QCD Factors $a_1$ and $a_2$ Beyond Leading Logarithms versus Factorization in Non-leptonic Heavy Meson Decays

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Abstract

We calculate the QCD factors $a_1$ and $a_2$, entering the tests of factorization in non-leptonic heavy meson decays, beyond the leading logarithmic approximation in three renormalization schemes (NDR, HV, DRED). We investigate their $\mu$-dependence and the renormalization scheme dependence. We point out that $a_1$ for B-decays depends very weakly on $\Lambda_{\overline{MS}}$, $\mu$ and the choice of the renormalization scheme. For $\Lambda_{\overline{MS}}^{(1)} = 225 \pm 85$ MeV and $4 \text{ GeV} \leq \mu \leq 8 \text{ GeV}$ we find $a_1 = 1.01 \pm 0.02$ in accordance with phenomenology. The $\Lambda_{\overline{MS}}$, $\mu$ and scheme dependences of $a_2$ are on the other hand sizable. Interestingly, for the NDR scheme we find $a_2^{NDR} = 0.20 \pm 0.05$, in the ball park of recent phenomenological results and substantially larger than leading order estimates. However $a_2^{HV} = 0.16 \pm 0.05$ and $a_2^{DRED} = 0.15 \pm 0.05$. Implications of these findings for the tests of factorization in B-decays and D-decays are critically discussed.

*Supported by the German Bundesministerium für Forschung und Technologie under contract 06 TM 732 and by the CEC science project SC1-CT91-0729.
1 Introduction

In the factorization approach to non-leptonic meson decays [1, 2] one can distinguish three classes of decays for which the amplitudes have the following general structure [3, 4]:

\[ A_I = \frac{G_F}{\sqrt{2}} V_{CKM} a_1(\mu) \langle O_1 \rangle_F \quad (\text{Class I}) \] (1)

\[ A_{II} = \frac{G_F}{\sqrt{2}} V_{CKM} a_2(\mu) \langle O_2 \rangle_F \quad (\text{Class II}) \] (2)

\[ A_{III} = \frac{G_F}{\sqrt{2}} V_{CKM} [a_1(\mu) + a_2(\mu)] \langle O_1 \rangle_F \quad (\text{Class III}) \] (3)

Here \( V_{CKM} \) denotes symbolically the CKM factor characteristic for a given decay. \( O_1 \) and \( O_2 \) are local four quark operators present in the relevant effective hamiltonian, \( \langle O_i \rangle_F \) are the hadronic matrix elements of these operators given as products of matrix elements of quark currents and \( x \) is a non-perturbative factor equal to unity in the flavour symmetry limit. Finally \( a_i(\mu) \) are QCD factors which are the main subject of this paper.

As an example consider the decay \( B^0 \rightarrow D^+ \pi^- \). Then the relevant effective hamiltonian is given by

\[ H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* [C_1(\mu) O_1 + C_2(\mu) O_2] \] (4)

where

\[ O_1 = (\bar{d}_i u_j)_{V-A}(\bar{e}_j b_i)_{V-A} \quad O_2 = (\bar{d}_i u_j)_{V-A}(\bar{e}_j b_i)_{V-A} \] (5)

with \( i, j = 1, 2, 3 \) denoting colour indices and \( V-A \) referring to \( \gamma_\mu(1 - \gamma_5) \). \( C_1(\mu) \) and \( C_2(\mu) \) are short distance Wilson coefficients computed at the renormalization scale \( \mu = O(m_b) \). We will neglect the contributions of penguin operators since their Wilson coefficients are numerically very small as compared to \( C_{1,2} \) [5, 6]. Exceptions are CP-violating decays and rare decays which are beyond the scope of this paper. Note that we use here the labeling of the operators as given in [3, 4] which differs from [5, 6] by the interchange \( 1 \leftrightarrow 2 \). \( C_i \) and \( a_i \) are related as follows:

\[ a_1(\mu) = C_1(\mu) + \frac{1}{N} C_2(\mu) \quad a_2(\mu) = C_2(\mu) + \frac{1}{N} C_1(\mu) \] (6)

where \( N \) is the number of colours. We will set \( N = 3 \) in what follows.

Application of the factorization method gives

\[ A(\bar{B}^0 \rightarrow D^+ \pi^-) = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* a_1(\mu) \langle \pi^- | (\bar{d}_i u_i)_{V-A} | 0 \rangle \langle D^+ | (\bar{e}_j b_j)_{V-A} | \bar{B}^0 \rangle \] (7)
where $\langle D^+ \pi^+ | O_1 | \bar{B}^0 \rangle$ has been factored into two quark current matrix elements and the second term in $a_i(\mu)$ represents the contribution of the operator $O_2$ in the factorization approach.

