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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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AND BLACK-HOLE INSTABILITIES

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ATOMIC ENERGY
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SCIENTIFIC
AND CULTURAL
ORGANIZATION

MIRAMARE-TRIESTE

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
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**STRING-LOOP CORRECTIONS TO EFFECTIVE ACTION
AND BLACK-HOLE INSTABILITIES¹**

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ABSTRACT

Following the ideas of ref. [3], to account for instability of $2D$ black-hole solution, we discuss possible imaginary string-loop corrections to the coefficient at the Einstein term in string effective action. In closed bosonic string theory, such corrections appear because of the tachyon contribution to the integration measure over the moduli. In superstring theory (in critical as well as in non-critical dimensions), no one-string-loop complex corrections to the Einstein term are generated and the mechanism for generating black-hole instabilities proposed in ref. [3] fails.

MIRAMARE – TRIESTE

July 1994

¹Supported in part by the Russian Fund for Fundamental Researches, Grant No.94-02-05869-a.

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1 Introduction

Recently considerable attention attracted two-dimensional black-hole solutions of two-dimensional string theories [1, 2]. Because of their relative simplicity, these solutions are expected to give insight into the long-standing problems of black-hole physics. An interesting idea put forward in this context is that string effective action (EA), from which the black-hole solution is derived, as a result of quantum effects, may acquire imaginary contributions [3]. These may yield an imaginary correction to the mass of black hole, i.e. result in instability of solution (static at the tree-level).

The aim of this paper is to investigate under what circumstances string-loop perturbation theory may provide imaginary corrections to the tree-level EA and, in particular, modify the tree-level coefficient at the Einstein term. In string theory, imaginary contributions can appear as a result of regularization of (exponentially) divergent integrals over the moduli [3-5].

In $2D$ string theory modular divergences are absent, but if $2D$ theory is considered as a limit from a theory in $D > 2$, in which case there can be modular divergences, the question requires more careful analysis.

Black hole appears as a solution of classical equations of motion derived from the graviton-dilaton part of the tree-level EA

$$S = \int d^D X \sqrt{G} e^{-2\phi} \left[\Lambda + \alpha' \left(R + 4(\nabla\phi)^2 \right) + \dots \right] \quad (1)$$

in two-dimensional space-time $D = 2$ [1, 2].

(1) is the $O(\alpha')$ part of EA corresponding to general closed bosonic string theory with the world-sheet action

$$I = \frac{1}{2\pi} \int d^2 z \sqrt{g} \left(g_{ab} G_{\mu\nu} \partial^a X^\mu \partial^b X^\nu + \alpha' R^{(2)} \phi(X) + T(X) \right). \quad (2)$$

Since the solution for the dilaton field is defined up to an additive constant a , the mass of the black hole, which is determined by the coefficient at the Einstein term is, correspondingly, defined up to a factor $M \sim (\alpha')^{-\frac{1}{2}} e^a$ [1]. If the string-loop effects produce complex corrections to the tree-level EA and, in particular, the coefficient at the Einstein term acquires an imaginary part, this can be interpreted as an imaginary mass shift to the mass of the classical black hole [3].

In non-critical dimensions $2D$ gravity does not decouple. In conformal gauge, in ansatz of [6], dynamics of $2D$ gravity is that of conformal mode and is given by the Liouville action. Liouville field φ can be considered as an additional coordinate in the target space, however, without translation invariance in this direction [7]. The resulting action is again of the form (2), but with the dilaton having a “classical part”

$$\phi(X, \varphi) = Q\varphi + d(X, \varphi).$$

Consistency of the theory requires that β -functions vanish, and, in particular, the equation $\beta_\phi = 0$ is

