SCAN-9409256

SW 34 39

WSP-IF 94-40,

July 1994.

Discussion of an alternative approach to a description of time evolution in neutral kaons system.

K.Urbanowski

Pedagogical University, Institute of Physics,
Plac Slowianski 6, PL-65-069 Zielona Gora, Poland

PACS numbers: 11.30.Er., 11.10.St., 03.65.Bz.

Abstract

Long time properties of the effective Hamiltonian $H_{\parallel}(t)$ governing the time evolution in a given subspace of the total state space of the system considered are discussed. The effects caused by the early time behaviour of the eigenstates $|i^{=0}\rangle$, $|s^{=0}\rangle$ for $H_{\parallel}(t)$ in the case of the neutral kaons (or other similar particles) subspace are discussed and suppositions as to how they should manifest themselves in experiments are made. The hypothesis that $|i^{=0}\rangle$, $|s^{=0}\rangle$ could be eigenstates for CP - transformation is formulated and a method for its verification is proposed. The comparison of parameters characterizing the properties of the neutral kaons system obtained within the LOY method and within the method based on the use of the Hamiltonian $H_{\parallel}(t)$ is made.

1 Introduction

The early time properties of the time evolution in neutral kaons subspace $\mathcal{H}_{\parallel} = P\mathcal{H}$ (P is the projection operator) of the state space $\mathcal{H}$ have been studied, e.g. in Refs. 1 — 3. A more detailed analysis of the problem³⁻⁵ shows that the eigenvectors of the effective Hamiltonian $H_{\parallel}(t)$ appearing in the evolution equation for $\mathcal{H}_{\parallel}$, (which is consistent with the evolution equation for the total state space $\mathcal{H}$, i.e. with the Schrödinger equation with total Hamiltonian H acting in $\mathcal{H}$) and therefore governing the time evolution in such a subspace (see: Refs. 3 — 12) differ from those given by LOY theory¹³, in the early time period, and coincide with them in the long time region $t \rightarrow \infty$. In Refs. 4 and 5 it has been shown that this property could manifest itself in a new CP- violation effect, where the exponential decay law of K_S and K_L type particles should be modified by multiplying it by some factor, which is equal to 1 only if there are no CP- violations. This conclusion has been derived only from the general properties of time evolution in the two-dimensional subspace $\mathcal{H}_{\parallel}$ in the cases of: (i) conserved CP-symmetry, and (ii) nonconserved CP- and conserved CPT-symmetries. The aim of this paper is to discuss the problem of experimental verification of the properties mentioned, to consider a relation between solutions of evolution equation in subspace $\mathcal{H}_{\parallel}$ with effective Hamiltonians $H_{\parallel}(t \ll \infty)$ and $H_{\parallel}(t \rightarrow \infty) \equiv H_{\parallel}$, and to compare the properties of matrix elements of the effective Hamiltonians $H_{\parallel}(t)$ and H_{LOY} (where H_{LOY} is the LOY effective Hamiltonian).

2 Time evolution in distinguished subspace of a state space

The natural tool for the description of time evolution in a given subspace $\mathcal{H}_{\parallel}$, in the case of irreversible processes such as the creation of neutral kaons in accelerators, is the equation for a projection of a state vector³⁻¹¹. The evolution operator $U_{\parallel}(t)$ acting in this subspace has the following form: $U_{\parallel}(t) \equiv P U(t) P$, where $U(t) = \exp(-i t H)$ and H is the total Hamiltonian of the system under consideration. In general, if the states in the subspace $\mathcal{H}_{\perp} \equiv (1 - P)\mathcal{H}$ orthogonal to $\mathcal{H}_{\parallel}$, are not occupied at the initial instant $t_0 = 0$, i.e. if $(1 - P) | \psi, t_0 \rangle = 0$ for every $| \psi, t_0 \rangle \in \mathcal{H}$, the operator $U_{\parallel}(t)$ fulfills the following equation:

$$\left(i \frac{\partial}{\partial t} - P H P\right) U_{\parallel}(t) = -i \int_0^{\infty} K(t - \tau) U_{\parallel}(\tau) d\tau, \quad (1)$$

$$U_{\parallel}(0) = P,$$

where:

$$(2) \quad \begin{aligned} K(t) &= \Theta(t)PHQ \exp(-iQH\Theta Q)QH P, \\ Q &= I - P, \\ \Theta(t) &= \begin{cases} 1 & \text{for } t \geq 0, \\ 0 & \text{for } t < 0. \end{cases} \end{aligned}$$

Vectors evolving in time in $\mathcal{H}_\parallel$ and having a particle interpretation can be found if one replaces this equation by the following equivalent differential, Schrödinger-like equation⁵⁻¹²:

$$(3) \quad \begin{aligned} i\frac{\partial}{\partial t}H_\parallel(t)U_\parallel(t) &= 0, \\ U_\parallel(0) &= P. \end{aligned}$$

It is reasonable to distinguish the subspace $\mathcal{H}_\parallel$ in a total state space $\mathcal{H}$, and thus to use the evolution equations (1) or (3) for determining and searching for $U_\parallel(t)$ if the transitions from subspace $\mathcal{H}_\parallel$ into subspace $\mathcal{H}_\perp$ are not too fast, i.e., if vectors in subspace $\mathcal{H}_\parallel$ describe bound states or quasistationary and similar states. In other words, some distinguished properties described by vectors in $\mathcal{H}_\parallel$ should rather be preserved by the system during a relatively long time interval. If they are not, there is no sense in applying equations (1) or, especially, (3) to investigations of $U_\parallel(t)$. These equations are effective if the transition probability from a state $|\alpha\rangle \in \mathcal{H}_\parallel$ into $\mathcal{H}_\perp$ during relatively short time interval $\tau = t_2 - t_1$, $\|QU(\tau)P|\alpha\rangle\|^2$ is much smaller than the survival probability in $\mathcal{H}_\parallel$, $\|PU(\tau)P|\alpha\rangle\|^2 \equiv \|U_\parallel(\tau)|\alpha\rangle\|^2$, during this same time interval, i.e., if for every normalized $|\alpha\rangle \in \mathcal{H}_\parallel$

$$(4) \quad \|QU(\tau)P|\alpha\rangle\|^2 \ll \|U_\parallel(\tau)|\alpha\rangle\|^2 \leq 1, \quad 0 < \tau \ll \infty.$$