Other decays can be handled in a similar manner [4]. Although the flavour structure of the corresponding local operators changes, the colour structure and the coefficients $C_i(\mu)$ remain unchanged. For instance $\bar{B}^0 \rightarrow \bar{K}^0 \psi$ and $B^- \rightarrow D^0 \bar{K}^-$ belong to class II and III respectively. Finally a similar procedure can be applied to D-decays with the coefficients $C_i$ evaluated at $\mu = O(m_\pi)$. Once the matrix elements have been expressed in terms of various meson decay constants and generally model dependent formfactors, predictions for non-leptonic heavy meson decays can be made. Moreover relations between non-leptonic and semi-leptonic decays can be found which allow to test factorization in a model independent manner.

Although the simplicity of this framework is rather appealing, it is well known that non-factorizable contributions must be present in the hadronic matrix elements of the current-current operators $O_1$ and $O_2$ in order to cancel the $\mu$ dependence of $C_i(\mu)$ or $a_i(\mu)$ so that the physical amplitudes do not depend on the arbitrary renormalization scale $\mu$. $\langle O_i \rangle_F$ being products of matrix elements of conserved currents are $\mu$ independent and the cancellation of the $\mu$ dependence in (1)-(3) does not take place. Consequently from the point of view of QCD the factorization approach can be at best correct at a single value of $\mu$, the so-called factorization scale $\mu_F$. Although the approach itself does not provide the value of $\mu_F$, the proponents of factorization expect $\mu_F = O(m_b)$ and $\mu_F = O(m_c)$ for B-decays and D-decays respectively.

Here we would like to point out that beyond the leading logarithmic approximation for $C_i(\mu)$ a new complication arises. As stressed in [7], at next to leading level in the renormalization group improved perturbation theory the coefficients $C_i(\mu)$ depend on the renormalization scheme for operators. Again only the presence of non-factorizable contributions in $\langle O_i \rangle$ can remove this scheme dependence in the physical amplitudes. However $\langle O_i \rangle_F$ are renormalization scheme independent and the factorization approach is of course unable to tell us whether it works better with an anti-commuting $\gamma_5$ in $D \neq 4$ dimensions (NDR scheme) or with another definition of $\gamma_5$ such as used in HIV (non-anticommuting $\gamma_5$ in $D \neq 4$) or DRED ($\gamma_5$ in $D = 4$) schemes. The renormalization scheme dependence of $a_i$ emphasized here is rather annoying from the factorization point of view as it precludes a unique phenomenological determination of $\mu_F$ as we will show explicitly below.
On the other hand, arguments have been given [8, 9, 4] that once $H_{ee}$ in (4) has been constructed, factorization could be approximately true in the case of two-body decays with high energy release [8], or in certain kinematic regions [9]. We will not repeat here these arguments, which can be found in the original papers as well as in a critical analysis of various aspects of factorization presented in [10]. Needless to say the issue of factorization does not only involve the short distance gluon corrections discussed here but also final state interactions which are discussed in these papers.

It is difficult to imagine that factorization can hold even approximately in all circumstances. In spite of this, it became fashionable these days to test this idea, to some extent, by using a certain set of formfactors to calculate $\langle O_i \rangle_F$ and by making global fits of the formulae (1)-(3) to the data treating $a_1$ and $a_2$ as free independent parameters. The most recent analyses of this type give for non-leptonic two-body B-decays [11]-[16]

$$a_1 \approx 1.05 \pm 0.10 \quad a_2 \approx 0.25 \pm 0.05$$

which is compatible with earlier analyses [4, 17]. The new CLEO II data [15] favour a positive value of $a_2$ in contrast to earlier expectations [3, 18] based on extrapolation from charm decays. At the level of accuracy of the existing experimental data and because of strong model dependence in the relevant formfactors it is not yet possible to conclude on the basis of these analyses whether the factorization approach is a useful approximation in general or not. It is certainly conceivable that factorization may apply better to some non-leptonic decays than to others [4, 8, 9, 10, 19, 20] and using all decays in a global fit may misrepresent the true situation.

Irrespective of all these reservations let us ask whether the numerical values in (8) agree with the QCD expectations for $\mu = 0(m_b)$?