$$0 = \Lambda + \alpha' \left(R + 4(\nabla\phi)^2 \right) + \dots \quad (3)$$

where

$$\Lambda = \frac{D + 3Q^2 - 25}{3}.$$

2D cosmological term in the Liouville action or its generalization in the form of tachyon background in the σ -model action (2) are required to be conformal tensors of weight (1, 1). In flat graviton background, this condition together with the requirement that the world-sheet cosmological term is real results in restriction $D \leq 1$ [6, 8]. Since we are interested in the range $1 < D \leq 25$ (or $1 < D \leq 10$ for superstring theory), we assume that restriction $D \leq 1$ can be somehow avoided. Specifically, we assume that either cosmological terms in the matter and Liouville action cancel, or, in the case of general σ -model action, the backgrounds are such that in the range $1 < D \leq 25$ tachyon equation has an admissible real solution (cf. [9]). The former possibility is supported by calculation of correlators in non-critical string theory [10].

In the next section we briefly review some basic facts concerning string EA. In Section 3, general expressions are applied to calculation of one-string-loop contribution to closed bosonic string EA. Although in bosonic string theory the coefficient at the Einstein term acquires imaginary correction, this is due to tachyonic divergence in the integrals over the moduli. In Section 4, we discuss the superstring theory in non-critical dimensions and argue that in this theory, no corrections to Einstein term are generated. Finally, in Section 5, we discuss the possible complex contributions to the superstring EA arising from decay of heavy massive states into the light ones.

2 String effective action

Before we discuss string-loop corrections to the tree-level action, we shall briefly review basic facts about the string EA. The full EA S is equal to renormalized generating functional of string amplitudes $\hat{Z}_R(G_R, \phi_R)$. Generating functional $\hat{Z}(G, \phi, \epsilon)$ is equal to the sum of generating functionals $\hat{Z}_n$, calculated on surfaces of genus n :

$$\hat{Z} = \sum_{n=0}^{\infty} \hat{Z}_n. \quad (4)$$

To implement renormalization of the theory consistently it is necessary that all the divergences should be regularized in a universal way with the same cutoff parameter ϵ [11, 12].

An explicit realization of such regularization is provided by Schottky parametrization [13] of the “extended” moduli space. In this parametrization, a surface of genus n (sphere with n handles) is mapped on the complex plane $\mathbb{C}$ with n pairs of holes with the boundary circles pairwise identified (see e.g. [14, 12]). $3n$ complex moduli correspond to coordinates of the centers of holes, radii and twist angles in identification of boundaries. On the complex plane $\mathbb{C}$, acts the $SL(2, \mathbb{C})$ group. If the corresponding (Möbius) symmetry is not fixed, then the volume of the group $SL(2, \mathbb{C})$ enters the amplitudes as a universal factor. Fixing 3 complex parameters of the group $SL(2, \mathbb{C})$, one reduces the number of independent moduli to $3n - 3$.

Singularities of the amplitudes can appear if coordinates of several vertex operators (punctures) tend to each other and also when the holes from the handles shrink to a point. The advantage of Schottky parametrization is in the fact that both types of singularities can be regularized in a universal way by introducing the short-distance cutoff ϵ which enters propagators as well as the integration measure over the moduli. Note that in this regularization “local” and “modular” divergences mix together [11, 12]. In this formalism

the regularized generating functional of string amplitudes is given by [11]⁴

$$\hat{Z} = \frac{\partial}{\partial \ln \varepsilon} (Z_0 + Z_1 + \dots). \quad (5)$$

The $O(\alpha')$ part of the generating function Z_n is

$$Z_n = \int [d\mu]_n \int d^D y \sqrt{G} e^{2(n-1)\phi} \left[1 + \alpha' (b_1^{(n)} R + b_2^{(n)} \nabla^2 \phi) + \dots \right]. \quad (6)$$

