89

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MESONS TO THE ORDER $1/M$

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The $1/M$ Expansion in the B-S Formalism and the decay of 1^- and 0^+ Heavy Mesons To The Order $1/M^1$

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ABSTRACT

We present the $1/M$ relativistic B-S expansion formalism for heavy mesons, explain its relation to HQET and give further simplified formulas. The numerical results for the decay rates of 1^- and 0^+ heavy mesons including the effects of the order $1/M$ are obtained with a model which has been used successfully to the spectra of heavy mesons.

I. Introduction

In last three years, a remarkable progress in the physics of hadrons containing a heavy quark has been achieved. On the one hand, it was pointed out that in the limit the heavy quark mass $M \rightarrow \infty$ a spin-heavy flavor symmetry emerges in such systems[1]. Systematic expansions in $1/M$ base on QCD can be carried out with the heavy quark effective theory(HQET)[2]. On the other hand, the combination of chiral symmetry and spin-heavy flavor symmetry has been made in the heavy meson chiral lagrangian(HMCL) which is applicable to the interaction of hadrons containing a single heavy quark with the π , K and η [2]. The general frameworks of such effective theories are model independent. They contain a series of coupling constants and form-factors which can not be determined from symmetry considerations alone. Nonperturbative methods are necessary for calculating such parameters and form-factors. QCD sum rules have been used to calculate some of them.

Systematic expansions in $1/M$ can also be carried out in the Bethe-Salpeter formalism[3-6]. In the reference[4] it has been shown that the number of components of the general relativistic covariant B-S wave function of a heavy meson with

arbitrary spin-parity is reduced from eight to two in the $M \rightarrow \infty$ limit. By using some model for the B-S kernel this formalism can be used to obtain various parameters in the $1/M$ expansions. The calculations in the lowest orders in $1/M$ are made simpler by the simple form of B-S wave functions in the leading order. The general formulas for the Isgur-Wise functions for the transitions from heavy meson states of arbitrary spin-parity to the ground state and the value of $r_{1/2}$, $r_{3/2}$ in the leading order have been obtained in [4]. By using the results of the reference[6], it is not difficult to calculate the parameters in HQET and HMCL to the order $1/M$.

In the section II we shall give a review of the formalism of the $1/M$ expansion of B-S equations. The main results have been contained in [4] and [6]. However there are some refinements and additional discussions. In section III, we shall apply our formalism to the processes $1^- \rightarrow 0^- + \pi$ and $0^+ \rightarrow 0^- + \pi$. Using the experimental data of the branch ratio of $1^- \rightarrow 0^- + \pi$ and $1^- \rightarrow 0^- + \gamma$, we can determine the coupling constant of chiral quark lagrangian in order $1/M$. Then the coupling constants of 0^- with 1^- and 0^- with 0^+ are obtained in leading order and next leading order and numerical results of decay width of 1^- and 0^+ are given. Finally the results obtained will be discussed in section IV.

II. $1/M$ expansion in the B-S equation

The B-S equation for a meson can be written as the following

$$\chi_P(p) = S_h(\lambda_1 P + p) \int G(P, p, q) \chi_P(q) \frac{d^4 q}{(2\pi)^4} S_l(-\lambda_2 P + p) \quad (1)$$

where $\lambda_1 = \frac{M_1}{M_1 + M_2}$, $\lambda_2 = \frac{M_2}{M_1 + M_2}$, M_1 , M_2 are masses of the two quarks, $P = Mv$, M is the meson mass, v is the meson velocity, p is the relative momentum. Let $p_l = v \cdot p$, $p_t = p - p_l v$. For convenience, we shall shift the p_t variable by defining $p'_t = p_t - \frac{M_2}{M_1 + M_2} M$ and omit the "t" in the following.

In the case $M_1 \gg p_t = O(\Lambda_{QCD})$ and $M_1 \gg M_2 = m$, the residue of $\chi_P(p)$ in (1) at the pole $p_t \approx -2M_1$ of the heavy quark propagator in the upper half p_t plane is of the order $1/M^2$. We assume that other pole of (1) on the p_t plane are of the order Λ_{QCD} or p_t . Working in the upper half p_t plane, we can disregard this pole and consider p_t as a quantity of the order Λ_{QCD} . Therefore, the heavy quark propagator can be expanded as

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$$S_h(\lambda_1 P + p) = \frac{1 + \hat{v}}{2(p_l + E_0 + m + i\epsilon)} - \frac{E_1(1 + \hat{v})}{2M_1(p_l + E_0 + m + i\epsilon)^2} + \frac{|p_t|^2(1 + \hat{v})}{4M_1(p_l + E_0 + m + i\epsilon)^2} - \frac{1 - \hat{v}}{4M_1} + \frac{\hat{p}_t}{2M_1(p_l + E_0 + m + i\epsilon)} + O\left(\frac{1}{M_1^2}\right) \quad (2)$$

where $M - M_1 - m = E_0 + E_1/M_1 + O(1/M_1^2)$, E_0 and E_1/M_1 are the first two terms in the expansion of the bound state energy. We decompose G into $G = G_0 + \frac{1}{M_1}G_1 + O(\frac{1}{M_1^2})$, $G_0 = G_+ + G_-$, where the vertex on the heavy quark line of G_+ and G_- commute and anti-commute with $\hat{v}$ respectively. Substituting (2) into (1) we obtain the following equations