A straightforward calculation of $a_i(\mu)$ with $C_i(\mu)$ in the leading logarithmic approximation [21] gives for $\mu = 5.0$ GeV and the QCD scale $\Lambda_{LO} = 225 \pm 85$ MeV

$$a_1^{LO} = 1.03 \pm 0.01 \quad a_2^{LO} = 0.10 \pm 0.02$$

Whereas the result for $a_1$ is compatible with the experimental findings, the theoretical value for $a_2$ disagrees roughly by a factor of two. The solution to this problem by dropping the $1/N$ terms in (6) suggested in [3] and argued for in [18, 22, 23] gives $a_1^{LO} = 1.12 \pm 0.02$ and $a_2^{LO} = -0.27 \pm 0.03$. Whereas the absolute magnitudes for $a_i$ are consistent with (8), the sign of $a_2$ is wrong. It has been remarked in [4] that the value of $a_2$ could be increased by using (6) with $\mu \gg m_b$. Indeed as shown in
table 1 for $\mu = 15 - 20$ GeV the calculated values for $a_1$ and $a_2$ are compatible with (8). The large value of $\mu = (3 - 4) m_\ell$ is, however, not really what the proponents of factorization would expect.

Table 1: Leading order coefficients $a_{1,2}^{LO}$ for B-decays.

<table>
<thead>
<tr>
<th>$\mu [GeV]$</th>
<th>$a_1^{(5)} = 140$ MeV</th>
<th>$a_{1,2}^{(5)} = 225$ MeV</th>
<th>$a_{1,2}^{(5)} = 310$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>1.024</td>
<td>0.124</td>
<td>1.030</td>
</tr>
<tr>
<td>10.0</td>
<td>1.011</td>
<td>0.191</td>
<td>1.014</td>
</tr>
<tr>
<td>15.0</td>
<td>1.007</td>
<td>0.224</td>
<td>1.008</td>
</tr>
<tr>
<td>20.0</td>
<td>1.004</td>
<td>0.246</td>
<td>1.005</td>
</tr>
</tbody>
</table>

Yet it should be recalled that in order to address the issue of the value of $\mu$ corresponding to the findings in (8) it is mandatory to go beyond the leading logarithmic approximation and to include at least the next-to-leading (NLO) terms. In particular only then one is able to use meaningfully the value for $\Lambda_{\overline{MS}}$ extracted from high energy processes. As an illustration we have used in (9) $\Lambda_{LO}^{(5)} = \Lambda_{\overline{MS}}$ which is of course rather arbitrary. To our surprise no NLO analysis of $a_1$ and $a_2$ has been presented in the literature in spite of the fact that the NLO corrections to $C_1$ and $C_2$ have been known for many years [24, 25].

At this point an important warning should be made. The coefficients $C_1$ and $C_2$ as given in [24, 25] and also in [7] cannot simply be inserted into (6) as done often in the literature. As stressed in [5] the coefficients given in [24, 25, 7] differ from the true coefficients of the operators $O_i$ by $O(\alpha_s)$ corrections which have been included in these papers in order to remove the renormalization scheme dependence. The only paper which gives the true $C_1$ and $C_2$ for B-decays is ref. [5], where these coefficients have been given for the NDR and HV renormalization schemes.

Now the main topic of ref. [5] was the ratio $\varepsilon'/\varepsilon$. Consequently the full set of ten operators including QCD-penguin and electroweak penguin operators had to be be considered which made the whole analysis rather technical. The penguin operators have, however, no impact on the coefficients $C_1$ and $C_2$ and also $O(\alpha_{QED})$ renormalization considered in [5] can be neglected here. On the other hand we are interested in the $\mu$ dependence of $a_1$ and $a_2$ around $\mu = O(m_\ell)$ and consequently we have to
generalize the numerical analysis of [5].

At this point it should be remarked that in the context of the leading logarithmic approximation, the sensitivity of \( a_2 \) to the precise values of \( C_i \) has been emphasised in ref. [26] long time ago. The expectation of Kühn and Rückl that higher order QCD corrections should have an important impact on the numerical values of \( a_2 \) turns out to be correct as we will demonstrate explicitly below.

The main objectives of the present paper are:

- The values of \( a_1(\mu) \) and \( a_2(\mu) \) beyond the leading logarithmic approximation,
- The analysis of their \( \mu \) and \( \Lambda_{\overline{\text{MS}}} \) dependences,
- The analysis of their renormalization scheme dependence in general terms, which we will illustrate here by calculating \( a_i(\mu) \) in three renormalization schemes: NDR, HV and DRED.