Here $[d\mu]_n$ is the measure on the moduli space in Schottky parametrization, G and ϕ are background metric and dilaton. The coefficients $b_{1,2}^{(n)}$ are functions of moduli and are given by the expressions [11]:

$$\begin{aligned} b_1 &= a_1 + a_2 + a_3, \\ a_1 &= \frac{\pi}{3} \int d^2 z \sqrt{g} \left[\nabla_a \nabla'^a G(z, z') G(z, z') - (\nabla^a G(z, z'))^2 \right]_{z=z'}, \\ a_2 &= -\frac{\pi}{3} \int d^2 z \sqrt{g} K_\varepsilon(z, z) G(z, z), \\ a_3 &= -\frac{2\pi}{3V} \int d^2 z \sqrt{g} G(z, z'); \quad V = \int d^2 z \sqrt{g}, \\ b_2 &= -\frac{1}{4} \int d^2 z \sqrt{g} R^{(2)} G(z, z). \end{aligned} \quad (7)$$

$G(z, z')$ is the regularized propagator, $K_\varepsilon(z, z')$ is the regularized δ -function on a surface of genus n equipped with a metric g . The coefficients $b_{1,2}^{(n)}$ have logarithmic divergence stemming from the limit of coinciding arguments in the integrands (7).

$$b_1^{(n)} = \frac{1}{2} \ln \varepsilon + \bar{b}_1^{(n)}; \quad b_2^{(n)} = (n-1) \ln \varepsilon + \bar{b}_2^{(n)}. \quad (8)$$

Here $\bar{b}_{1,2}^{(n)}$ are the moduli-dependent finite parts.

3 One-loop closed bosonic string EA

The tree-level expression for the functional Z_0 contains no integration over the moduli. At the one-loop level (topology of the torus), the “extended” moduli space is parametrized by three complex parameters ξ , η and k . The measure on the “extended” moduli space is ⁵

$$d\mu_1 = \frac{d^2 \xi d^2 \eta}{|\xi - \eta|^4} [d^2 k].$$

In parametrization $k = e^{2\pi i \tau}$, the measure $[d^2 k]$ is

$$[d^2 \tau] = \frac{d^2 \tau}{\tau_2^2} \left(\tau_2 |\eta(\tau)|^4 \right)^{-\frac{D-2}{2}} \quad (9)$$

⁴Derivative with respect to $\ln \varepsilon$ removes Möbius infinities.

⁵The shape of the integration domain over the moduli is parametrization-dependent. However, for our discussion this is irrelevant.

where $\eta(\tau)$ is the Dedekind η -function $\eta(\tau) = k^{\frac{1}{24}} \prod_1^\infty (1 - k^m)$.

Note that $k \sim e^{-2\pi\tau_2}$ as $\tau_2 \rightarrow \infty$.

Consider the Green function for the scalar Laplacian on the torus in the form [12]

$$G(z_1, z_2) = G_0(z_1 - z_2) - \ln |\tilde{E}(k, \lambda)|^2 + \frac{(\ln |\lambda|)^2}{2\pi\tau_2}, \quad (10)$$

where

$$\begin{aligned} \tilde{E}(k, \lambda) &= \prod_{m=1}^{\infty} \frac{(1 - \lambda k^m)(1 - \lambda^{-1} k^m)}{(1 - k^m)^2}, \\ \lambda &= (z_1 - \xi)(z_2 - \eta)(z_1 - \eta)^{-1}(z_2 - \xi)^{-1}, \\ G_0(z_1 - z_2) &= -\ln(|z_1 - z_2|^2 + \varepsilon^2). \end{aligned} \quad (11)$$

The standard expression for the propagator [15, 16, 17] is obtained if one (partially) fixes the "Möbius" gauge by setting $\xi = 0, \eta = 0$:

$$G(z|\tau) = -\ln \left| \frac{\vartheta_1(z, \tau)}{\vartheta_1'(0)} \right|^2 + \frac{2\pi}{\tau_2} (\text{Im } z)^2.$$

In the limit $z_1 \sim z_2$, the asymptotics of the propagator (10) is

$$\begin{aligned} G(z_1, z_2) &\simeq -\ln(|z_1 - z_2|^2 + \varepsilon^2) + \frac{1}{2\pi\tau_2} \left[\text{Re} \left((z_1 - z_2) \left(\frac{1}{z_1 - \xi} - \frac{1}{z_1 - \eta} \right) \right) \right]^2 \\ &+ \sum_{m=1}^{\infty} \left[\frac{mk^m}{(1 - k^m)} (z_1 - z_2)^2 \left(\frac{1}{z_1 - \xi} - \frac{1}{z_1 - \eta} \right)^2 + \text{c.c.} \right]. \end{aligned} \quad (12)$$