$$\chi(p) = \chi_0 + \chi_1^+ + \chi_1^-, \quad \chi_0(p) = -\frac{1 + \hat{v}}{2(p_l + E_0 + m + i\epsilon)} \int G_+(P, p, q) \chi_0(q) \frac{d^4 q}{(2\pi)^4} \times \frac{p_l \hat{v} + \hat{p}_t + m}{p_l^2 - W^2 + i\epsilon}, \quad (3)$$

$$\chi_1^+(p) = -\frac{1 + \hat{v}}{2(p_l + E_0 + m + i\epsilon)} \int G_+(P, p, q) \chi_1^+(q) \frac{d^4 q}{(2\pi)^4} \times \frac{p_l \hat{v} + \hat{p}_t + m}{p_l^2 - W^2 + i\epsilon} + J[\chi_0], \quad (4)$$

$$\chi_1^-(p) = \frac{1 - \hat{v}}{4M_1} \int G_-(P, p, q) \chi_0(q) \frac{d^4 q}{(2\pi)^4} \times \frac{p_l \hat{v} + \hat{p}_t + m}{p_l^2 - W^2 + i\epsilon} + \frac{\hat{p}_t}{2M_1} \chi_0(p), \quad (5)$$

where χ_0 is the zero-th order wave function, $\chi_1^\pm$ are first order wave functions satisfying $\hat{v}\chi_1^\pm = \pm \chi_1^\pm$ and

$$J[\chi_0] = \frac{1 + \hat{v}}{4M_1(p_l + E_0 + m + i\epsilon)} \{ [|p_t|^2 - 2E_1] \chi_0 - \int [\hat{p}_t G_-(P, p, q) + G_-(P, p, q) \hat{q}_t] \chi_0(q) \frac{d^4 q}{(2\pi)^4} \frac{p_l \hat{v} + \hat{p}_t + m}{p_l^2 - W_p^2 + i\epsilon} - \int \int G_-(P, p, q') G_-(P, q', q) \chi_0(q) \frac{q_l \hat{v} + \hat{q}_t + m}{q_l^2 - W_q^2 + i\epsilon} \frac{p_l \hat{v} + \hat{p}_t + m}{p_l^2 - W_p^2 + i\epsilon} \frac{d^4 q}{(2\pi)^4} \frac{d^4 q'}{(2\pi)^4} - 2 \int G_1(P, p, q) \chi_0(q) \frac{d^4 q}{(2\pi)^4} \frac{p_l \hat{v} + \hat{p}_t + m}{p_l^2 - W_p^2 + i\epsilon} \}, \quad (6)$$

where $W_p = \sqrt{|p_t|^2 + m^2}$.

It has been shown in [4] that the zero-th order equation (3) has two series of solutions corresponding to degenerate doublet states of the following form

$$\chi_{0,j^{(-1)^{j+1}},1}(p) = \sqrt{\frac{2j+1}{2j+2}} \frac{1 + \hat{v}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_j} p_t^{\alpha_2} \dots p_t^{\alpha_j} (p_t^{\alpha_1} - \frac{j}{2j+1} \gamma^{\alpha_1} \hat{p}_t) (\phi_{1j} - \hat{p}_t \phi_{2j}), \quad \chi_{0,j^{(-1)^{j+1}},1}(p) = \frac{1}{\sqrt{2}} \frac{1 + \hat{v}}{2} \eta_{\alpha_1 \dots \alpha_{j+1}} \gamma^{\alpha_1} p_t^{\alpha_2} \dots p_t^{\alpha_{j+1}} (\phi_{1j} - \hat{p}_t \phi_{2j}) \quad (7)$$

and

$$\chi_{0,j^{(-1)^j},2}(p) = \frac{1}{\sqrt{2}} \frac{1 + \hat{v}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_{j+1}} \gamma^{\alpha_1} p_t^{\alpha_2} \dots p_t^{\alpha_{j+1}} (\psi_{1j} - \hat{p}_t \psi_{2j}), \quad \chi_{0,j^{(-1)^j},2}(p) = \sqrt{\frac{2j+1}{2j+2}} \frac{1 + \hat{v}}{2} \eta_{\alpha_1 \dots \alpha_j} p_t^{\alpha_2} \dots p_t^{\alpha_j} (p_t^{\alpha_1} - \frac{j}{2j+1} \gamma^{\alpha_1} \hat{p}_t) (\psi_{1j} - \hat{p}_t \psi_{2j}). \quad (8)$$

They correspond to meson doublets of the spin-parity $[j^{(-1)^{j+1}}, (j+1)^{(-1)^{j+1}}]$ and $[j^{(-1)^j}, (j+1)^{(-1)^j}]$ respectively. These results can be proved under the most general condition that G_0 contains only the Dirac matrices I and γ^μ on the heavy quark line. This is equivalent to the condition that in the limit $M_1 \rightarrow \infty$ the interaction does not depend on the spin of the heavy quark. This spectra pattern is in agreement with that obtained from the general considerations with the spin-flavor symmetry of the heavy quarks. The partial wave equations for ϕ_{1j} , ϕ_{2j} , ψ_{1j} and ψ_{2j} can be found in [4]. One will note that the two states of (7) and the corresponding states of (8) have the same spin-parity after making the substitution $j \rightarrow j-1$ or $j \rightarrow j+1$ in the latter, but they have different eigenstate equations and energies[4]. In order to distinguish these states, we use the third subscript (1 or 2).