Since the \( \mu, \Lambda_{\overline{\text{MS}}} \) and the renormalization scheme dependences of \( a_i(\mu) \) are caused by the non-factorizable hard gluon contributions, this analysis should give us some estimate of the expected departures from factorization. It will also give us the answer whether, within the theoretical uncertainties, the problem of the small value of \( a_2 \), stressed by many authors in the past, can be avoided.

Our paper is organized as follows. In section 2 we give a set of compact expressions for \( C_1(\mu) \) and \( C_2(\mu) \) which clearly exhibit the \( \mu \) and renormalization scheme dependences. Subsequently in sections 3 and 4 we will critically analyse \( a_i \) for B-decays and D-decays respectively. Our main findings and conclusions are given in section 5.

## 2 Master Formulae

The coefficients \( C_i(\mu) \) can be written as follows:

\[
C_1(\mu) = \frac{z_+(\mu) + z_-(\mu)}{2} \quad C_2(\mu) = \frac{z_+(\mu) - z_-(\mu)}{2}
\]

where

\[
z_{\pm}(\mu) = \left[ 1 + \frac{\alpha_s(\mu)}{4\pi} J_\pm \right] \left[ \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{d_\pm} \left[ 1 + \frac{\alpha_s(M_W)}{4\pi} (B_\pm - J_\pm) \right] \quad (11)
\]

with

\[
J_\pm = \frac{d_\pm}{\beta_0} \beta_1 - \frac{\gamma_\pm^{(1)}}{2\beta_0}, \quad d_\pm = \frac{\gamma_\pm^{(0)}}{2\beta_0} \quad (12)
\]
Here we have introduced the parameter $\kappa_\pm$ which distinguishes between various renormalization schemes:

$$\kappa_\pm = \begin{cases} 
0 & (NDR) \\
\mp 4 & (HV) \\
\mp 6 - 3 & (DRED) 
\end{cases}$$

Thus $J_\pm$ in (12) can also be written as

$$J_\pm = (J_\pm)_{NDR} + \frac{3 \mp 1}{6} \kappa_\pm = (J_\pm)_{NDR} \pm \frac{\gamma_\pm^{(0)}}{12} \kappa_\pm$$

Setting $\gamma_\pm^{(1)}$, $B_\pm$ and $\beta_1$ to zero gives the leading logarithmic approximation [21]. The NLO corrections in the dimensional reduction scheme (DRED) have been first considered in [24]. The corresponding calculations in the NDR scheme and in the HV scheme have been presented in [25], where the DRED-results of [24] have been confirmed. In writing (14) we have incorporated the $-2\gamma_\pm^{(1)}$ correction in the HV scheme resulting from the non-vanishing two-loop anomalous dimension of the weak current. Similarly we have incorporated in $\gamma_\pm^{(1)}$ a finite renormalization of $\alpha_s$ in the case of the DRED scheme in order to work in all schemes with the usual $\overline{MS}$ coupling [27]. For the latter we take

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\overline{MS}}^2)} \left[ 1 - \frac{\beta_1}{\beta_0^2} \frac{\ln(\mu^2/\Lambda_{\overline{MS}}^2)}{\ln(\mu^2/\Lambda_{\overline{MS}}^2)} \right].$$

The formulae given above depend on $f$, the number of active flavours. In the case of B–decays $f = 5$. According to the most recent world average [28] we have:

$$\alpha_s(M_Z) = 0.117 \pm 0.007 \quad \Lambda_{\overline{MS}}^{(i)} = (225 \pm 85) \text{ MeV}$$

where the superscript stands for $f = 5$.

In the case of D–decays the relevant scale is $\mu = O(m_c)$. In order to calculate $C_i(\mu)$ for this case one has to evolve these coefficients from $\mu = O(m_b)$ down to $\mu = O(m_c)$ in an effective theory with $f = 4$. Matching $\alpha_s^{(i)}(m_b) = \alpha_s^{(4)}(m_b)$ we find to a very good approximation $\Lambda_{\overline{MS}}^{(4)} = (325 \pm 110) \text{ MeV}$. Unfortunately the necessity to evolve
$C_i(\mu)$ from $\mu = M_W$ down to $\mu = m_c$ in two different theories ($f = 5$ and $f = 4$) and eventually with $f = 3$ for $\mu < m_c$ makes the formulae for $C_i(\mu)$ in D–decays rather complicated. They can be found in [5]. Fortunately all these complications can be avoided by a simple trick, which reproduces the results of [5] to better than 0.5%. In order to find $C_i(\mu)$ for $1 \text{ GeV} \leq \mu \leq 2 \text{ GeV}$ one can simply use the master formulae given above with $\Lambda_{\overline{MS}}^{(5)}$ replaced by $\Lambda_{\overline{MS}}^{(4)}$ and $f = 4.15$. The latter "effective" value for $f$ allows to obtain a very good agreement with [5]. The nice feature of this method is that the $\mu$ and renormalization scheme dependences of $C_i(\mu)$ can be studied in simple terms.

