i=1}^8 [m_i^2(\phi, T)]^{\frac{3}{2}} + 4 \left[\frac{g^2}{4} \sum_{i=1}^8 \phi_i^2 \right]^{\frac{3}{2}} + 2 \left[\frac{g^2 + g'^2}{4} \sum_{i=1}^8 \phi_i^2 \right]^{\frac{3}{2}} \right] \quad (8)$$

In Eq.(7), the expression in curly brackets is the trace of the scalar mass squared matrix $\partial_i \partial_j V_0$. The g_t, g_b and g_τ are the third generation Yukawa couplings and g, g' the usual gauge couplings. The smaller masses of the other quarks and leptons are neglected. In Eq.(8), the first term is the contribution from the scalar masses. Here we have not just used the tree level masses squared, which are eigenvalues of $\partial_i \partial_j V_0$ as used in Eq.(7), but we use the eigenvalues of $\partial_i \partial_j (V_0(\phi) + V_2(\phi, T))$. It has been shown [2, 16] that this is equivalent to correcting the one-loop potential by adding to the loop all ring diagrams ('daisy diagrams') which makes what would otherwise be imaginary values of $[m_i^2]^{\frac{3}{2}}$, for some values of ϕ , into real values [15, 17]. Also in Eq.(8), a correction to the gauge boson contribution has been included; in the contribution of the longitudinal polarisation excitations, there is suppression due to a temperature dependent 'Debye mass' factor. A simple treatment of this due to Dine et al. [2] is just to drop the longitudinal contribution, and we follow this prescription.

Up to this stage it has been necessary to retain all of the fields ϕ_i to evaluate the eigenvalues of the second derivative matrices.² But now if we wish to find a spontaneous breaking minimum, away from the origin, as a function of T , we can insert into Eq.(6) the values of Eq.(4), namely $\phi_1 = \phi_2 = \phi_5 = \phi_6 = \phi_8 = 0$, $\phi_3 = v_1 \cos \theta$, $\phi_4 = v_1 \sin \theta$, $\phi_7 = v_2$ and minimise Eq.(6) as a function of these 3 variables.

²This requirement was not observed in the similar work of Ref. [18]. Note also that this same work couples the up-type quarks to both doublets and hence results in flavour violating scalar mediated neutral currents.

3 Electroweak Baryogenesis

3.1 The Basis Parameter Space

Before testing for electroweak baryogenesis we have to restrict the parameters to be compatible with fundamental and experimental requirements at zero temperature. Firstly, the potential must be bounded below - this leads to a set of constraints

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad 2\sqrt{\lambda_1\lambda_2} + \lambda_3 > 0, \quad 2\sqrt{\lambda_1\lambda_2} + \Sigma_{\pm} > 0 \quad (9)$$

where $\Sigma_{\pm} = \lambda_3 + \lambda_4 \pm \lambda_5$ [12, 19]. Secondly we require the eigenvalues of the scalar mass-squared matrix Eq.(5) to be positive.

The other restrictions come from the present experimental bounds on the scalar masses [20, 21, 22, 23]. For the charged scalars, $H^{\pm}$, which might be produced in a reaction $e^+e^- \rightarrow Z \rightarrow H^+H^-$, a mass lower bound of 42 GeV is well established. In the standard model with one Higgs doublet there is a mass lower bound of about 64 GeV on the Higgs scalar, but the situation is more complicated for the 3 neutral scalars of a two Higgs doublet model. Firstly, the commonly treated case is that with CP invariance (which applies for μ_3^2 being zero, real or pure imaginary) when there are two CP-even states ('scalars'), denoted as h (the lighter) and H (the heavier), and one CP-odd state ('pseudoscalar') denoted as A . The CP-even states mix ϕ_3 and ϕ_7 with mixing angle α and there is a reduction in the rate of $Z \rightarrow hZ^*$ of $\sin^2(\beta - \alpha)$ over the one doublet case, so that generally lower values of m_h are experimentally compatible. The experimental bound can be expressed as a curve in the m_h, m_A -plane such as that given in Ref.[24]. For us, μ_3^2 is in general complex ($\mu_R^2 \neq 0, \mu_I^2 \neq 0$), there is no CP invariance, and three mixing angles ($\alpha_1, \alpha_2, \alpha_3$) specify the scalar particles. In this situation, to lessen complications, we have chosen to exclude, from our basis parameter space, parameters giving any neutral masses less than 40 GeV. While this will exclude some cases which are experimentally allowed, and in principle could include some cases which are experimentally excluded, there is no significant effect on the overall picture from our statistical sample.

Finally we limit our basis space to the region where the perturbative treatment, implicit in the loop expansion for the effective potential, is likely to be valid. We choose the restriction $|\lambda_i| < \frac{1}{2}$. If we also require the perturbative two doublet model to be valid up to some grand unification

energy scale Λ , then the top quark mass is restricted to be less than or equal to its quasi-fixed point value m_t^{FP} determined by the renormalisation group equations. For $\Lambda = 10^{15}$ Gev, $m_t^{FP} \simeq 230 \sin \beta$ Gev, giving [14, 19] an upper limit of 150 Gev on the top quark mass for $\tan \beta = 0.9$. We therefore take $\tan \beta > 0.9$.