The $1/M_1$ correction term J also has an interpretation in HQET. In the following we shall take $G_1 \sim g_s^2 \gamma_t^\mu \otimes \gamma_t^\nu \Delta_{\mu\nu}$, where $\Delta_{\mu\nu}$ is the gluon propagator. At the heavy quark vertex, we have the correspondence

$$\hat{p}_t G_-(P, p, q) + G_-(P, p, q) \hat{q}_t \rightarrow g_s i (\hat{\partial}_t \hat{A}_t + \hat{A}_t \hat{\partial}_t), \quad G_- G_- \rightarrow g_s \hat{A}_t \hat{A}_t,$$

$$\frac{1}{2M_1} [|p_t|^2 - (\hat{p}_t G_-(P, p, q) + G_-(P, p, q) \hat{q}_t) - G_- G_-] \rightarrow \frac{1}{2M_1} \{ |p_t|^2 - \frac{g_s}{2} [\gamma_t^\mu, \gamma_t^\nu] (\partial_\mu A_\nu - \partial_\nu A_\mu - g_s^2 [A_\mu, A_\nu]) \},$$

where A is the gluon field. Thus, we can see that the term $\frac{1}{2M_1} [|p_t|^2 - (\hat{p}_t G_-(P, p, q) + G_-(P, p, q) \hat{q}_t) - G_- G_-]$ in J can be obtained with the $O(1/M_1)$ interaction term $\bar{h}_v (\frac{D_t^2}{2M_1} + \frac{g_s}{4M_1} \sigma_{\mu\nu} G^{\mu\nu}) h_v$ in the heavy quark effective lagrangian by connecting A^μ with the light quark propagator.

Since the convolution integral $\int G_+ \chi(q) \frac{d^4 q}{(2\pi)^4}$ leaves invariant the form of the r.h.s. of (7), (8) when χ has one of such forms[4], we write $\chi_1^+(p)$ and $J[\chi_0]$ in the following form

$$\chi_{1,j^P,k}^+(p) = \frac{1}{M_1}(\xi_{j^P,k}(p) + \zeta_{j^P,k}(p)) \quad (9)$$

and

$$J[\chi_{0,j^P,k}] = \frac{1}{M_1}(B_{j^P,k} + C_{j^P,k}) \quad (10)$$

where

$$\begin{aligned} \xi_{j(-1)^{j+1},k}(p) &= \sqrt{\frac{2j+1}{2j+2}} \frac{1+\hat{v}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_j} p_t^{\alpha_1} \dots p_t^{\alpha_j} (p_t^{\alpha_1} - \frac{j}{2j+1} \gamma^{\alpha_1} \hat{p}_t) (f_{1j,k} - \hat{p}_t f_{2j,k}), \\ \zeta_{j(-1)^{j+1},k}(p) &= \frac{1}{\sqrt{2}} \frac{1+\hat{v}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_j} \gamma^{\alpha_1} p_t^{\alpha_2} \dots p_t^{\alpha_j} (h_{1j-1,k} - \hat{p}_t h_{2j-1,k}), \end{aligned} \quad (11)$$

$$\begin{aligned} B_{j(-1)^{j+1},k} &= \sqrt{\frac{2j+1}{2j+2}} \frac{1+\hat{v}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_j} p_t^{\alpha_2} \dots p_t^{\alpha_j} (p_t^{\alpha_1} - \frac{j}{2j+1} \gamma^{\alpha_1} \hat{p}_t) (b_{1j,k} - \hat{p}_t b_{2j,k}), \\ C_{j(-1)^{j+1},k} &= \frac{1}{\sqrt{2}} \frac{1+\hat{v}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_j} \gamma^{\alpha_1} p_t^{\alpha_2} \dots p_t^{\alpha_j} (c_{1j-1,k} - \hat{p}_t c_{2j-1,k}), \end{aligned} \quad (12)$$

and $k=1,2$. There are similar forms for $j^{(-1)^j}$ states. Then, substituting (9) and (10) into (4), we get the following equations in which $\xi_{j(-1)^{j+1},k}(p)$, $\zeta_{j(-1)^{j+1},k}(p)$ and $\xi_{j(-1)^j,k}(p)$, $\zeta_{j(-1)^j,k}(p)$ are not mixed

$$\begin{aligned} \xi_{j(-1)^{j+1},k}(p) &= -\frac{1+\hat{v}}{2(p_t + E_0 + m + i\epsilon)} \int G_+(P, p, q) \xi_{j(-1)^{j+1},k}(q) \frac{d^4 q}{(2\pi)^4} \\ &\quad \times \frac{p_t \hat{v} + \hat{p}_t + m}{p_t^2 - W^2 + i\epsilon} + B_{j(-1)^{j+1},k} \end{aligned} \quad (13)$$

$$\begin{aligned} \zeta_{j(-1)^{j+1},k}(p) &= -\frac{1+\hat{v}}{2(p_t + E_0 + m + i\epsilon)} \int G_+(P, p, q) \zeta_{j(-1)^{j+1},k}(q) \frac{d^4 q}{(2\pi)^4} \\ &\quad \times \frac{p_t \hat{v} + \hat{p}_t + m}{p_t^2 - W^2 + i\epsilon} + C_{j(-1)^{j+1},k} \end{aligned} \quad (14)$$

and similar equations for $\xi_{j(-1)^j,k}(p)$, $\zeta_{j(-1)^j,k}(p)$.