Returning to (11) we note that $(B_\pm - J_\pm)$ is scheme independent. The scheme dependence of $z_{\pm}(\mu)$ originates then entirely from the scheme dependence of $J_\pm$ which has been explicitly shown in (17). We should stress that by the scheme dependence we always mean the one related to the operator renormalization. The scheme for $\alpha_s$ is always $\overline{MS}$. The scheme dependence present in the first factor in (11) has been removed in [25] by multiplying $z_{\pm}(\mu)$ by $(1 - B_\pm \alpha_s(\mu)/4\pi)$ and the corresponding hadronic matrix elements by $(1 + B_\pm \alpha_s(\mu)/4\pi)$. Although this procedure is valid in general, it is not useful in the case of the factorization approach which precisely omits the non-factorizable, scheme dependent corrections such as $B_\pm$ or $J_\pm$ in the hadronic matrix elements. Consequently in what follows we will work with the true coefficients $C_i(\mu)$ of the operators $O_i$ as given in (10) and (11).

In order to exhibit the $\mu$ dependence on the same footing as the scheme dependence, it is useful to rewrite (11) as follows:

$$z_{\pm}(\mu) = \left[1 + \frac{\alpha_s(m_b)}{4\pi} J_{\pm}(\mu) \right] \left[ \frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right] \left[ 1 + \frac{\alpha_s(M_W)}{4\pi} (B_\pm - J_\pm) \right]$$

(20)

with

$$J_{\pm}(\mu) = (J_{\pm})_{\text{NDR}} \pm \frac{\gamma_{\pm}^{(0)}}{12} \kappa_{\pm} + \frac{\gamma_{\pm}^{(0)}}{2} \ln(\frac{\mu^2}{m_b^2})$$

(21)

summarizing both the renormalization scheme dependence and the $\mu$–dependence. Note that in the first parenthesis in (20) we have set $\alpha_s(\mu) = \alpha_s(m_b)$ as the difference in the scales in this correction is still of a higher order. We also note that the scheme and the $\mu$–dependent terms are both proportional to $\gamma_{\pm}^{(0)}$. This implies that a change of the renormalization scheme can be compensated by a change in $\mu$. From (21) we find generally

$$\mu_{\pm}^{\pi} = \mu_{\text{NDR}} \exp\left( \pm \frac{\gamma_{\pm}^{(i)}}{12} \right)$$

(22)
where \( i \) denotes a given scheme. From (16) we have then
\[
\mu_{HV} = \mu_{NDR} \exp \left( \frac{1}{3} \right) \quad \mu_{DRED}^\pm = \mu_{NDR} \exp \left( \frac{2 \pm 1}{4} \right)
\] (23)

Evidently whereas the change in \( \mu \) relating HV and NDR is the same for \( z_+ \) and \( z_- \) and consequently for \( a_i(\mu) \) and \( C_i(\mu) \), the relation between NDR and DRED is more involved. In any case \( \mu_{HV} \) and \( \mu_{DRED}^\pm \) are larger than \( \mu_{NDR} \). This discussion shows that a meaningful analysis of the \( \mu \) dependence of \( C_i(\mu) \) can only be made simultaneously with the analysis of the scheme dependence.

Using (20) and (21) we can find the explicit dependence of \( a_i \) on \( \mu \) and the renormalization scheme:
\[
\Delta a_{1,2}(\mu) = \frac{\alpha_s(m_b)}{3\pi} [F_+ \mp F_-] \ln \left( \frac{\mu^2}{m_b^2} \right) + \frac{\alpha_s(m_b)}{18\pi} [F_+ \kappa_+ \pm F_- \kappa_-]
\] (24)

where \( F_\pm \) denotes the product of the last two factors in (20) which are scheme independent. For \( m_b = 4.8 \text{ GeV} \), \( \Lambda_{\overline{MS}}^{(5)} = 225 \pm 85 \text{ MeV} \) we have \( F_+ = 0.88 \pm 0.01 \) and \( F_- = 1.28 \pm 0.03 \). It is evident from (24) that the \( \mu \) and renormalization scheme dependences are much smaller for \( a_1 \) than for \( a_2 \). We will verify this numerically below.