Substituting the asymptotics (12) into (7), one gets the following expression for Z_1 :

$$Z_1 = \lambda_1 \int d^D y \sqrt{G} \int d\mu_1 [1 + \alpha' R \left(\frac{1}{2} \ln \varepsilon + \bar{b}_1^{(1)}(\xi, \eta, k) \right) + \dots]. \quad (13)$$

In the region $\xi \sim \eta$, the measure is singular, but the finite parts $\bar{b}_1^{(1)}(\xi, \eta, k)$ are regular as $\xi \rightarrow \eta$ and their expressions can be obtained from (12) and (7). Omitting the power divergence in (13) and keeping the logarithmic term from the measure which is regularized by the same parameter ε as the propagator, one has

$$Z_1 = \lambda_1 \int d^D y \sqrt{G} \int [d^2 \tau] \left[\ln \varepsilon \left(1 + \frac{\alpha' R}{2} \ln \varepsilon \right) + \alpha' R (C_1(\tau) \ln \varepsilon + C_2(\tau)) + \dots \right] \quad (14)$$

where $C_{1,2}(\tau)$ are obtained from $\bar{b}_1^{(1)}$ by integration over ξ, η . Notice, that $C_{1,2}(\tau)$ are polynomially bounded functions of τ . Combining the tree-level and one-loop contributions, the generating functional is obtained as

$$\hat{Z} = \int d^D y \sqrt{G} \left\{ e^{-2\phi} \alpha' R + \varrho \int [d^2 \tau] [(1 + \alpha' R \ln \varepsilon) + \alpha' R C_1(\tau)] \right\} \quad (15)$$

where $\varrho = \frac{\lambda_1}{\lambda_0}$ and λ_0 is a normalization coefficient at the tree-level contribution. Introducing the renormalized fields $G_{\mu\nu}^{(R)}$ and $\phi^{(R)}$ by the relations

$$G_{\mu\nu} = G_{\mu\nu}^{(R)} - \alpha' R_{\mu\nu}^{(R)} \ln \varepsilon + \frac{\varrho}{4} e^{2\phi^{(R)}} G_{\mu\nu}^{(R)} \ln \varepsilon + \dots,$$

$$\phi = \phi^R + \frac{\alpha'}{2} \partial^2 \phi^R \ln \varepsilon + \frac{\varrho}{16} (D+2) e^{2\phi^{(R)}} \ln \varepsilon, \quad (16)$$

one obtains the EA [11, 12]

$$S = \int d^D y \sqrt{G} e^{-2\phi} \left\{ \alpha' R \left(1 + \lambda e^{2\phi} \int [d^2 \tau] C_1(\tau) \right) + \lambda e^{2\phi} \int [d^2 \tau] \right\} \quad (17)$$

(here all the fields are renormalized, but to simplify notations we omitted the subscript (R)).

In the EA (17), the integrals over τ diverge exponentially and can be defined by analytic continuation [3, 4]. The exponential growth of the measure is due to the presence of tachyon in the spectrum. In superstring theory, however, there is no tachyon and the integral over the moduli is finite.

4 Fermionic string in non-critical dimension

At the tree level, closed fermionic string theory in flat backgrounds in non-critical dimensions is very similar to non-critical bosonic theory [6]. With necessary modifications, tree-level amplitudes are calculated essentially in the same way as in bosonic theory. A new element appears at the one-loop level where amplitudes are obtained as the sum of contributions from different spin structures [16].