Up to now, all the above analyses are model-independent. From now on, we will confine our discussions in a special model. We assume that the kernel is the sum of a static scalar confinement potential and a gluon exchange term in the Coulomb gauge in the rest frame of the meson. In order to obtain wave functions of moving mesons we generalize the kernel to the following covariant form $G=G_++G_-$, where

$$\begin{aligned} -iG_+ &= I \otimes I \left[\frac{8\pi\kappa}{(|p_t - q_t|^2 + \mu^2)^2} - (2\pi)^3 \delta^3(p_t - q_t) \right. \\ &\quad \left. \times \int \frac{8\pi\kappa}{[k^2 + \mu^2]^2} \frac{d^3 k}{(2\pi)^3} \right] - I \otimes \hat{v} \frac{16\pi\alpha_{seff}}{3(|p_t - q_t|^2)}, \end{aligned} \quad (15)$$

$$\begin{aligned} -iG_- &= -[\gamma^\mu \otimes \gamma_\mu - \hat{v} \otimes \hat{v} + \frac{(\hat{p}_t - \hat{q}_t) \otimes (\hat{p}_t - \hat{q}_t)}{|p_t - q_t|^2}] \\ &\quad \times \frac{16\pi\alpha_{seff}}{3[|p_t - q_t|^2 - (p_t - q_t)^2 - i\epsilon]}, \end{aligned} \quad (16)$$

μ is taken to approach zero after solving the equation numerically. The running coupling constant α_{seff} is taken as

$$\begin{aligned} \alpha_{seff}(p_t, q_t) &= \frac{12\pi}{27 \ln \frac{\text{Max}(|p_t|^2, |q_t|^2)}{\Lambda_{QCD}^2}}, \quad \text{when } \alpha_{seff} < 1, \\ \alpha_{seff}(p_t, q_t) &= 1 \quad \text{otherwise.} \end{aligned} \quad (17)$$

The kernel G_+ in (15) is independent of p_t and q_t . Defining $\tilde{f}(p) = f(p) \frac{d^4 p}{2\pi}$ for arbitrary function $f(p)$, we find from (3) the following 3-dim equation

$$\tilde{\chi}_0(p_t) = \frac{-i}{E_0 + m - W} \int G_+ \tilde{\chi}_0 \frac{d^3 q_t}{(2\pi)^3} \frac{W \hat{v} - \hat{p}_t - m}{2W}. \quad (18)$$

From the equation (18) we know that the solution $\tilde{\chi}_0(p_t)$ satisfies the condition

$$\tilde{\chi}_0(p_t) P_- = \tilde{\chi}_0(p_t) \frac{-W \hat{v} + \hat{p}_t + m}{2W} = \frac{m}{W} \tilde{\chi}_0(p_t). \quad (19)$$

From (7), (8) and (15) we obtain the relations

$$\tilde{\phi}_{1j} = -(W + m) \tilde{\phi}_{2j}, \quad \tilde{\psi}_{1j} = (W - m) \tilde{\psi}_{2j}. \quad (20)$$

Therefore, only one independent component remains in the zero-th order wave function. Equations satisfied by $\tilde{\phi}_{2j}$ and $\tilde{\psi}_{1j}$ can be found in [4]

$$\begin{aligned} \tilde{\phi}_{2j} &= \frac{1}{2W_p(E + m - W_p)} \{ -A_-^j [(m + W) \tilde{\phi}_{2j}] - (W_p - m) A_+^{j+1} [\tilde{\phi}_{2j}] \}, \\ \tilde{\psi}_{1j} &= \frac{1}{2W_p(E' + m - W_p)} \{ p_t^2 A_-^j [(W - m)^{-1} \tilde{\psi}_{1j}] + (W_p - m) A_+^{j-1} [\tilde{\psi}_{1j}] \}, \end{aligned} \quad (21)$$

where $A_{\pm}^j[\tilde{f}]$ can be defined as

$$A_{\pm}^j[\tilde{f}] = \frac{1}{|p_t|^j} \int \frac{d^3 q_t}{(2\pi)^3} |q_t|^j P_j(\cos\theta) (\tilde{V}_1 \pm \tilde{V}_2) \tilde{f}(q_t^2), \quad (22)$$

and

$$\begin{aligned} \tilde{V}_1 &= \frac{8\pi\kappa}{(|p_t - q_t|^2 + \mu^2)^2} - (2\pi)^3 \delta^3(p_t - q_t) \times \int \frac{8\pi\kappa}{[k^2 + \mu^2]^2} \frac{d^3 k}{(2\pi)^3}, \\ \tilde{V}_2 &= \frac{16\pi\alpha_{seff}}{3(|p_t - q_t|^2)}. \end{aligned} \quad (23)$$

We now proceed to the $1/M_1$ corrections. The 4-dim wave function χ_0 can be expressed in term of the 3-dim wave function as the following

$$\chi_0(p) = \frac{-i}{E_0 + m + p_t + i\epsilon} \left[\frac{E_0 + m - W}{p_t + W - i\epsilon} \tilde{\chi}_0(p_t) + \frac{i}{p_t - W + i\epsilon} \int \frac{d^3 q_t}{(2\pi)^3} G_0 \tilde{\chi}_0(q_t) \frac{W\hat{v} + \hat{p}_t + m}{2W} \right]. \quad (24)$$