We have written all the formulae without invoking heavy quark effective theory (HQET). It is sometimes stated in the literature that for \( \mu < m_b \) in the case of B-decays one has to switch to HQET. In this case for \( \mu < m_b \) the anomalous dimensions \( \gamma_\pm \) differ from those given above [29]. We should however stress that switching to HQET can be done at any \( \mu < m_b \) provided the logarithms \( \ln(m_b/\mu) \) in \( \langle O_i \rangle \) do not become too large. Similar comments apply to D-decays with respect to \( \mu = m_c \).

Of course the coefficients \( C_i \) calculated in HQET for \( \mu < m_b \) are different from the coefficients presented here. However the corresponding matrix elements \( \langle O_i \rangle \) in HQET are also different so that the physical amplitudes remain unchanged. Again, if factorization for \( \langle O_i \rangle \) is used, it matters to some extent at which \( \mu \) the HQET is invoked. For the range of \( \mu \) considered here this turns out to be inessential.

### 3 B-Decays

The coefficients \( C_i(\mu) \) are shown in tables 2 and 3 for different \( \mu \), \( \Lambda_{\overline{MS}}^{(5)} \) and the three renormalization schemes in question. We include these results because they should be useful independently of the factorization issue. The corresponding values for \( a_i(\mu) \) are given in tables 4 and 5. We observe:
The coefficient $C_1(\mu)$ for B-decays.

<table>
<thead>
<tr>
<th>$\mu$ [GeV]</th>
<th>$\Lambda_{MS}^{(5)} = 140$ MeV</th>
<th>$\Lambda_{MS}^{(5)} = 225$ MeV</th>
<th>$\Lambda_{MS}^{(5)} = 310$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NDR</td>
<td>HV</td>
<td>DRED</td>
</tr>
<tr>
<td>4.0</td>
<td>1.074</td>
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<td>1.073</td>
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<td>5.0</td>
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<td>7.0</td>
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</tr>
<tr>
<td>8.0</td>
<td>1.042</td>
<td>1.055</td>
<td>1.039</td>
</tr>
</tbody>
</table>

The coefficient $C_2(\mu)$ for B-decays.

<table>
<thead>
<tr>
<th>$\mu$ [GeV]</th>
<th>$\Lambda_{MS}^{(5)} = 140$ MeV</th>
<th>$\Lambda_{MS}^{(5)} = 225$ MeV</th>
<th>$\Lambda_{MS}^{(5)} = 310$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NDR</td>
<td>HV</td>
<td>DRED</td>
</tr>
<tr>
<td>4.0</td>
<td>-0.175</td>
<td>-0.211</td>
<td>-0.216</td>
</tr>
<tr>
<td>5.0</td>
<td>-0.151</td>
<td>-0.184</td>
<td>-0.189</td>
</tr>
<tr>
<td>6.0</td>
<td>-0.133</td>
<td>-0.164</td>
<td>-0.169</td>
</tr>
<tr>
<td>7.0</td>
<td>-0.118</td>
<td>-0.148</td>
<td>-0.153</td>
</tr>
<tr>
<td>8.0</td>
<td>-0.106</td>
<td>-0.135</td>
<td>-0.140</td>
</tr>
</tbody>
</table>

- The coefficient $a_1$ is very weakly dependent on $\mu, \Lambda_{MS}^{(5)}$ and the choice of the renormalization scheme. In the full range of parameters considered we find:

$$a_1 = 1.01 \pm 0.02$$

in an excellent agreement with (8). The weak dependence of $a_1$ on the parameters considered can be understood by inspecting (24).

- The coefficient $a_2$ depends much stronger on $\mu, \Lambda_{MS}^{(5)}$ and the choice of the renormalization scheme. Interestingly, for the NDR scheme we find

$$a_2^{NDR} = 0.20 \pm 0.05$$

which is in the ball park of the experimental findings in (8). Smaller values are found for HV and DRED schemes:

$$a_2^{HV} = 0.16 \pm 0.05 \quad a_2^{DRED} = 0.15 \pm 0.05$$
This exercise shows that by including NLO QCD corrections and choosing "appropriately" the renormalization scheme for the operators $O_i$, one can achieve the agreement of the QCD factor $a_2$ in (6) evaluated at $\mu = O(m_b)$ with the phenomenological findings. No high scales as found in the leading logarithmic approximation are necessary. Moreover, as it is clear from (22), by choosing a scheme with positive $\kappa_+$ and negative $\kappa_-$ even higher values for $a_2$ at $\mu = m_b$ can be obtained.