Assuming that this condition is fulfilled only the eigenvectors of $H_\parallel(t)$ may be considered as suitable candidates for a description of such states of the subsystem described by vectors in $\mathcal{H}_\parallel$, which possess particle-like behaviour. It is convenient to write $H_\parallel(t)$ as follows

$$(5) \quad H_\parallel(t) \equiv PHP + V_\parallel(t).$$

3

4

The "quasipotential" $V_\parallel(t)$ has the following properties:

$$(6) \quad V_\parallel(0) = 0, \quad V_\parallel(t \rightarrow 0) \approx -iPHQH P.$$

One finds from (1), (3) and (5)

$$(7) \quad V_\parallel(t)U_\parallel(t) = -i \int_0^t K(t-\tau)U_\parallel(\tau)d\tau \stackrel{\text{def}}{=} -iK * U_\parallel(t).$$

(Here the star * denotes the convolution: $f * g(t) = \int_0^t f(t-\tau)g(\tau)d\tau$). Next, using this relation and a retarded Green's operator $G(t)$ for the equation (1)

$$(8) \quad G(t) = -i\Theta(t) \exp(-iPH P)P,$$

and solving equation (1), to the lowest order of

$$(9) \quad L(t) = G * K(t),$$

one obtains⁶ from (4)

$$(10) \quad V_\parallel(t) \stackrel{\text{def}}{=} -i \int_0^t K(t-\tau) \exp[i(t-\tau)PH P]d\tau.$$

This approximation is valid for $\|L(t)\| \ll 1$.

3 Long time properties of evolution equation for subspace $\mathcal{H}_\parallel$

In practical applications, e.g., in searching for properties of quasistationary or bound states, a knowledge of properties of long time solutions of Eqs (1) and, especially (3) is essential. This problem has the following solution according to the approach described above.

If $V_\parallel$, i.e.,

$$(11) \quad H_\parallel \equiv PH P + V_\parallel,$$

and $T \gg t_0 \equiv 0$ exist, such that

$$\| |\alpha\rangle \|^{-1} \| [H_{\parallel}(t \geq T) - H_{\parallel}] |\alpha\rangle \| \ll 1, \quad (12)$$

for every $|\alpha\rangle \in \mathcal{H}_{\parallel}$, then one can show that the solution of the following equation

$$[i \frac{\partial}{\partial t} - H_{\parallel}] U_{\parallel}^{as}(t) = 0 \quad (13)$$

(with the initial condition at $t \equiv T$, $U_{\parallel}^{as}(T) \neq U_{\parallel}(0)$), which is valid for $t \geq T \gg t_0$, i.e., the operator

$$U_{\parallel}^{as}(t) = \exp[-i(t-T)H_{\parallel}] U_{\parallel}^{as}(T), \quad (14)$$

approximates very well to the solutions $U_{\parallel}(t)$ of Eq. (3) for long times $t \geq T$, at least in the case of weak decays (4). Strictly speaking, assuming (12) and

$$U_{\parallel}^{as}(T) = U_{\parallel}(T), \quad (15)$$

one can show that for every $|\alpha\rangle \in \mathcal{H}_{\parallel}$

$$\| |\alpha\rangle \|^{-1} \| [U_{\parallel}(t) - U_{\parallel}^{as}(t)] |\alpha\rangle \| \Big|_{t \geq T} \ll 1, \quad (16)$$

which means that for $t > T \gg t_0 \equiv 0$ (see Appendix)

$$U_{\parallel}(t) \simeq \exp(-itH_{\parallel}), \quad (t \geq T). \quad (17)$$

4 The neutral kaons case — general considerations

The subspace of states of neutral kaons is spanned by orthonormal vectors $|K^0\rangle$, $|\bar{K}^0\rangle$. Let vectors $|1\rangle$, $|2\rangle$ form an orthonormal and time-independent basis in $\mathcal{H}_{\parallel}$ — these vectors can be identified with the neutral kaon states $|K^0\rangle = |1\rangle$, $|\bar{K}^0\rangle = |2\rangle$, or $B^0, \bar{B}^0$ mesons and so on. Here:

$$P = |1\rangle\langle 1| + |2\rangle\langle 2|. \quad (18)$$

Operators $U_{\parallel}(t)$, $H_{\parallel}(t)$, $V_{\parallel}(t)$ acting in this subspace can be written as 2×2 matrices: $U_{\parallel}(t) = (u_{jk}(t))$, $H_{\parallel}(t) = (h_{jk}(t))$, ..., $j, k = 1, 2$. The eigenvectors of $U_{\parallel}(t)$ will be denoted as $|L^s\rangle$ and $|S^s\rangle$. The eigenvectors of $H_{\parallel}(t)$ for the eigenvalues $\mu_l(t)$, $\mu_s(t)$ will be denoted as $|l^s\rangle$ and $|s^s\rangle$ respectively. Assuming

$$[CP, H] = 0, \quad (19)$$

$$[CP, P] = 0, \quad (19')$$

or

$$[CPT, H] = [CPT, P] = 0, \quad (20)$$

(where C, P and T denote operators describing C-, P- and T-transformations respectively) which imply, e.g. for the neutral K system, the equality of the diagonal matrix elements of H^n , $n = 1, 2, \dots$, in the $|K^0\rangle$, $|\bar{K}^0\rangle$ basis or

$$\langle 1 | H | 1 \rangle = \langle 2 | H | 2 \rangle, \quad \langle 1 | H Q H | 1 \rangle = \langle 2 | H Q H | 2 \rangle \quad (21)$$

in the $|1\rangle$, $|2\rangle$ basis, it has been shown in Refs.3-5 that $U_{\parallel}(t)$ and $H_{\parallel}(t)$ have common eigenvectors at early time period:

$$\begin{aligned} |L(S)^{t=0}\rangle &\equiv |l(s)^{t=0}\rangle \\ &= \rho_{l(s)}(0) \left[|1\rangle - (+) \left(\frac{\langle 2 | H | 1 \rangle}{\langle 1 | H | 2 \rangle} \right)^{1/2} |2\rangle \right], \quad (22) \end{aligned}$$

which are orthogonal quite independently of whether CP-symmetry is violated or not. Here $\rho_{S(L)}(t)$, $\rho_{s(l)}(t)$ denote the normalization factors for vectors $|L(S)^t\rangle$ and $|l(s)^t\rangle$ respectively — we have $|\rho_{S(L)}(0)|^2 = |\rho_{s(l)}(0)|^2 = \frac{1}{2}$ and therefore we can choose

$h \cdot s = h_x \sigma_x + h_y \sigma_y + h_z \sigma_z$; $\sigma_x, \sigma_y, \sigma_z$ — are the Pauli matrices, $I_{\parallel}$ — is the unit operator in $\mathcal{H}_{\parallel}$, $h^2 = h_x^2 + h_y^2 + h_z^2 \equiv h_1^2 + h_2^2 + h_3^2$. Eigenvalues corresponding to these eigenvectors of $H_{\parallel}$ will be denote as μ_1 and μ_2 respectively.