From(4) we obtain the 3-dim equation

$$\tilde{\chi}_1^+(p_t) = \frac{-i(1 + \hat{v})}{2(E_0 + m - W)} \int G_+(P, p, q) \tilde{\chi}_1^+(q_t) \frac{d^3 q_t}{(2\pi)^3} \times \frac{W\hat{v} - \hat{p}_t - m}{2W} + \tilde{J}[\tilde{\chi}_0]. \quad (25)$$

By using projection operators

$$\begin{aligned} \Lambda_{\pm}(p_t) &= \frac{W \pm \hat{v}(-\hat{p}_t + m)}{2W}, \\ \Lambda_+(p_t) + \Lambda_-(p_t) &= 1, \quad \Lambda_-(p_t) \Lambda_+(p_t) = 0, \end{aligned} \quad (26)$$

we can divide $\tilde{\chi}_1^+(p)$ and $\tilde{J}[\tilde{\chi}_0]$ into two components

$$\Phi^{\pm}(p_t) = \tilde{\chi}_1^{\pm}(p) \Lambda_{\mp}(-p_t), \quad \tilde{J}^{\pm}[\tilde{\chi}_0] = \tilde{J}[\tilde{\chi}_0] \Lambda_{\mp}(-p_t), \quad (27)$$

which satisfy the following equations by using equations (25)

$$\begin{aligned} \Phi^+(p_t) &= \frac{-i(1 + \hat{v})}{2(E_0 + m - W)} \int G_+(P, p, q) [\Phi^+(q_t) + \Phi^-(q_t)] \frac{d^3 q_t}{(2\pi)^3} \\ &\quad \times \frac{W\hat{v} - \hat{p}_t - m}{2W} + \tilde{J}^+[\tilde{\chi}_0], \\ \Phi^-(p_t) &= \tilde{J}^-[\tilde{\chi}_0]. \end{aligned} \quad (28)$$

Same as (9) and (10), We define

$$\Phi^{\pm}(p_t) = (\tilde{\xi}^{\pm} + \tilde{\zeta}^{\pm})/M_1, \quad \tilde{J}^{\pm}(p_t) = (\tilde{B}^{\pm} + \tilde{C}^{\pm})/M_1, \quad (29)$$

where

$$\begin{aligned} \tilde{\xi}_{j(-1),j+1}^{\pm}(p_t) &= \sqrt{\frac{2j+1}{2j+2}} \frac{1 + \hat{v}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_j} p_t^{\alpha_1} \dots p_t^{\alpha_j} (p_t^{\alpha_{j+1}} - \frac{j}{2j+1} \gamma^{\alpha_{j+1}} \hat{p}_t) (\tilde{f}_1^{\pm} - \hat{p}_t \tilde{f}_2^{\pm}) \\ \tilde{\zeta}_{j(-1),j+1}^{\pm}(p) &= \frac{1}{\sqrt{2}} \frac{1 + \hat{v}}{2} \gamma^5 \eta_{\alpha_1 \dots \alpha_j} \gamma^{\alpha_{j+1}} p_t^{\alpha_1} \dots p_t^{\alpha_j} (\tilde{h}_1^{\pm} - \hat{p}_t \tilde{h}_2^{\pm}), \end{aligned} \quad (30)$$

and so on. Here, for convenience we only keep the first subscript of equations (9) and (11). Combining (29), (30), with (9), (10), and (11), , we have

$$\begin{aligned} \tilde{f}_1^{\pm} &= \frac{W \pm m}{2W} \tilde{f}_1 \mp \frac{p_t^2}{2W} \tilde{f}_2, & \tilde{f}_2^{\pm} &= \mp \frac{1}{2W} \tilde{f}_1 + \frac{W \mp m}{2W} \tilde{f}_2, \\ \tilde{h}_1^{\pm} &= \frac{W \mp m}{2W} \tilde{h}_1 \pm \frac{p_t^2}{2W} \tilde{h}_2, & \tilde{h}_2^{\pm} &= \pm \frac{1}{2W} \tilde{h}_1 + \frac{W \pm m}{2W} \tilde{h}_2, \end{aligned} \quad (31)$$

and similarly relations holds for $\tilde{b}$ and $\tilde{c}$, which obviously satisfy

$$\tilde{f}_1^{\pm} = \mp(W \pm m) \tilde{f}_2^{\pm}, \quad \tilde{h}_1^{\pm} = \pm(W \mp m) \tilde{h}_2^{\pm}. \quad (32)$$

Multiplying $\tilde{\chi}_0 W$ in both sides of equation (25) and using the adjoint equation of (18)satisfied by $\tilde{\chi}_0$, we obtain

$$\int \tilde{\chi}_0(p_t) \tilde{J}[\tilde{\chi}_0] W_p \frac{d^3 p_t}{(2\pi)^3} = 0. \quad (33)$$