In spite of the possibility of "fitting" the phenomenological values for $a_2$ by choosing appropriately the renormalization scheme, the sizable dependence of $a_2$ on $\mu$ and the renormalization scheme is rather disturbing from the point of view of the factorization approach. On the other hand it is interesting that within $2 - 3\%$ we find $a_1 = 1$ in the full range of the parameters considered. We will return to these issues in the final section.
4 Charm Decays

The phenomenological analyses of (1)-(3) give in the case of two-body D meson decays [4]:

\[ a_1 \approx 1.2 \pm 0.10 \quad a_2 \approx -0.5 \pm 0.10 \]  \hspace{1cm} (28)

The different sign of \( a_2 \) compared with the case of B-decays shows that the structure of non-leptonic D decays differs considerably from the one in B decays. Calculating \( a_i \) according to our master formulae for scales \( 1.0 \text{ GeV} \leq \mu \leq 2.0 \text{ GeV} \) we find that \( a_1 \) roughly agrees with (28). On the other hand as already found in the leading order [3, 18, 4], the coefficient \( a_2 \) is generally substantially smaller than its phenomenological value (28) due to strong cancellation between \( C_2 \) and \( C_1/3 \). Only for \( \mu = 1.0 \text{ GeV} \), the largest \( \Lambda_{\overline{MS}} \) and HV and DRED schemes it is possible to obtain \( a_2 \) within a factor of two from the value in (28). Otherwise one finds typically \( a_2 = O(0.1) \) and consequently branching ratios for class II decays by an order of magnitude smaller than the experimental branching ratios.

Because of these findings, a “new factorization” [3] approach has been proposed in which the “1/N” terms in (6) are discarded. Some arguments for this modified approach can be given in the frameworks of 1/N expansion [18] and QCD sum rules [22]. Yet ”the rule of discarding 1/N terms” is certainly not established both theoretically [23] and phenomenologically [30]. Moreover as we already mentioned in the introduction, it does not work for B decays giving wrong sign for \( a_2 \). For completeness however we show in tables 6 and 7 the values of \( a_1 = C_1 \) and \( a_2 = C_2 \) relevant for D-decays. We observe:

- the coefficient \( a_1 \) is weakly dependent on the choice of the renormalization scheme for fixed \( \mu \) and \( \Lambda_{\overline{MS}}^{(4)} \). The dependence on \( \mu \) and \( \Lambda_{\overline{MS}}^{(4)} \) is sizable. In the full range of parameters we find

\[ a_1 = 1.31 \pm 0.19 \]  \hspace{1cm} (29)

which is compatible with phenomenology.

- the coefficient \( a_2 \) depends much stronger on the renormalization scheme than \( a_1 \) and the dependence on \( \mu \) and \( \Lambda_{\overline{MS}}^{(4)} \) is really large. Restricting the range of \( \mu \) to \( \mu = 1.25 \pm 0.25 \text{ GeV} \) we find

\[ a_2^{NDR} = -0.47 \pm 0.15 \quad a_2^{HV} \approx a_2^{DRED} \approx -0.60 \pm 0.22 \]  \hspace{1cm} (30)
in the ball park of (28).

- the dependences of $a_1$ and $a_2$ on the parameters considered are stronger in the charm sector than in B decays because of the larger QCD coupling involved.

Table 6: The coefficient $C_1(\mu)$ for D-decays.

<table>
<thead>
<tr>
<th>$\mu$ [GeV]</th>
<th>$\Lambda_{\overline{MS}}^{(4)} = 215$ MeV</th>
<th>$\Lambda_{\overline{MS}}^{(4)} = 325$ MeV</th>
<th>$\Lambda_{\overline{MS}}^{(4)} = 435$ MeV</th>
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<td></td>
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<td>HV</td>
<td>DRED</td>
</tr>
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<td>1.259</td>
<td>1.224</td>
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<tr>
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<td>1.152</td>
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<td>1.152</td>
<td>1.128</td>
</tr>
</tbody>
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Table 7: The coefficient $C_2(\mu)$ for D-decays.