Let us consider correlators of the Liouville-dressed vertex operators

$$V(k, \zeta) = \int d^2 z \sqrt{g} e^{ikX + i\zeta DX} \quad (18)$$

where

$$\begin{aligned} ikX &= ik_a X^a = ik_\mu X^\mu + \beta\varphi, \\ \zeta DX &= \zeta_a^\alpha D_\alpha X^a, \end{aligned} \quad (19)$$

and k_a satisfy the condition

$$k_\mu^2 - \beta^2 + \beta Q = 0. \quad (20)$$

Let the coefficients at contributions from different spin structures be chosen so that the resulting amplitudes are modular-invariant and one-loop kinematic structures are the same as at the tree level. In this case, it can be shown that one-loop amplitudes have the same functional form as in critical dimension with the only difference that the scalar products $k_i^\mu k_{j\mu}$, $k_i^\mu \zeta_{j\mu}$ and $\zeta_i^\mu \zeta_{j\mu}$ are substituted by $k_i^\mu k_{ja} = k_i^\mu k_{j\mu} - \beta_i \beta_j$, etc. However, in contrast to the theory in critical dimension, coefficients in the sum over spin structures in non-critical dimensions are moduli-dependent [18]. It should be noted that these coefficients are not determined uniquely, but are defined up to a common modular-invariant factor. Taking this, for example, as a power of a modular-invariant combination $(\tau_2 |\eta(\tau)|^4)^{a(D-9)}$ ($a = \text{const}$) one can obtain a measure which has an exponential growth as $\tau_2 \rightarrow \infty$. However, as it will be clear below, this is of no importance for our goals, because in superstring theory at one-loop level no corrections to Einstein term in EA are generated. For simplicity and also to keep similarity to superstring theory in critical dimension as close as possible, the arbitrary factor will be set to a constant.

Let us consider the correlator of four generalized graviton vertex functions

$$V = \int d^2 z \sqrt{g} \zeta_a \bar{\zeta}_b \partial X^a \partial X^b e^{ikX}, \quad (21)$$

$$k_\mu \zeta^\mu = 0; \quad \zeta^0 = 0 \text{ (the same for } \bar{\zeta})$$

The sum of tree- and one-loop terms is [17, 16]:

$$A_4^{(D)} \sim \kappa_D^2 K \left\{ -\frac{4}{stu} \frac{\Gamma(1 - \frac{\alpha' s}{4}) \Gamma(1 - \frac{\alpha' t}{4}) \Gamma(1 - \frac{\alpha' u}{4})}{\Gamma(1 + \frac{\alpha' s}{4}) \Gamma(1 + \frac{\alpha' t}{4}) \Gamma(1 + \frac{\alpha' u}{4})} + \kappa_D^2 (\alpha')^{4 - \frac{D}{2}} g^{(1)} \right\}. \quad (22)$$

Here κ_D is the D -dimensional gravitation constant defined in terms of the string loop constant g as $\kappa_D = g(\alpha')^{\frac{D-2}{4}}$, K is the kinematic factor which has the same functional form as in critical dimension, and $g^{(1)}$ is the one-loop amplitude ⁶

$$g^{(1)} = \int_F \frac{d^2 \tau}{\tau_2^2} \int \prod_{i=1}^4 \left(\frac{d^2 z_i}{\tau_2} \right) \exp \left\{ \sum \alpha' k_i k_j G(z_i, z_j) \right\}. \quad (23)$$

Here G is the Green function on the torus (10). Integration over z_i extends over the parallelogram $(0, 1, \tau, 1 + \tau)$. The factor $(\alpha')^{4 - \frac{D}{2}}$ makes both terms in (22) to have the same dimension. Kinematic factor K corresponds to the R^4 structure [19], and the tree-level EA is ⁷

$$S^{(0)} \sim \int d^D x \sqrt{G} \frac{1}{\kappa_D^2} (R + \alpha'^3 R^4 + \dots). \quad (24)$$