Using(33) and(19), we find after some algebraic manipulations the expression of eigen-energy of mesons in the order $1/M$

$$\frac{E_1}{M_1} = \frac{\text{tr}[\int^{\Lambda} \int^{\Lambda} \tilde{\chi}_0(p_t) \hat{\Gamma} \tilde{\chi}_0(q_t) \frac{d^4 q}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4}]}{\text{tr}[\int^{\Lambda} \tilde{\chi}_0(p_t) W_p \tilde{\chi}_0(p_t) \frac{d^3 p_t}{(2\pi)^3}]}, \quad (34)$$

where

$$\begin{aligned} \hat{\Gamma} &= \left\{ \frac{W_p |p_t|^2}{2M_1} (2\pi)^4 \delta^4(p - q) + \frac{m}{2M_1} [\sigma^{\mu\nu} (p_t - q_t)_{\mu} \otimes \gamma_{\nu} + (W_p + W_q) 1 \otimes \hat{v} \right. \\ &\quad + \frac{(|q_t|^2 - |p_t|^2)(W_p - W_q)}{|p_t - q_t|^2} 1 \otimes \hat{v}] \times \frac{16\pi\alpha_{seff}}{3[|p_t - q_t|^2 - (p_t - q_t)^2 - i\epsilon]} \\ &\quad \times \frac{E_0 + m - W_q}{(q_t + E_0 + m + i\epsilon)(q_t + W_q - i\epsilon)} \left. \right\} \frac{E_0 + m - W_p}{(p_t + W_p - i\epsilon)(p_t + E_0 + m + i\epsilon)}. \end{aligned} \quad (35)$$

One will notices that only the $\sigma^{\mu\nu}$ term in (35) has a contribution to the energy split of degenerate doublets. Since the integral is convergent slowly, we introduce a cut-off Λ of the order of M_1 . E_1 is not sensitive to the value of Λ . In (35) we have neglected the $G_- G_-$ term proportional to α_s^2 since we already neglected terms in G_+ of the same order.

We fit the experimental data with the following values of the parameters. The masses of ground and low excited states in B, B_s, D, D_s series were calculated with these parameters in [6].

$$\begin{aligned} M_b &= 5.024 \text{ GeV}, & M_c &= 1.58 \text{ GeV}, & m_\mu &= 0.35 \text{ GeV}, \\ m_s &= 0.465 \text{ GeV}, & \Lambda_{QCD} &= 0.38 \text{ GeV}, & \kappa &= 0.2 \text{ GeV}^2. \end{aligned} \quad (36)$$

The agreement with experimental data are good when they are available.

Using the parameters above we can solve the equations(28) numerically. As mentioned above, eq.(4) distinguishes from (3) only by the inhomogeneous term $J[\chi_0]$, so the solution of eq.(28) will have an indeterminate term which is proportional to $\tilde{\chi}_0$. Since we have identity (33), we can discard this term by using the orthogonal condition

$$\int \tilde{\chi}_1 \tilde{\chi}_0 W_p \frac{d^3 p_i}{(2\pi)^3} = 0. \quad (37)$$

This does not affect the ratio of coefficients of $\tilde{\chi}_0$ and $\tilde{\chi}_1$. The overall coefficient of $\tilde{\chi}_0 + \tilde{\chi}_1$ can be determined by the normalization condition

$$\langle D(v) | \bar{Q} \gamma^\mu Q | D(v) \rangle = 2v^\mu. \quad (38)$$

Similar to (9) and (10), the solution of the equation (28) for $\tilde{\chi}_1^+$ can be simplified by decomposing it into several decoupling parts each contains only one unknown function. However, the decomposition of $\tilde{J}$ into $\tilde{b}_i^\pm, \tilde{c}_i^\pm$ is tedious. We will not explicitly write down these equations.

III. Processes $1^- \rightarrow 0^- + \pi$, $1^- \rightarrow 0^- + \gamma$, $0^+ \rightarrow 0^- + \pi$

Let V be the 1^- heavy meson in the lowest doublet states. We assume that the electro-magnetic current vertex of the light quark has the form

$$e_q \left\{ \gamma_\mu + \frac{\mu}{2m} \sigma_{\mu\nu} q^\nu \right\}, \quad (39)$$

where e_q and μ are the charge and anomalous magnetic moment factor of the light quark. The $V \rightarrow P + \gamma$ amplitude can be written as

$$A(V \rightarrow P + \gamma) = i \varepsilon_{\mu\nu\sigma\rho} v^\mu \eta^\nu e^{*\rho} q^\sigma \left[(-R + \frac{\mu}{2m} S) e_q + \frac{eQ}{2M} \right], \quad (40)$$

where η^μ is the polarization vector of V, q^σ and $e^{*\rho}$ are the momentum and the polarization vector of γ respectively. The last term in (40) is the contribution of the heavy quark, which is directly from the corrections of $1/M$ wave function. The

partial width of this decay channel is

$$\Gamma(V \rightarrow P + \gamma) = \frac{|q_i|^3}{3\pi} |(-R + S \frac{\mu}{2m}) e_q + \frac{eQ}{2M}|^2. \quad (41)$$

R and S can be written in the form

$$R = r_0 + r_1/M_1, \quad S = s_0 + s_1/M_1, \quad (42)$$

where r_1 and s_1 are contributions from corrections to wave function in the order $1/M_1$.