We obtain the following picture: if the K^0 meson is created at the instant $t = 0$, e.g. in the reaction $\pi^- + p \rightarrow K^0 + \Lambda^0$, it immediately transforms into the linear combination of orthonormal vectors $|l(s)^{t=0}\rangle$; $|K^0, t=0\rangle \equiv (2\rho(0))^{-1/2}(|l^{t=0}\rangle + |s^{t=0}\rangle)$, and then, in the case of

$$H_{\parallel} \equiv H_{\parallel}(t \rightarrow \infty) = (h_0 I_{\parallel} + h \cdot s) P, \quad (28)$$

and h_x and h are defined by means of $H_{\parallel}$ (11) as follows

$$\alpha(s) = \frac{h_x + (-)h}{h_2}, \quad (27)$$

$$|l(s)\rangle \equiv p(s) |1\rangle - \alpha(s) |2\rangle, \quad (26)$$

where:

$$|l^{t \rightarrow \infty}\rangle \equiv |L^{t \rightarrow \infty}\rangle \stackrel{\text{def}}{=} |l\rangle, \quad |s^{t \rightarrow \infty}\rangle \equiv |S^{t \rightarrow \infty}\rangle \stackrel{\text{def}}{=} |s\rangle, \quad (25)$$

for the intermediate time region ($0 < t \ll \infty$):

at short time ($t \rightarrow 0$) and long time: ($t \rightarrow \infty$) regions, and is not generally true conclusion is true for CPT-conserved and CP-nonconserved symmetries only common eigenvectors in this case. Next, it has been shown there that such a case of the conserved CP-symmetry and that operators $U_{\parallel}(t)$ and $H_{\parallel}(t)$ have $t > 0$, vectors $|S^t\rangle$ and $|L^t\rangle$ (or $|s^t\rangle$, $|l^t\rangle$) are orthogonal only in the

It has also been found in Refs.4,5 that beyond the short time region - for every $|\psi\rangle \in \mathcal{H}_{\parallel}$.

$$|< l^{t=0} | \psi >|^2 + |< s^{t=0} | \psi >|^2 = < \psi | \psi >, \quad (24)$$

Vectors $|L(S)^{t=0}\rangle \equiv |l(s)^{t=0}\rangle$ form another, orthonormal and complete basis in $\mathcal{H}_{\parallel}$. The completeness condition for these vectors can be written as follows:

$$p_{S(L)}(0) \equiv p_{S(L)}(0) \stackrel{\text{def}}{=} p(0) \equiv 2^{-1/2}. \quad (23)$$

Figure 1: The plausible configuration of eigenstates $|l^t\rangle$, $|s^t\rangle$ for the effective Hamiltonian $H_{\parallel}(t)$ in the subspace $\mathcal{H}_{\parallel}$: (a) at the initial instant $t = t_0 \equiv 0$, (b) at the instant $t \gg t_0 \equiv 0$ ($t \rightarrow \infty$).

CP-nonconserved symmetry, vectors $|l(s)^{t=0}\rangle$ evolve smoothly in time into nonorthogonal eigenstates $|l(s)^{t=\infty}\rangle \equiv |l(s)\rangle$ of the effective Hamiltonian $H_{II}(t \rightarrow \infty)$ (see Fig. 1). The vectors $|l(s)\rangle$ correspond with the states $|K_L\rangle$ and $|K_S\rangle$ obtained by the use of the LOY approach.

From (25) and (26) it has been found that the probability $p(|s(l)^{t=0}\rangle \rightarrow |s(l)^t\rangle; t)$ of finding the system in the particle-like state of type $|s^t\rangle$ (or $|l^t\rangle$) at relatively long time $t \rightarrow \infty$, if it were in the state of this same type at $t = 0$ (i.e. in $|s^{t=0}\rangle$ or $|l^{t=0}\rangle$) equals

$$\begin{aligned} p(|s(l)^{t=0}\rangle \rightarrow |s(l)^t\rangle; t \rightarrow \infty) &= |\langle s(l)^{t=0} | U_{II}(t \rightarrow \infty) | s(l)^{t=0} \rangle|^2 \\ &\equiv |\langle s(l)^{t=0} | \exp(-itH_{II}) | s(l) \rangle|^2 \Big|_{t \rightarrow \infty} \\ &\equiv \exp(-t\gamma_s(l)) |\langle s(l)^{t=0} | s(l) \rangle|^2, \end{aligned} \quad (29)$$

where:

$$\gamma_s(l) = -2\text{Im} \mu_s(l), \quad (30)$$

$$\mu_s(l) \equiv \mu_s(l)(t \rightarrow \infty) = h_0 + (-)h. \quad (31)$$

There is $|\langle s(l)^{t=0} | s(l) \rangle|^2 \neq 1$ in the case of CPT-conserved but CP-nonconserved symmetry — this is the main result of Refs.4,5. From this the following conclusion has been derived: if it were possible to determine the function $p(|s(l)^{t=0}\rangle \rightarrow |s(l)^t\rangle; t)$ directly from the experiment, then starting from this experimentally obtained $p(|s(l)^{t=0}\rangle \rightarrow |s(l)^t\rangle; t)$, one could draw a segment of a straight line

$$\ln p(|s(l)^{t=0}\rangle \rightarrow |s(l)^t\rangle; t) \equiv -\gamma_s(l)t + b_s(l), \quad (32)$$

where:

$$b_s(l) = \ln |\langle s(l)^{t=0} | s(l) \rangle|^2, \quad (33)$$

$$\begin{aligned} \langle s(l)^{t=0} | s(l) \rangle &= \\ \rho_{s(l)}^*(0) \rho_{s(l)} &\left\{ 1 - (+) \left[\left(\frac{\langle 2 | H | 1 \rangle}{\langle 1 | H | 2 \rangle} \right)^* \right]^{1/2} \alpha_{s(l)} \right\} \end{aligned} \quad (34)$$

and thus one could determine the values of the parameters $b_{s(l)}$, which should be different from zero in the case of CPT-conserved but not CP-conserved symmetry. When there is no CP-violation it should be $b_{s(l)}^{CP} \equiv 0$.