One-loop EA was discussed in [20, 21]. In ref. [21] it was noted that the “field-theory” limit of string amplitudes is obtained in the limit $\alpha' \rightarrow 0$ with Λ^2 fixed, where Λ^2 is the UV cutoff. In an operator approach, the proper times t_i in propagators

$$\Delta^{-1} = \int_0^\infty dt e^{-t(L_0 + \bar{L}_0 - 2)} \int d\phi e^{(L_0 - \bar{L}_0)\phi}$$

are bounded by intervals $0 \leq (\alpha')^{-1} t_i \leq \Lambda^2$. The total proper time $t = \sum t_i$ is connected to the modulus τ as $t = 2\tau_2 \pi \alpha'$. Integration over the modular parameter τ is performed over the fundamental domain $F = \{\tau : |\tau| \leq 1; |\tau_1| \leq \frac{1}{2}\}$. Thus, Λ^2 and α' are not independent, but $\Lambda^2 \sim (\alpha')^{-1}$. In contrast to bosonic case, the τ -integral in (23) is finite, and in the limit $\alpha' \rightarrow 0$ the one-loop part of the amplitude (22) is

$$A_4^{(1L)} \sim \kappa_D^4 K (\alpha')^{4 - \frac{D}{2}}.$$

Thus, the sum of the tree and one-loop contributions has the following structure

$$S \sim \int d^D X \sqrt{G} e^{-2\phi} \left[\frac{1}{\kappa_D^2} (R + \alpha'^3 R^4 + \dots) + R^4 (\Lambda^2)^{\frac{D}{2} - 4} \right]. \quad (25)$$

It is seen that the one-loop part produces no corrections to the Einstein term in the EA.

⁶The amplitude is calculated with propagators in which zero-mode parts of x^μ and ϕ are projected out.

⁷The invariants s , t and u defined with the use of $(D+1)$ -dimensional scalar products $k_i^\alpha k_{j\alpha}$ satisfy the kinematic relation $s + t + u = 0$. This implies that the second non-zero term in $O(\alpha')$ expansion of the tree-level amplitude is $O(\alpha'^3)$.

5 Decay of heavy states in superstring theory

Finally, let us discuss a potential source of imaginary terms in EA from exponentially divergent expressions connected with decay of heavy massive states in the string spectrum into the light states [4, 5, 21].

For definiteness, let us consider the four-graviton amplitude in closed fermionic string theory in critical dimension. Exponentially divergent expressions may appear from integration over domains where the variables z_i are clustered in such a way that the resulting contribution to the amplitude has a pole structure.

Let, for instance, $z_1 \sim z_2$ and $z_3 \sim z_4$. Then the integral over this domain of z_i has the double pole in variable $(k_1 + k_2)^2 = (k_3 + k_4)^2$ and the residue in the pole is effectively reduced to two-point correlator of massive states. To see this in an explicit although in an approximate way, let us consider the one-loop amplitude (23). Using the asymptotics of the function $\chi(z|\tau) = \exp G(z|\tau)$ at small $|z| \leq \frac{1}{\pi}$ [17] :

$$\chi(z|\tau) \approx |2\pi z|,$$

the amplitude is effectively reduced to the following integral

$$\begin{aligned} \int \prod_{i=1}^4 d^2 z_i \prod_{i < j} \chi(z_i - z_j)^{\frac{\alpha' k_i k_j}{2}} &\approx \int d^2 z_2 d^2 z_3 |z_1 - z_2|^{\frac{\alpha' k_1 k_2}{2}} |z_3 - z_4|^{\frac{\alpha' k_3 k_4}{2}} \int d^2 z_1 d^2 z_4 \\ \chi(z_1 - z_4|\tau)^{\frac{\alpha'}{2}(k_1 k_3 + k_1 k_4 + k_2 k_3 + k_2 k_4)} &\approx \frac{1}{(\frac{\alpha'}{2} k_1 k_2 + 2)^2} \tau_2 \int d^2 z \chi(z|\tau)^{-\frac{\alpha'}{2}(k_1 + k_2)^2}. \end{aligned} \quad (26)$$

Integrations over z_i are performed over the parallelogram $(0, \tau, 1, 1 + \tau)$, but because of invariance of χ under shifts $\chi(z + 1|\tau) = \chi(z|\tau)$, integration over z can be extended over the rectangle with the sides 1 and τ_2 .