<table>
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<tr>
<th>$\mu$ [GeV]</th>
<th>$\Lambda_{\overline{MS}}^{(4)} = 215$ MeV</th>
<th>$\Lambda_{\overline{MS}}^{(4)} = 325$ MeV</th>
<th>$\Lambda_{\overline{MS}}^{(4)} = 435$ MeV</th>
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<tbody>
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5 Final Remarks

We have calculated the QCD factors $a_1$ and $a_2$, entering the tests of factorization in non-leptonic heavy meson decays, beyond the leading logarithmic approximation. In particular we have pointed out that $a_i$ in QCD depend not only on $\mu$ and $\Lambda_{\overline{MS}}$, but also on the renormalization scheme for the operators. The latter dependence precludes a unique determination of the factorization scale $\mu_F$, if such a scale exists.
at all, at which the factorization approach would give results identical to QCD. For instance going from the NDR scheme to the HV scheme is equivalent, in the case of current-current operators $O_i$, to a change of $\mu_F$ by 40%. Simultaneously we would like to emphasize the strong dependence of a possible $\mu_F$ on $\Lambda_{\overline{MS}}$. The latter uncertainty can however be considerably reduced in the future by reducing the uncertainty in $\Lambda_{\overline{MS}}$ extracted from high energy processes. The NLO calculations of $a_1$ and $C_i$ presented here, allow a meaningful use of $\Lambda_{\overline{MS}}$, extracted from high energy processes, in the non-leptonic decays in question.

On the phenomenological side the following results are in our opinion interesting. In the simplest renormalization scheme with anti-commuting $\gamma_5$ (NDR), $\Lambda^{(1)}_{\overline{MS}} = (225 \pm 85) \text{ MeV}$ and $\mu = 6 \pm 2 \text{ GeV}$, we find in the case of B-decays

$$a_1^{NDR} = 1.02 \pm 0.01 \quad a_2^{NDR} = 0.20 \pm 0.05$$

which are in the ball park of the results of phenomenological analyses. In particular, the inclusion of NLO corrections in the NDR scheme appears to "solve" the problem of the small value of $a_2$ obtained in the leading order.

In the case of D-decays, $\Lambda^{(4)}_{\overline{MS}} = (325 \pm 110) \text{ MeV}$, $\mu = 1.25 \pm 0.25 \text{ GeV}$ and using the "new factorization" approach we find

$$a_1^{NDR} = 1.26 \pm 0.10 \quad a_2^{NDR} = -0.47 \pm 0.15$$

again in the ball park of phenomenological analyses. The standard factorization gives for D-decays $a_1^{NDR} \approx 1.10 \pm 0.05$ and $a_2^{NDR} \approx -0.06 \pm 0.12$ for the same range of parameters. The result for $a_2$ is phenomenologically unacceptable.

We have also stressed that similar results for $a_1$ in B-decays are obtained in HV and DRED schemes. Moreover the very weak dependence of $a_1$ on $\mu$ and $\Lambda_{\overline{MS}}$ indicates that $a_1$ is predicted to be close to unity in agreement with phenomenology of factorization. However the $\mu$, $\Lambda_{\overline{MS}}$ and scheme dependences of $a_2$ for B decays and in particular for D decays are rather sizable.

In our opinion the failure of the usual factorization approach in D decays and the strong dependence of $a_2$ on $\mu$, $\Lambda_{\overline{MS}}$ and the choice of the renormalization scheme indicate that non-factorizable contributions must play generally an important role in heavy meson non-leptonic decays if QCD is the correct description of these decays. In K meson decays the non-factorizable contributions are known to be very important anyway [23, 31]. Consequently we expect that, when the experimental data improves,
sizable departures from factorization should become visible in particular in decays belonging to class II. An exception could be the class I in B decays where an accidental approximate cancellations of \( \mu \) and renormalization scheme dependences takes place in \( a_1 \). It should however be stressed that the stability of \( a_1 \) with respect to changes of \( \mu \) and the renormalization scheme is only a necessary condition for an "effective" validity of factorization in class I decays. It certainly does not imply that factorization of matrix elements indeed takes place.

In spite of these critical remarks the tests of factorization in non-leptonic decays are important because the patterns of the expected departures from factorization will teach us about the non-factorizable contributions. Recent discussion of such contributions can be found in [20]. In this connection, once the data and the models for formfactors improve, it would be useful to investigate in detail how the phenomenologically extracted parameters \( a_1 \) and \( a_2 \) depend on the decay channel considered.

I would like to thank Gerhard Buchalla and Robert Fleischer for critical reading of the manuscript. I also thank Reinhold Rückl for a discussion related to his work.

References