From the results of Refs.4 and 5 it follows also that

$$b_s - b_l \equiv \ln \frac{|\langle s^{t=0} | s \rangle|^2}{|\langle l^{t=0} | l \rangle|^2} \neq 0, \quad (35)$$

in the CP-noninvariant case.

The problem of verifying the above results experimentally is that the states K_L and K_S are not directly observed in known experiments with neutral kaons but only via their decay products. Therefore the results obtained in Refs.4,5 should be rewritten in the language of decay products. So, if the number of unstable objects $N(t)$ observed at the instant t in a given experiment is large, then there is the following, standard relation between the nondecay (survival) probability $p(t)$ at time t and $N(t)$:

$$p(t) \cong N(t)/N_0, \quad (36)$$

where $N_0 \equiv N(t \equiv t_0 = 0)$. In the case of experiments with neutral kaons, e.g. when the reaction $\pi^- + p \rightarrow K^0 + \Lambda^0$ is used, simply $N_0 \equiv N(|s^{t=0}\rangle) = N(|l^{t=0}\rangle) \equiv N(K^0; t=0) \equiv N(\Lambda^0)$. The number $N(\Lambda^0)$ can be determined by registering the decay products of the particle Λ^0 . Thus, if one has a large statistic of decays, one can use relation (36) to obtain from (29) an expression for the number of decays of the state $|l^{t=\infty}\rangle$ or $|s^{t=\infty}\rangle$ per unit of time:

$$\begin{aligned} & - \frac{dN_{l(s) \rightarrow l(s)}(t)}{dt} \Big|_{t \rightarrow \infty} \\ &= \gamma_{l(s)} p(|l(s)^{t=0}\rangle \rightarrow |l(s)^t\rangle; t \rightarrow \infty) N_0 \\ &\equiv \gamma_{l(s)} \exp(-t\gamma_{l(s)}) |\langle l(s)^{t=0} | l(s) \rangle|^2 N_0. \end{aligned} \quad (37)$$

The straight line

$$\ln \left\{ - \frac{1}{N_0 \gamma_{l(s)}} \frac{dN_{l(s) \rightarrow l(s)}(t)}{dt} \Big|_{t \rightarrow \infty} \right\} \equiv -\gamma_{l(s)}t + b_{l(s)}, \quad (38)$$

corresponds exactly to (32). Unfortunately this relation does not solve the problem - from Fig. 1) and from the results of Ref. 5 it can be seen that the state $|\epsilon^0\rangle$ ($|\epsilon^0\rangle$) can give a nonzero contribution to the decays of the state $|\epsilon^0\rangle$ ($|\epsilon^0\rangle$). We have

$$p(|\epsilon(t)\rangle_{\epsilon^0} \rightarrow |\epsilon(s)\rangle_{\epsilon^0}) = \exp(-\gamma(s)) \langle \epsilon(s) | \epsilon(t) \rangle. \quad (39)$$

Let us notice that the decay widths for the processes both, e.g. $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ and $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$, are the same: γ (and similarly for the processes $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ and $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$). Therefore the problem arises: how to distinguish and identify in the experiment the decays caused by both these processes. Formula (38) solves our problem only if the decay products coming from these processes are distinguishable. From formula (39) one can calculate, e.g. the number of decays per unit of time $\left(-dN_{\epsilon^0-\epsilon^0}(t)/dt \right)$ of the state $|\epsilon^0\rangle$ in the $|\epsilon^0\rangle$ channel. If the decay products arising from the transitions, e.g. $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ and $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$, are indistinguishable, then the number of decays per unit of time registered in the $|\epsilon^0\rangle$ channel: $\left(-dN_{\epsilon^0-\epsilon^0}(t)/dt \right)$ is given by the sum of both: $\left(-dN_{\epsilon^0-\epsilon^0}(t)/dt \right)$ and $\left(-dN_{\epsilon^0-\epsilon^0}(t)/dt \right)$. Thus by means of the completeness relation (24) one obtains simply that

$$\left. \frac{dN_{\epsilon^0}(t)}{dt} \right|_{t \rightarrow \infty} = - \left[\frac{dN_{\epsilon^0-\epsilon^0}(t)}{dt} + \frac{dN_{\epsilon^0-\epsilon^0}(t)}{dt} \right] \Big|_{t \rightarrow \infty} \approx \gamma \exp(-\gamma t) N_0. \quad (40)$$

which means that in the considered case of indistinguishable decay products, the effect produced by formulae (28) and (32) cannot be observed when only the decay products can be registered and the states of type $|\epsilon^0\rangle$ or $|\epsilon^0\rangle$ cannot be observed directly. Decay products caused by processes of type $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ and $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ (or: $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ and $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$) have the chance of being distinguishable, e.g. if the states $|\epsilon^0\rangle$ and $|\epsilon^0\rangle$ are additionally the eigenstates of the operator, say Ω , representing another physical, measurable quantity, for the different eigenvalues ω and ω . If

$$[\Omega, P] = [\Omega, H_0(0)] = 0. \quad (41)$$

In such a case, the decay products arising from the processes: (i) $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ and (ii) $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$, are distinguishable (and analogously for the processes $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ and $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$) — see Refs. 15, 16. The value of $CP = (-1)$ corresponds with the decays accompanied with process (i), and $CP = (+1)$ with process (ii). The summed value of the eigenvalue of CP -operator for decay products, e.g. in the case of neutral kaons, is experimentally distinguishable and measurable. Therefore, if relations (43), (45) were valid, one could state in the case considered, that the decay products corresponding with $CP = (-1)$ are caused by the transition $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ as $t \rightarrow \infty$, and these, which correspond with $CP = (+1)$, are generated by the process $|\epsilon^0\rangle \rightarrow |\epsilon^0\rangle$ as $t \rightarrow \infty$.