We find the following expressions for r_i and s_i ($i=0,1$) in terms of the B-S wave functions

$$\begin{aligned} r_0 &= i \int \frac{d^4 p}{(2\pi)^4} (E + p_i) \left[\phi_1 \phi_2 - \frac{2}{3} \left(\phi_2 \frac{\partial \phi_1}{\partial p_i^2} - \phi_1 \frac{\partial \phi_2}{\partial p_i^2} \right) p_i^2 \right], \\ s_0 &= i \int \frac{d^4 p}{(2\pi)^4} (E + p_i) \left[\phi_1^2 + \frac{1}{3} \phi_2^2 p_i^2 \right], \\ r_1 &= i \int \frac{d^4 p}{(2\pi)^4} (E + p_i) \left[\phi_1 f_2 - \frac{2}{3} \left(\phi_2 \frac{\partial f_1}{\partial p_i^2} - \phi_1 \frac{\partial f_2}{\partial p_i^2} \right) p_i^2 \right] \\ &\quad + i \int \frac{d^4 p}{(2\pi)^4} (E + p_i) \left[\phi_2 f_3 - \frac{2}{3} \left(f_4 \frac{\partial \phi_1}{\partial p_i^2} - f_3 \frac{\partial \phi_2}{\partial p_i^2} \right) p_i^2 \right] \\ &\quad + i \int \frac{d^4 p}{(2\pi)^4} (E + p_i) \frac{2\sqrt{3}}{3} \left(h_3 \frac{\partial \phi_1}{\partial p_i^2} + h_4 \frac{\partial \phi_2}{\partial p_i^2} \right) p_i^2, \\ s_1 &= i \int \frac{d^4 p}{(2\pi)^4} (E + p_i) \left[\phi_1 (f_1 + f_3) + \frac{1}{3} p_i^2 \phi_2 (f_2 + f_4) \right] \\ &\quad - i \int \frac{d^4 p}{(2\pi)^4} (E + p_i) \frac{2\sqrt{3}}{3} p_i^2 (\phi_1 h_4 + \phi_2 h_3), \end{aligned} \quad (43)$$

where f_i and h_i ($i=3,4$) are the wave functions of states $j^{(-1)^j}$ corresponding to f_i and h_i ($i=1,2$) in (11) and the approximation $q=0$ has been taken in R and S. (43) also can be expressed in term of 3-dim wave functions by using equations (4) and (15), we will not write down these formulas here since they are very tedious.

Using PCAC, we assume that the pion vertex of the light quark has the form

$$\frac{g_q}{f_\pi} q_\mu \gamma^5 \gamma^\mu \frac{\tau_i}{\sqrt{2}}, \quad (44)$$

where q is the pion momentum, g_q is the renormalization factor for quark axial current. The amplitude of the $V \rightarrow P + \pi$ process has the form

$$A(V \rightarrow P + \pi) = \frac{g}{f_\pi} \eta \cdot q_t I, \quad (45)$$

in the lowest order of q , where I is an iso-spin factor, $I = 1$ for $V^+ \rightarrow P^0 + \pi^+$, and $I = 1/\sqrt{2}$ for $V^+ \rightarrow P^+ + \pi^0$, $V^0 \rightarrow P^0 + \pi^0$. The partial width of $V \rightarrow P + \pi$, is given by

$$\Gamma(V \rightarrow P + \pi) = \frac{|q_t|^3 g^2}{6\pi f_\pi^2} I^2. \quad (46)$$

where g can be written in the form $g = g_0 + g_1/M_1$. We find the following expression for g_0 and g_1 in terms of the B-S wave functions in $q \rightarrow 0$ limit

$$\begin{aligned} g_0 &= g_q \times i \int \frac{d^4 p}{(2\pi)^4} (E + p_t) [\phi_1^2 + \frac{1}{3} p_t^2 \phi_2^2], \\ g_1 &= g_q \times i \int \frac{d^4 p}{(2\pi)^4} (E + p_t) [\phi_1(f_1 + f_3) + \frac{1}{3} p_t^2 \phi_2(f_2 + f_4 - h_3)], \end{aligned} \quad (47)$$

which also can be expressed in term of 3-dim wave functions.

The process $D^* \rightarrow D + \gamma$ and $D^* \rightarrow D + \pi$ have been observed experimentally. We use the formulas obtained above for R , S and g to these processes. For fixed μ , the parameter g is determined by the experimental data for the branching ratio of $D^{*0} \rightarrow D^0 + \pi^0$ and $D^{*0} \rightarrow D^0 + \gamma$. μ is assumed to be small in the non-relativistic constituent quark model. This is supported by calculations based on the chiral quark lagrangian [7]. The numerical results for branching ratios, g_q^2 and g^2 corresponding to (46) together with the experimental data [7] are shown in the table 2

table 2. the branching ratio for process $D^* \rightarrow D + \pi$ and $D^* \rightarrow D + \gamma$ for $m = 0.35 \text{ GeV}$

	calculation	experiment(CLEO)
g_q^2	0.32	
g^2	0.21	
$\text{BR}(D^{*+} \rightarrow D^0 + \pi^+)$	67.7	$68.1 \pm 1.1 \pm 1.3$
$\text{BR}(D^{*+} \rightarrow D^+ + \pi^0)$	31.8	$30.8 \pm 0.4 \pm 0.8$
$\text{BR}(D^{*+} \rightarrow D^+ + \gamma)$	0.49	$1.1 \pm 1.4 \pm 1.6$
$\text{BR}(D^{*0} \rightarrow D^0 + \pi^0)$	63.6	$63.6 \pm 2.3 \pm 3.3$
$\text{BR}(D^{*0} \rightarrow D^0 + \gamma)$	36.4	$36.4 \pm 2.3 \pm 3.3$

The corresponding value of the total width of D^{*+} is

$$\Gamma(D^{*+} \rightarrow D + \pi^+) = 56 \text{ Kev}, \quad (48)$$

which is smaller than the experimental upper limit $\Gamma(D^{*+} \rightarrow D + \pi^+) < 105 \text{ Kev}$.