Without loss of generality we take $\mu_R^2 > 0$, $\mu_I^2 > 0$ and take a grid of 11×11 equally spaced points in the μ_R^2, μ_I^2 -plane such that

$$0 \leq \mu_R^2 \leq 1.5 \times 10^4 \quad \& \quad 0 \leq \mu_I^2 \leq 1.5 \times 10^4 \text{ GeV}^2 \quad (10)$$

For each square on the grid we test for electroweak baryogenesis, as described below, at each point of a random sample of 500 points in the basis space.

3.2 Sphaleron Transitions and Baryon Preservation

Having obtained the effective potential as a function of ϕ and T in Eq.(6), we can evaluate the curvature matrix at the origin $R_{ij} \equiv \partial_i \partial_j V(\phi, T)|_{\phi=0}$; retaining only the T^2 terms in the high temperature expansion of V . This eliminates the possibility of imaginary terms (as noted above after Eq.(8)) which can particularly occur near the origin of ϕ . At high enough temperature all of the eigenvalues will be positive, corresponding to a minimum at the origin. As the temperature descends, one of the eigenvalues is the first to become zero and we shall name the temperature, T_c , at which this happens as the critical temperature. At T_c there is no longer any barrier in some direction between what was the minimum at the origin and the new minimum away from the origin, and condensation of the scalar fields can progress rapidly without any suppression from a tunnelling factor. As mentioned in the introduction, we have to impose as a necessary condition that, after the phase transition, thermal transitions from one vacuum to another over the sphaleron energy barrier, E^{sph} , not be so great as to wash out the net baryon number gained previously.

Calculations with the kinetic equations for the dilution of baryonic charge just after the phase transition give a baryon preservation condition [1, 2, 3, 25]

$$\frac{v(T_c)}{T_c} \geq \xi \equiv \frac{45g}{4\pi B} \quad (11)$$

where B is a λ_i, μ_i^2 and g dependent number, and ξ varies between about 0.9 and 1.5 according to the value of B . We take $\xi = 1$. The condition, Eq.(11),

is a necessary condition for electroweak baryogenesis to succeed. This paper investigates what proportion of the basis parameter space satisfies Eq.(11) , as a function of the symmetry breaking parameter μ_3^2 .

3.3 Potentials Allowing Baryogenesis

In finding the proportion of basis parameter space which is baryon preserving, we must take note that in evaluating the one loop addition to the effective potential we used the high temperature expansion to develop a power series in m/T . We need to verify that this is not invalidated at temperature T_c , which of course is different for different potentials. It is the value of the masses, symbolically m , near the minimum of $V(\phi, T_c)$ which determine the vacuum expectation values $v_1(T_c)$, $v_2(T_c)$ and $\theta(T_c)$ and hence $v(T_c)$ in the baryon preservation condition Eq.(11). The masses which enter are the scalar, the gauge boson and the top, bottom and tau masses, and we impose the condition

$$\frac{m(T_c)}{T_c} < 1.6 \quad (12)$$

to ensure that we are safe in our expansion of the exponential in the Bose-Einstein/Fermi-Dirac term.

Proceeding now to the results, we present them as a function of μ_3^2 on a grid of 121 points bounded in the μ_3^2 -plane as in Eq.(10). The number of baryon preserving cases, out of the 500 starting cases for each point, is shown in Fig. 1. The outstanding feature is a significant and steep rise away from $\mu_3^2 = 0$. Over the whole region, the average proportion of successful cases to the number of initial random selection of basis cases (500 per grid point) is about 5%.

A previous investigation [18], in the same statistical spirit as the present one, just examined $\mu_3^2 = 0$ and so did not reveal the interesting dependence on μ_3^2 . Unfortunately, also that investigation was flawed by an invalid gauge choice (as noted in §(2) above) and by coupling the top quark equally to both Higgs doublets Φ_1 and Φ_2 in violation of the discrete symmetry chosen to ensure tree level flavour conserving neutral currents.

Plotted in Fig. 2 are the *average* values of T_c for the *successful* cases at each grid point. The (zero temperature) masses of the five physical scalars in the allowed baryon preserving cases of Fig. 1, range from about 50 GeV to 300 GeV. The frequency of occurrence of the various masses in all the allowed

cases is shown in Fig. 3. The general shape and range of the frequency curves is also present in the initial basis space sample; the baryon preservation condition sharpens up the shape, tending to cut off any low mass tail, to give the results shown in the Figures. We note that the proportion of lightest mass scalars below 60 GeV is small and other neutral masses are greater than 90 GeV; also there is a rate reduction factor (depending on β and the mixing angles) for the process $Z \rightarrow hZ^*$ compared to the minimal standard model case. So we conclude there is no significant contradiction with experimental bounds in the baryon preserving cases found.

4 Supersymmetry

The predictions of supersymmetry for the parameters of the scalar potential are not well prescribed because we do not know which of a number of supersymmetric candidate theories is correct and we do not know how and at what energy breaking occurs. Here we investigate, for electroweak baryogenesis, the theory which is closest to, and is a particular case of, our previous investigations - that is the Minimal Supersymmetric Standard Model (MSSM) which has just two Higgs doublets [13, 26, 27, 28]. Supersymmetry requires the quarks to couple to the doublets in the same way as we have previously assumed.