Then, if (43) holds, we find

$$CP |\epsilon^0\rangle_{\epsilon^0} = (-1) |\epsilon^0\rangle_{\epsilon^0}, \quad CP |\epsilon^0\rangle_{\epsilon^0} = (+1) |\epsilon^0\rangle_{\epsilon^0}. \quad (45)$$

define the CP -transformation as follows⁵

$$CP |1\rangle = e^{-i\phi_{CP}} |2\rangle. \quad (44)$$

which does not contradict (42), then they are. Here simply $\Omega \equiv CP$. Let us

vectors for the CP -transformation, but if

$$[CP, H_0(0)] \equiv [CP, P H P] = 0, \quad (43)$$

In general, the eigenvectors $|\epsilon(s)\rangle_{\epsilon^0}$ ($|\epsilon(s)\rangle_{\epsilon^0}$) need not be the eigenvectors for the CP -transformation, but if

$$[CP, H] \neq 0, \quad (42)$$

and $H_0(0)$ may have common eigenvectors in subspace $\mathcal{H}_0$ (for times $t > t_0 \equiv 0$ such as the creation of neutral kaons in an accelerator and described by time evolution of vectors in the given subspace $\mathcal{H}_0$, i.e. the moment $t = t_0 \equiv 0$ is the moment at which the operators CP, CPT for which the relations (19') and (20) hold but (19) does not:

then, in some cases, there may exist a possibility of classifying the decay products with respect to the values of ω and ω . Let us notice that the beginning of irreversible processes, such as the creation of neutral kaons in an accelerator and described by time evolution of vectors in the given subspace $\mathcal{H}_0$, i.e. the moment $t = t_0 \equiv 0$ is the moment at which the operators CP, CPT for which the relations (19') and (20) hold but (19) does not:

and then one would be able to draw the segment of the straight line (38) using experimental data.

It seems that there should not be problems with verifying the hypothesis (43) - one needs only in a large statistic of decays. In other words, if this hypothesis were true, then counting the decay products at given moment of time t , e.g. of the state $|l\rangle$, one obtains separately: $(-dN_{l \rightarrow l}(t))/dt \equiv (-dN_l(t; CP = -1))/dt$ and $(-dN_{s \rightarrow l}(t))/dt \equiv (-dN_l(t; CP = +1))/dt$. From (36) and (29), (39) it follows that the functions of time:

$$\ln \left[-\frac{dN_{l \rightarrow l}(t)}{dt} \right] \equiv A_{l,l} - \gamma_l t, \quad (46)$$

$$\ln \left[-\frac{dN_{s \rightarrow l}(t)}{dt} \right] \equiv A_{s,l} - \gamma_l t, \quad (47)$$

where $A_{l(s),l} = \ln \langle l(s)^{t=0} | l \rangle^2 + \ln(\gamma_l N_0)$ (one should expect that $A_{l,l} \neq A_{s,l}$ significantly), form straight, parallel lines. So, if logarithms of the experimentally obtained numbers of decays per unit of time: $\ln [(-dN_l(t; CP = -1))/dt]$ and $\ln [(-dN_l(t; CP = +1))/dt]$ form parallel lines (see Fig. 2), we can conclude that there is a high probability that the hypothesis expressed by formulae (43) and (45) is true (there is a problem, if the eventual positive result of such a test can be considered as the crucial for a verification of this hypothesis). The analogous construction can be made for the process $|l^{t=0}\rangle \rightarrow |s\rangle$ and $|s^{t=0}\rangle \rightarrow |s\rangle$.

In the case, when the hypothesis mentioned were experimentally confirmed, the following relations should hold: from experimentally obtained segments of straight lines expressed in Fig. 2, one can find

$$\Delta_l \stackrel{\text{def}}{=} A_{s,l} - A_{l,l} \equiv \ln \frac{\langle s^{t=0} | l \rangle^2}{\langle l^{t=0} | l \rangle^2}, \quad (48)$$

(and analogously $\Delta_s \stackrel{\text{def}}{=} A_{l,s} - A_{s,s}$). This quantity together with the completeness relation (24) enables us to calculate both: $\langle s^{t=0} | l \rangle^2$ and $\langle l^{t=0} | l \rangle^2$ (or $\langle s(l)^{t=0} | s \rangle^2$ — by means of Δ_s), and thus to reconstruct the states $|l\rangle$ (or $|s\rangle$) from the experimental data. Also, if properties (43) and (45) occur, then using the relations (29), (36), (37) and (39) together with (24) we conclude that, e.g.:

$$\frac{(-dN_l(t; CP = +1))/dt}{(-dN_l(t; CP = -1))/dt} \Big|_{t \gg t_0 \approx 0}$$

$$\begin{aligned} & \equiv \frac{\Delta N_l(t_2 - t_1; CP = +1)}{\Delta N_l(t_2 - t_1; CP = -1)} \Big|_{t_2 \gg t_1 \gg t_0 \approx 0} \\ & \equiv \frac{\langle s^{t=0} | l \rangle^2}{\langle l^{t=0} | l \rangle^2} \equiv \frac{1 - \langle l^{t=0} | l \rangle^2}{\langle l^{t=0} | l \rangle^2}, \end{aligned} \quad (49)$$

Figure 2: The supposed picture of registered decays of $|l\rangle$ state in the hypothetical case of $|l^{t=0}\rangle, |s^{t=0}\rangle$ being eigenstates for CP -transformation: (1) $y(t) = \ln [(-dN_l(t; CP = -1))/dt]$, (2) $y(t) = \ln [(-dN_l(t; CP = +1))/dt]$, $\tan \varphi \equiv \gamma_l$.

where $\Delta N(t_2 - t_1)$ denotes the number of decays during time interval $(t_2 - t_1)$. In a similar manner, if (43), (45) are valid, other useful relations can be found. On the other hand, if the relations (43), (45) do not occur, but there exists a quantity Ω fulfilling (41), relations (46) — (49) are valid for ϕ_l, ϕ_s replacing $CP = -1, +1$.

5 A comparison of properties of effective Hamiltonians $H_{\parallel}$ and H_{LOY}

In the case of the projector P (18), the approximate formula (10) for $V_H(t)$ leads to the following expressions⁶ for the matrix elements $v_{jk}(t \rightarrow \infty) \equiv v_{jk}$ of $V_H(t \rightarrow \infty) \equiv V_H \cong V_H^{(1)}(\infty)$,

Therefore matrix elements v_{jk}^θ of operator V_θ^θ (here V_θ^θ and H_θ^θ denote the operators V_θ^θ and H_θ^θ respectively when the property (20) occurs, $\theta \equiv CPT$) take the following form

$$\Sigma_{11}(\epsilon = \epsilon^*) \equiv \Sigma_{22}(\epsilon = \epsilon^*) \stackrel{\text{def}}{=} \Sigma_0(\epsilon = \epsilon^*). \quad (56)$$