The upper limit 105 Kev of $\Gamma(D^{*+} \rightarrow D + \pi^+)$ corresponds roughly to $g^2 = 0.4$, which implies $g_q^2 \approx 0.6$ in our calculation. If we assume this value of g_q^2 , μ will be determined by the branching ratio of $D^{*0} \rightarrow D^0 + \pi$ and $D^{*0} \rightarrow D^0 + \gamma$ to be $\mu \approx 0.4$. A precise measurement of the branching ratio of $D^{*+} \rightarrow D^+ + \gamma$ will be helpful for determining μ . Same as [7], the contribution of the magnetic moment of the heavy quark is very important for the branch ratio of $D^* \rightarrow D^0 + \gamma$.

g_q^2 obtained from the experimental value of g_A of the nucleon and the simple model of static light quark is 0.75^2 which is larger than the value obtained by us in table 2. In the reference [7] the static quark model is used to determine the value of g^2 in the chiral lagrangian for heavy mesons which is used to calculate decay rates of $D^* \rightarrow D + \pi$ and $D^* \rightarrow D + \gamma$.

On the other hand, the value of g_q^2 obtained with the method of chiral lagrangian in [8, 9] is $g_q^2 > (0.8)^2$. Therefore there is a discrepancy between this approach and the static constituent quark model. From our results obtained above we can see that the discrepancy becomes sharper as we use the relativistic quark model instead of the non-relativistic one.

The amplitude and width of the decay $S \rightarrow P + \pi$ are

$$A(S \rightarrow P + \pi) = \frac{g'}{f_\pi} I, \quad (49)$$

$$\Gamma(S \rightarrow P + \pi) = \frac{|q_t| q_t^2 g'^2}{2\pi f_\pi^2} I^2, \quad (50)$$

where $g' = g'_0 + g'_1/M_1$ and

$$\begin{aligned} g'_0 &= g_q \times i \int \frac{d^4 p}{(2\pi)^4} (E + p_t) [\phi_1 \psi_1 + p_t^2 \phi_2 \psi_2], \\ g'_1 &= g_q \times i \int \frac{d^4 p}{(2\pi)^4} (E + p_t) [f_1 \psi_1 + p_t^2 f_2 \psi_2 + \phi_1 h_1 + p_t^2 \phi_2 h_2], \end{aligned} \quad (51)$$

where q^π is momentum of π . In (51), we neglected terms of $O(q_t^2)$. We find

$$g'^2 = 0.72. \quad (52)$$

Using (50) and (52) we obtain

$$\Gamma(S \rightarrow P + \pi) = 79 \text{ MeV}. \quad (53)$$

IV. Conclusions

In summary, we have presented in this paper the $1/M$ expansion in the B-S formalism. It can be seen from Section II that this approach has the following features: (1) It is covariant and can be used for moving heavy mesons. (2) It has close analogy with HQET. In a sense, the heavy quark sector is treated exactly up to the $1/M$ order. The renormalization factor in the heavy quark sector neglected in the calculation can be implemented without difficulties if needed. Only the treatment of the light components is model dependent. (3) The formalism is fully relativistic in the internal moving of the light components. The retardation effect of the gluon interaction has been taken into account. (4) Due to the simple form of the B-S wave functions of heavy mesons in the zeroth order, the calculation of $O(1/M)$ correction is not very difficult and can be applied to heavy mesons of arbitrary spin-parity.

We have applied this formalism to the decay rates of 1^- and 0^+ heavy mesons. This improves our calculations in [4] to the $1/M$ order. In a forthcoming paper we shall apply this approach to the heavy mesons of arbitrary spin-parity.

References

1. M.B.Voloshin and M.A.Shifman, Sov. J. Nucl. Phys. 47(1988)199. N.Isgur and M.B. Wise, Phys. Lett. B232(1989)113; Phys. Lett. B237(1990)51. H.Georgi, Phys.Lett. B240(1990)447.
2. B.Grinstein, Nucl. Phys. B339(1990)253, E.Eichten and B. Hill, Phys. Lett. B234(1990)511, A.F.Falk, H.Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. B343(1990)1, F. Hussain, J.G. Körner and R. Migneron, Phys.Lett. B248(1990)299.
3. H.Y.Jin, C.S.Huang and Y.B.Dai, Zeit.Phys.C56(1992)707.
4. Y.B.Dai, H.Y.Jin and C.S.Huang, Zeit. Phys.C60(1993)527.
5. F.Hussain, D.Liu, M.Kramer, J.G. Körner and S.Tawfig, Nucl.Phys.B370 (1992) 259. S.Balk, J.G.Körner, G.Thompson and F.Hussain, Zeit.Phys.C59 (1993)283.
6. Y.B.Dai, H.Y.Jin and C.S.Huang, to be published in Phys.Lett.B.
7. T.M.Yan, H.Y.Cheng, C.Y.Cheng, G.L.Lin, Y.C.Lin and H.L.Yu, Phys.Rev.D46, 1148(1992).
8. S.Weinberg, Phys.Lett.67, 3473(1191).
9. D.A.Dicus, D.Minic, U.V.Kolck and R.Vega, Phys.Lett.284, 384(1992).