The residue at the double pole in (26) can be estimated as follows. As it will be clear below, if $s = (k_1 + k_2)^2 \leq 0$, then the integral over z is dominated by large $z_2 \sim \tau_2 \gg 1$. Up to exponentially small corrections, at $\tau_2 \geq z_2 = \text{Im } z \gg 1$, we have

$$\chi(z|\tau) \sim \exp \left[\pi \left(\frac{z_2^2}{\tau_2} - z_2 \right) \right] \quad (27)$$

Substituting (27) into (26), one has

$$\frac{1}{(\alpha' s + 8)^2} \tau_2 \int d^2 z \exp \left[-\frac{\pi \alpha' s}{2} \left(\frac{z_2^2}{\tau_2} - z_2 \right) \right] \sim \frac{1}{(\alpha' s + 8)^2} \left(\frac{\tau_2}{\alpha' s} \right)^{\frac{1}{2}} e^{\frac{\pi \alpha' s}{8} \tau_2} \quad (28)$$

up to exponentially small corrections $O \left(e^{-\frac{\pi \alpha' s}{8} \tau_2} \right)$.

The integral over the modular parameter τ is dominated by large τ_2 and exponentially diverges⁸ :

$$\int_F \frac{d^2 \tau}{\tau_2^5} \frac{1}{(\alpha' s + 8)^2} \left(\frac{\tau_2}{\alpha' s} \right)^{\frac{1}{2}} e^{\frac{\pi \alpha' s}{8} \tau_2}. \quad (29)$$

⁸At the pole $\alpha' s = -8$, the integrand in (29) has the same exponential factor as in [4], but differs in the power of the factor τ_2 .

To estimate (29), we simplify the integration domain over τ by taking it as a strip $\tau : \{-\frac{1}{2} \leq \tau_1 \leq \frac{1}{2}; 1 \leq \tau_2 < \infty\}$. The integral over τ_2 is divided into two parts

$$\int_1^{\frac{\pi\alpha's}{8}} \dots + \int_{\frac{\pi\alpha's}{8}}^{\infty} \dots \quad (30)$$

The first integral converges and does not produce an imaginary part. The second integral is divergent and can be written as

$$\frac{1}{(\alpha's + 8)^2} (\alpha's)^3 \int_1^{\infty} dx x^{-\frac{9}{2}} e^x. \quad (31)$$

By analytic continuation from the convergence domain, to the integral (31) can be assigned a finite (complex) value. For our discussion it is important to note that (31) is proportional to $(\alpha's + 8)^{-2}(\alpha's)^3$ and the kinematic structure of the double-pole contribution is

$$K(\alpha's + 8)^{-2}(\alpha's)^3 \quad (32)$$

where K is the kinematic factor in (22). Since K corresponds to the R^4 structure, the expression (32) cannot contribute to the coefficient at the Einstein term in EA.

6 Conclusions

In this paper, following the ideas of ref. [3] we studied possibilities to obtain an imaginary part in the coefficient at the Einstein term in the string EA as a result of string-loop effects. Since in two-dimensional string theory (one-dimensional matter and Liouville field) modular divergences are absent, we considered string theory in dimensions $D > 2$ having in mind the limit to $D = 2$.

In closed bosonic field theory, one-loop modular divergences generate imaginary shifts to the coefficient at the Einstein term in EA. However, these divergences are due to tachyon.

In closed fermionic string theory in non-critical dimension, using an arbitrariness in definition of the modular measure, it is possible to define modular-invariant amplitudes so that, as in critical dimension, modular divergences are absent. Kinematic structures of correlators is very similar to that in critical theory. As in critical dimension, in non-critical theory, no one-loop corrections to the Einstein term in EA are generated.

Finally, we considered divergent contributions to the 4-graviton amplitude in critical closed fermionic string theory connected with decay of heavy string states into the light states. Again, it appeared that relevant contributions to the EA does not have the Einstein-term structure.

Acknowledgements

One of the authors (V.V.B.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, where the work was completed.

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