$\kappa \equiv |H_{12}|$, $H_2 \equiv 0$ and $H_0 \equiv H_{11} \equiv H_{22}$, and⁵
In the case of preserved CPT-symmetry (20), properties (21) imply that of the expressions (50), (51) have the same order with respect to $\Sigma(\epsilon)$. H of the system considered. It should also be emphasized that all components any symmetries of a type CP -, T -, or CPT -symmetry for the total Hamiltonian and thus for h_{jk} have been derived without assuming

$$h_{jk} = H_{jk} + v_{jk}. \quad (55)$$

where for ϵ real, $\Sigma_R(\epsilon) = \Sigma'_R(\epsilon)$ and $\Sigma'_I(\epsilon) = \Sigma_I(\epsilon)^+$. Hence, by (5)

$$\Sigma(\epsilon) = PHQ \frac{1}{1 - QH\bar{Q} - \epsilon - i0} QHP \stackrel{\text{def}}{=} \Sigma_R(\epsilon) + i\Sigma_I(\epsilon), \quad (54)$$

and

$$\kappa = (|H_{12}|^2 + H_2^2)^{1/2}, \quad (53)$$

$$H_{jk} = \langle j | H | k \rangle, \quad H_0 = \frac{1}{2}(H_{11} + H_{22}), \quad H_2 = \frac{1}{2}(H_{11} - H_{22}), \quad (52)$$

where $j, k = 1, 2$ and $\Sigma_{jk}(\epsilon) = \langle j | \Sigma(\epsilon) | k \rangle$,

$$v_{j2} = -\frac{1}{2} \left(1 - \frac{\kappa}{H_2} \right) \Sigma_{j2}(H_0 + \kappa) - \frac{1}{2} \left(1 + \frac{\kappa}{H_2} \right) \Sigma_{j2}(H_0 - \kappa) - \frac{H_{12}}{2\kappa} \Sigma_{j1}(H_0 + \kappa) + \frac{H_{12}}{2\kappa} \Sigma_{j1}(H_0 - \kappa), \quad (51)$$

$$v_{j1} = -\frac{1}{2} \left(1 + \frac{\kappa}{H_2} \right) \Sigma_{j1}(H_0 + \kappa) - \frac{1}{2} \left(1 - \frac{\kappa}{H_2} \right) \Sigma_{j1}(H_0 - \kappa) - \frac{H_{21}}{2\kappa} \Sigma_{j2}(H_0 + \kappa) + \frac{H_{21}}{2\kappa} \Sigma_{j2}(H_0 - \kappa), \quad (50)$$

$$[H^{(0)}, P] = [H^{(0)}, Q] = 0. \quad (61)$$

commute with $H^{(0)}$:

$$Q \equiv 1 - P \equiv \sum_{F \in \sigma_c} |F\rangle \langle F| \quad (60)$$

This means that projectors P (18) and

$$H^{(0)}|F\rangle = E_F|F\rangle$$

and let us denote by $|F\rangle$ vectors orthogonal to $|1\rangle, |2\rangle$ and corresponding to the continuous part σ_c of the spectrum σ of $H^{(0)}$: $\langle F|F'\rangle = \delta(F - F')$:

$$H^{(0)}|j\rangle = m_j|j\rangle, \quad j = 1, 2, \quad (59)$$

$|1\rangle, |2\rangle$ considered above be eigenstates of $H^{(0)}$
transitions between eigenstates of $H^{(0)}$, i.e., for decay processes. So, let vectors free Hamiltonian, and H_1 denotes a weak interaction, which is responsible for tonian H splits into two parts^{13,15-19}: $H = H^{(0)} + H_1$, where $H^{(0)}$ is the LOY effective Hamiltonian^{13,15-19} H_{LOY} .

In the course of deriving of H_{LOY} one usually assumes that the total Hamiltonian H governing the time evolution, e.g., in the neutral kaons subspace in the system with preserved CPT-symmetry, can be related to those obtained within the Weiskopf-Wigner approximation, i.e., defining the effective Hamiltonian H_θ^θ of the approximate

$$v_{j2}^\theta = -\frac{1}{2} \left\{ \Sigma_{j2}(H_0 + |H_{12}|) + \Sigma_{j2}(H_0 - |H_{12}|) + \frac{H_{12}}{2} \Sigma_{j1}(H_0 + |H_{12}|) - \frac{H_{12}}{2} \Sigma_{j1}(H_0 - |H_{12}|) \right\}, \quad (58)$$

$$v_{j1}^\theta = -\frac{1}{2} \left\{ \Sigma_{j1}(H_0 + |H_{12}|) + \Sigma_{j1}(H_0 - |H_{12}|) + \frac{H_{21}}{2} \Sigma_{j2}(H_0 + |H_{12}|) - \frac{H_{21}}{2} \Sigma_{j2}(H_0 - |H_{12}|) \right\}, \quad (57)$$

It is also usually assumed that

$$[H^{(0)}, CPT] = 0, \quad (62)$$

and, for simplicity,

$$\langle 1|H_1|1 \rangle \equiv \langle 2|H_1|2 \rangle \equiv 0, \quad (63)$$

the consequence of which is that in (59):

$$m_1 = m_2 \stackrel{\text{def}}{=} m_0 \equiv \langle 1|H|1 \rangle \equiv \langle 2|H|2 \rangle \equiv H_0.$$

Within assumptions (61) - (63) the Weisskopf-Wigner approach leads to the following formula for H_{LOY} (e.g., see Ref. 18):

$$H_{\text{LOY}} = m_0 P + P H_1 P - \Sigma(m_0) \stackrel{\text{def}}{=} M - \frac{i}{2} \Gamma. \quad (64)$$

So, the matrix elements h_{jk}^{LOY} of H_{LOY} are

$$h_{jk}^{\text{LOY}} = H_{jk} - \Sigma_{jk}(m_0), \quad (j, k = 1, 2), \quad (65)$$

where, in this case,

$$H_{jk} \equiv \langle j|(H^{(0)} + H_1)|k \rangle \equiv m_0 \delta_{jk} + \langle j|H_1|k \rangle. \quad (66)$$

If additionally $[CPT, H_1] = 0$, then, by (56),

$$\Sigma_{11}(m_0) \equiv \Sigma_{22}(m_0), \quad (67)$$

which, together with (63) mean that

$$h_{11}^{\text{LOY}} \equiv h_{22}^{\text{LOY}}. \quad (68)$$

in CPT-invariant system (20).