We shall model the supersymmetry breaking in the simplest possible way by taking the breaking to occur at an energy TS above which there is a complete spectrum of supersymmetric particles and supersymmetry is perfect, and below which there are only the known particles and the scalar Higgs particles. We follow a number of papers over recent years in taking the breaking to occur at fairly low energy and we investigate supersymmetry breaking energies between 0.5 TeV and 2.5 TeV. Here we take the top quark mass as 150 GeV. So, in our model, the top squarks have a mass TS and the λ parameters of the scalar potential are given at the scale TS by

$$\lambda_1 = \lambda_2 = \frac{(g^2 + g'^2)}{8}, \lambda_3 = \frac{(g^2 - g'^2)}{4}, \lambda_4 = \frac{-g^2}{2}, \lambda_5 = 0 \quad (13)$$

where g^2 and g'^2 are the electroweak couplings at the scale TS . We find the values of g^2 and g'^2 at TS by using the two Higgs doublet renormalisation group equations (RGE) to scale up from their known values at the electroweak

scale. We then use the RGE for the λ_i to scale down from TS to 246 GeV and take these as the λ_i of our scalar potential in Eq.(1); the λ_i are rather insensitive to further scaling to lower energies which would anyway depend on the details of the scalar masses. In the λ_i -RGE the top quark mass enters importantly and the bottom quark mass enters unimportantly except when $\tan\beta$ is very large, and this results in λ_2 being considerably greater than λ_1 . Since λ_5 remains zero, we take μ_3^2 to be real without loss of generality and it is a free parameter of the theory. The turning point equations at zero temperature, Eq.(3), determine μ_1^2 and μ_2^2 . Thus, for a given top mass, we have a theory with only two real free parameters, μ_3^2 and $\tan\beta = v_2/v_1$.

The results found indicate, for the range of supersymmetry breaking energy (TS) and μ_3^2 considered, that the baryon preservation condition rejects $\tan\beta > 1$ by finding a too low lowest scalar mass, m_h . (It turns out that in all cases found, $\sin^2(\beta - \alpha) \geq .7$, α being the scalar mass mixing angle, so that the experimental bound on m_h is similar to that in the standard model.) For $\tan\beta < 1$ and $TS > 1$ TeV there is a range of μ_3^2 which satisfies the baryon preservation condition and has m_h consistent with the experimental bound. For a given μ_3^2 , m_h then increases with $\cot\beta$ and also, though fairly slowly, with TS . In Fig. 4 we show, for $TS = 1.5$ TeV, lines of constant $\tan\beta$ for which the baryon preservation condition, Eq.(11), is satisfied on the μ_3^2, m_h -plane. (A kink occurs due to level repulsion when there is a near degeneracy in the mass matrix eigenvalues.)

However for $m_t \simeq 150$ GeV the parameter range $\tan\beta < 1$ is disfavoured, since the top Yukawa coupling is then large threatening the validity of perturbation theory at high energy. In particular if there are no new particles beyond those of the MSSM up to some very high energy (e.g. SUSY-GUT) scale Λ , the Yukawa RGEs imply that m_t is less than the infrared fixed point value m_t^{FP} . For $TS = 1.5$ TeV and $\Lambda = 10^{16}$ GeV, $m_t^{FP} \simeq 200 \sin\beta$ GeV and then for $\tan\beta \leq 1$ the top quark mass must be less than 140 GeV. The baryon preserving solutions of Fig. 4 require a smaller desert and new interactions must be introduced at a scale $\Lambda < 10^6$ (10^5) GeV for $\tan\beta = 2/3$ ($1/2$). Thus electroweak baryogenesis with the MSSM seems unattractive; in this we agree with the results of Ref.[28], which found only a small region of parameter space, $\tan\beta \approx 2$ and $m_A \gg m_Z$ (therefore outside of our basis parameter space), allowing baryogenesis, but did not report any results for $\tan\beta < 1$. Our conclusion is that investigations should be extended to non-minimal supersymmetric theories.

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Figure Captions

Figure 1. Frequency distribution of baryon number preserving cases in the complex μ_3^2 plane. Each block represents 500 initial cases. Note that the success rate is lowest (at about 3%) around $\mu_3^2 = 0$.

Figure 2. The average critical temperature T_c for the same cases as shown in Fig. 1. T_c ranges from $90 \rightarrow 160$ GeV, being lowest around $\mu_3^2 = 0$.

Figure 3. The frequency distribution of the scalar masses for the baryon preserving cases. m_{h_1}, m_{h_2} and m_{h_3} represent the scalar masses in increasing order.

Figure 4. Lines showing the lightest scalar mass (m_h) as a function of μ_R^2 in *baryon number preserving cases*, for 3 values of $\tan \beta$ with $m_t = 150$ GeV and $TS = 1500$ GeV in the MSSM case.

Figure 1.

Figure 2.

Figure 3.

Figure 4.