Matrix elements of operators M and Γ have the following form

$$M_{jk} = m_0 \delta_{jk} + \langle j|H_1|k \rangle - \sum_F \langle j|H_1|F \rangle \frac{P}{E_F - m_0} \langle F|H_1|k \rangle + \dots \quad (69)$$

$$\Gamma_{jk} = 2\pi \sum_F \langle j|H_1|F \rangle \delta(E_F - m_0) \langle F|H_1|k \rangle + \dots, \quad (70)$$

(where P denotes a principal value) and, for the CPT-invariant system - in the case (20), by (68)

$$M_{11} \equiv M_{22} \stackrel{\text{def}}{=} M_0, \quad \Gamma_{11} \equiv \Gamma_{22} \stackrel{\text{def}}{=} \Gamma_0. \quad (71)$$

Let us notice, that $v_{11}^0 \neq v_{22}^0$ (57), (58) and thus $h_{11}^0 \neq h_{22}^0$ in contradistinction to property (68) obtained within the LOY theory.

Assuming that,

$$|H_{12}| \ll |H_0|, \quad (72)$$

in formulae for v_{jk}^0 , we find

$$v_{j1}^0 \simeq -\Sigma_{j1}(H_0) - H_{21} \frac{\partial \Sigma_{j2}(x)}{\partial x} \Big|_{x=H_0}, \quad (73)$$

$$v_{j2}^0 \simeq -\Sigma_{j2}(H_0) - H_{12} \frac{\partial \Sigma_{j1}(x)}{\partial x} \Big|_{x=H_0}, \quad (74)$$

where $j = 1, 2$. One should stress that due to a presence of resonance terms, derivatives $\frac{\partial}{\partial x} \Sigma_{jk}(x)$ need not be small, and so the products $H_{jk} \frac{\partial}{\partial x} \Sigma_{jk}(x)$ in (73), (74).

Finally, assuming that (72) holds and using relations (73), (74), the definition (55) and the result (65), we obtain for the CPT-invariant system

$$h_{j1}^0 = h_{j1}^{\text{LOY}} - H_{21} \frac{\partial \Sigma_{j2}(x)}{\partial x} \Big|_{x=H_0}, \quad (75)$$

$$\alpha_{\phi}^* \approx -a + (2h_{\text{LOY}}^{12})^{-1} \left\{ H_{12} \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0} + 3a \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0} \right\} - H_{21} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} - a^{-1} \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0} \left\{ \right\}, \quad (82)$$

those appearing in the LOY approach as follows
 α_{ϕ}^* , α_{ϕ}^* , and μ_{ϕ}^* , discussed in Sec. 3 and in Refs. 3 — 5, can be related to order of $|H_{12}|$, the typical parameters characterizing the neutral kaons system Thus, in the CPT-invariant system, from (27) and (75), (76), to the lowest

$$\alpha_{\text{LOY}}^{*(i)} \equiv -(+) \left(\frac{h_{\text{LOY}}^{12}}{h_{\text{LOY}}^{12}} \right)^{1/2} \stackrel{\text{def}}{=} -(+)a. \quad (81)$$

where a is defined by means (27) and (65), (68):

$$h_{\text{LOY}} \equiv (h_{\text{LOY}}^{12} h_{\text{LOY}}^{21})^{1/2}, \quad (80)$$

$$h_{\phi} \approx h_{\text{LOY}} - \frac{1}{2} (a H_{12} + a^{-1} H_{21}) \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0}, \quad (79)$$

$$h_{\phi}^0 \approx h_{\text{LOY}} - \frac{1}{2} \left(H_{12} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} + H_{21} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} \right), \quad (78)$$

and, analogously, to the lowest order of $|H_{12}|$, for quantities h , h_0 defining coefficients α_i , α_{ϕ} , (26), (27) and eigenvalues μ_i , μ_{ϕ} (31) of $H_{\parallel}$, we obtain

$$h_{\phi}^{11} - h_{\phi}^{22} \equiv H_{12} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} - H_{21} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} \equiv h_{\text{LOY}}^{11} - h_{\text{LOY}}^{22} \equiv 0, \quad 2h_{\phi}^2 \neq 2h_{\text{LOY}}^2 \equiv h_{\text{LOY}}^{11} - h_{\text{LOY}}^{22} \equiv 0, \quad (77)$$

CPT-invariant tests for the neutral kaons system, equals diagonal matrix elements of $H_{\parallel}$, which plays an important role in designing where $j = 1, 2$. From this formulae we conclude that, e.g., the difference between

$$h_{j2}^{\phi} = h_{\text{LOY}}^{j2} - H_{12} \left[\frac{\partial}{\partial \Sigma_j(x)} \right]_{x=H_0}, \quad (76)$$

Summing up, the description on neutral kaons and similar systems based on the effective Hamiltonian $H_{\parallel}$ and described in Sec. 1 — 3, in the case when the matrix elements $H_{jk} \equiv \langle j | H | k \rangle$, ($j, k = 1, 2$) of the total Hamiltonian H are relatively small, is equivalent to introducing (maybe small) correction terms into the LOY theory, and, in such a case, there is the chance that the difference between the results obtained within both these theories is small. If $|H_{12}|$ is not suitably small, the difference between parameters mentioned above can be significant, and then, the method based on the effective Hamiltonian $H_{\parallel}$ should rather be used instead the LOY method. The difference between these two approaches both is especially noticeable when one examines the CPT-invariance of neutral kaons subsystem induced by such an invariance of the total system, i.e., by relation (20). In these cases both the conclusions following from the

$$\mu_{\phi}^* \approx \lambda_1 - \frac{1}{2} \left[H_{12} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} - a \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0} \right] + H_{21} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} - a^{-1} \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0} \left\{ \right\}, \quad (86)$$

$$\mu_{\phi}^* \approx \lambda_2 - \frac{1}{2} \left[H_{12} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} + a \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0} \right] + H_{21} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} + a^{-1} \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0} \left\{ \right\}, \quad (85)$$

we find

$$\lambda_{\phi}^{*(i)} \stackrel{\text{def}}{=} \mu_{\text{LOY}}^{*(i)} \equiv h_{\text{LOY}}^0 + (-) h_{\text{LOY}} \quad (84)$$

Defining (see (31))

$$v_{\phi}^* \approx a + (2h_{\text{LOY}}^{12})^{-1} \left\{ H_{12} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} + a \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0} \right\} - H_{21} \left[\frac{\partial}{\partial \Sigma_{12}(x)} \right]_{x=H_0} - a^{-1} \left[\frac{\partial}{\partial \Sigma_0(x)} \right]_{x=H_0} \left\{ \right\}, \quad (83)$$

assumed CPT-invariance of the total system are qualitatively different (even if $|H_{12}|$ is small) — see (77). Similar effect was observed in Ref.19: it has been also found there that relation (20) does not imply of the equality of diagonal elements of the effective Hamiltonian derived in that paper. So, it seems, that the LOY method should not be used for designing the CPT-invariance tests in neutral kaons and similar systems²⁰⁻²⁷ — the method based on $H_{\parallel}$ seems to be more convincing there. Also, contrary to the approach based on $H_{\parallel}(t)$ (5), there is no place in the LOY method for the effects described in Sec. 3 and Refs 4,5 and for searching the early time properties of neutral kaons subsystem together with possible early time evolution effects.

Acknowledgements The author is grateful to A.A.Grib for stimulating discussions and to L.A.Khalfin for helpful remarks during the author's visit to St-Petersburg in November 1992.

Appendix

From (3) it follows that

$$\begin{aligned} U_{\parallel}(t) - P &\equiv -i \int_0^t H_{\parallel}(\tau) U_{\parallel}(\tau) d\tau \\ &\equiv -i \int_0^T H_{\parallel}(\tau) U_{\parallel}(\tau) d\tau - i \int_T^t H_{\parallel}(\tau) U_{\parallel}(\tau) d\tau. \end{aligned} \quad (A1)$$

Next, from (12), for $t > T$, we find

$$U_{\parallel}(t) - P \simeq -i \int_0^T H_{\parallel}(\tau) U_{\parallel}(\tau) d\tau - i H_{\parallel} \int_T^t U_{\parallel}(\tau) d\tau, \quad (A2)$$

which, by (3), leads to

$$U_{\parallel}(t) \simeq U_{\parallel}(T) - i H_{\parallel} \int_T^t U_{\parallel}(\tau) d\tau, \quad (t > T). \quad (A3)$$

This equation has the following solution

$$U_{\parallel}(t) \Big|_{t>T} \simeq \exp[-i(t-T)H_{\parallel}] U_{\parallel}(T). \quad (A4)$$

Now, it is sufficient to show that

$$B \stackrel{\text{def}}{=} \exp(iTH_{\parallel}) U_{\parallel}(T) \simeq P \equiv U_{\parallel}(0). \quad (A5)$$

From (A4) it follows that

$$U_{\parallel}(2T) \simeq \exp(-iTH_{\parallel}) U_{\parallel}(T). \quad (A6)$$

On the other hand

$$U_{\parallel}(2T) \equiv U_{\parallel}(T) U_{\parallel}(T) + PU(T)Q QU(T)P, \quad (A7)$$

hence, taking into account that in $\mathcal{H}_{\parallel}$

$$U_{\parallel}(t) U_{\parallel}(t)^{-1} = U_{\parallel}(t)^{-1} U_{\parallel}(t) = I_{\mathcal{H}_{\parallel}} \equiv P, \quad (A8)$$

we find

$$P \simeq B[P + \epsilon(T)], \quad (A9)$$

where

$$\epsilon(T) \stackrel{\text{def}}{=} U_{\parallel}(T)^{-1} PU(T)Q QU(T)P U_{\parallel}(T)^{-1}. \quad (A10)$$

Therefore we can conclude that in the case of weak processes (4), the relation (A4) holds and the properties (16), (17) are true.

References

- [1] L.A.Khalfin, Preprints of the CPT, The University of Texas at Austin: DOE-ER-40200-211, February 1990 and DOE-ER- 40200-247, February 1991.
- [2] C.B.Chui and E.C.G.Sudarshan, *Phys.Rev. D* **42**, 3712(1990).
- [3] K.Urbanowski, *Int. Journ. Mod. Phys. A* **7**, 6299(1992).
- [4] K.Urbanowski, *Phys. Lett. A* **171**, 151(1992).
- [5] K.Urbanowski, *Int. Journ. Mod. Phys. A* **8**, 3721(1993).

- [6] W. Krolikowski and J. Rzewuski, *Bull. Acad. Polon. Sci.*, **4**, 19 (1956).
- [7] W. Krolikowski and J. Rzewuski, *Nuovo Cim.*, **25B**, 739 (1975), and references one can find therein.
- [8] K. Urbanowski, *Acta. Phys. Polon.*, **B14**, 485 (1983).
- [9] W. Garczynski, *Phys. Lett. A* **56**, 434 (1976).
- [10] K. Urbanowski, *Int. Journ. Mod. Phys. A* **6**, 1051 (1991).
- [11] M. A. Braun and K. Urbanowski, *Found. of Phys.*, **22**, 617 (1992).
- [12] K. Urbanowski, *Phys. Lett. B* **313**, 374 (1993).
- [13] e.g.: T. D. Lee, R. Oehme and C. N. Yang, *Phys. Rev.* **106**, 340, (1957); P. K. Kabir, *The CP-puzzle* (Academic Press, New York 1968).
- [14] L. A. Khaliln, *Zh. Eksp. Teor. Fiz. - Pisma v Red.*, **8**, 106 (1968), (in Russian).
- [15] e.g.: E. D. Comins and P. H. Bucksbaum, *Weak interactions of leptons and quarks* (University Press, Cambridge 1983).
- [16] T. Nakada, Preprint of the Paul Scherrer Institute: PSI-PR-91-02, Jan. 1991.
- [17] *Weak interactions* (Inst. National de Physique Nucleaire et de Physique des Particules, Paris, 1977), Eds. M. K. Gaillard and M. N. Kholm.
- [18] S. M. Bilenkiij, *Particles and nucleus*, vol. 1., No 1 (Dubna 1970), p. 227, (in Russian).
- [19] L. A. Khaliln, The theory of K, K (D, D and T, T) mesons beyond the Weiskopf-Wigner approximation and the CP- problem, Preprint LOMI P-4-80, Leningrad, February 1980 (in Russian).
- [20] J. W. Cronin, *Rev. Mod. Phys.*, **53**, 373 (1981).
- [21] J. W. Cronin, *Acta Phys. Polon.*, **B15**, 419 (1984).
- [22] V. V. Barmin, et al, *Nucl. Phys. B* **247**, 293 (1984).
- [23] L. Lavoura, *Ann. Phys. (NY)*, **207**, 428 (1991).
- [24] C. Buchanan, et al, *Ann. Phys. Rev.* **D45**, 4088 (1992).
- [25] C. O. Dib and R. D. Peccei, *Phys. Rev.*, **D46**, 2265 (1992).
- [26] R. D. Peccei, Preprint UCLA/93/TEP/19, University of California, June 1993.