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A three-flavour analysis of neutrino mixing

G. Conforto

Università degli Studi, 61029 Urbino, Italy
Istituto Nazionale di Fisica Nucleare, 50125 Firenze, Italy

A comprehensive three-flavour analysis of the existing experimental information on accelerator, atmospheric and solar neutrinos allows to determine the mass eigenstates as well as the complete mixing matrix relating mass and flavour eigenstates. The masses are found to be in agreement with the leptonic quadratic hierarchy of the see-saw model and large enough to render energy-independent any oscillation-induced observable in solar neutrino physics, in agreement with all available data. The solution obtained for the mixing matrix is unique. The most relevant implications are discussed.

1. INTRODUCTION

Several anomalous experimental results in neutrino physics have appeared in recent years. Although they have been taken as indications of the existence of non-zero neutrino masses and mixings, no satisfactory over-all interpretation has been obtained so far. We present here a comprehensive three-flavour analysis resulting in an unique and complete description of neutrino mixing [1,2].

2. THREE-FLAVOUR FORMALISM

In the complete three-flavour approach, the weak eigenstates $|v_\alpha\rangle = v_e, v_\mu, v_\tau$ and the mass eigenstates $|v_i\rangle = v_1, v_2, v_3$ are related by a unitary transformation matrix U , in terms of which the probability of an initial neutrino v_α of energy E being equal to another neutrino v_β at a distance L can be written as

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{j>i} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2(\Delta_{ij}/2) \quad (1)$$

with $\Delta_{ij} = \delta m_{ij}^2 L/2E$, where $\delta m_{ij}^2 = |m_i|^2 -$

$|m_j|^2$, $m_i = m(v_i)$.

Assuming CP-invariance, the U -matrix can be parametrized as

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{bmatrix} \quad (2)$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$, where θ_{12}, θ_{13} and θ_{23} are three independent real angles lying in the first quadrant.

Of the three δm_{ij}^2 's appearing in eq. (1), only two are independent. The complete solution of the problem consists therefore in determining five unknowns: two δm_{ij}^2 's and three θ_{ij} 's.

For $m_1 \ll m_2 \ll m_3$, $\delta m_{23}^2 \approx \delta m_{13}^2$ and the oscillatory behaviour of eq. (1) is characterized by δm_{12}^2 and δm_{13}^2 ($\gg \delta m_{12}^2$) corresponding, respectively, to a "slow" and a "fast" oscillation. Thus, for any given experimental situation, depending on the range of L/E under study, either only the fast or both the fast and slow oscillations may be occurring.

This allows to define the two ranges $1/\delta m_{13}^2 \text{ (eV}^{-2}\text{)} \ll L/E \text{ (m/MeV)} \ll 1/\delta m_{12}^2 \text{ (eV}^{-2}\text{)}$ ("short base") and $L/E \text{ (m/MeV)} \gg 1/\delta m_{12}^2 \text{ (eV}^{-2}\text{)}$ ("long base") in which the average transition probabilities $P_{\alpha\beta}^S$ and $P_{\alpha\beta}^L$ are calculated from eq. (1) for $\sin^2(\Delta_{12}/2)=0$, $\langle \sin^2(\Delta_{13}/2) \rangle = 0.5$ and $\langle \sin^2(\Delta_{12}/2) \rangle = 0.5$, $\langle \sin^2(\Delta_{13}/2) \rangle = 0.5$, respectively.

3. INPUT DATA AND MASSES

3.1 Prompt neutrinos

It is well known that, assuming neutrinos to be the main component of hot dark matter, the flatness of the universe ($\Omega_0=1$) implies, for $m_1 \ll m_2 \ll m_3$, m_3 of the order of 10 eV [3,4].

Remarkably, some evidence for the existence of such a heavy neutrino actually exists. In prompt neutrinos from charm decay, the equality between the ν_e and ν_μ spectra prescribed by e- μ universality appears to be violated [5]. When analysed in terms of neutrino mixing, this result leads to the existence of ν_e - ν_τ oscillations with the parameters [5,6]

$$\sin^2(2\alpha) = 0.48 \pm 0.10 \pm 0.05 \quad (3)$$

$$\delta m_{13}^2 = (377 \pm 27 \pm 7) \text{ eV}^2 \quad (4)$$

The result (3) is 4σ from zero and 2.5σ from the most stringent upper limit.

3.2 Atmospheric neutrinos

Neutrinos are also produced in the interactions of the primary component of cosmic rays in the Earth's atmosphere and in the subsequent decays of the produced secondaries. As almost all these decays involve a muon, which in turn also decays, the

ratio R between the ν_μ and ν_e fluxes can be safely predicted to be approximately 2. Several experiments have addressed this question. [7] They all consistently find values of R which are smaller than 2 by a factor of r , approximately equal to 0.6.

In fact, the weighted average of all available data gives

$$r = 0.64 \pm 0.06 \text{ with } \chi^2/\text{d.o.f} = 1.0 \quad (5)$$

If interpreted as being due to the superposition of the slow and fast oscillations, this result implies approximately

$$4 \times 10^{-3} < \delta m_{12}^2 < 3 \times 10^{-2} \text{ eV}^2 \quad (6)$$

Taking $m_1 \ll m_2$, eqs. (4) and (6) yield

$$\begin{aligned} m_1 &\ll 10^{-1} \text{ eV} \\ m_2 &= (0.12 \pm 0.05) \text{ eV} \\ m_3 &= (19.4 \pm 0.7) \text{ eV}. \end{aligned} \quad (7)$$

From the leptonic quadratic hierarchy of the see-saw model and the value of m_3 above, one has $m_2 = (0.068 \pm 0.002) \text{ eV}$, consistently with the lower limit of eq. (7).

3.3 Solar neutrinos

Solar neutrinos have been known for some time to deviate from expectations. The experimental results [8-11], coming from three different types of detectors, as well as the predictions of the Princeton-Yale [12] and Saclay [13] solar models are summarized in table I.

With neutrino masses as large as those of eqs. (7), all solar neutrino oscillation-induced phenomena observable on Earth are bound to be energy-independent. In particular, in all experiments the ν_e flux must be reduced by the same factor F , independently of the detection threshold.

Table 1

Solar neutrino experimental results and theoretical predictions (1σ).

For the Ga^{71} entry, the quoted experimental value is the weighted average of the SAGE

$(58^{+17}_{-24} \pm 14 \text{ SNU})$ [10] and Gallex $(87 \pm 14 \pm 7 \text{ SNU})$ [11] results.

Whenever appropriate, statistical and systematic errors have been added in quadrature.

	DETECTOR		
	Cl^{37} (SNU)	Kamiokande ($10^6 \text{cm}^{-2}\text{s}^{-1}$)	Ga^{71} (SNU)
Experimental result	2.33 ± 0.25	2.91 ± 0.41	79 ± 13
Princeton/Yale model	8 ± 1	5.7 ± 0.8	132 ± 6
Saclay model	6.4 ± 1.4	4.4 ± 1.1	123 ± 7

The data are consistent with this expectation. Fig. 1 shows the two χ^2 's obtained by comparing the experimental data with the theoretical predictions of the Princeton-Yale and Saclay solar models as a function of F . The $\chi^2_{\text{P(rinceton)Y(ale)}}$ and $\chi^2_{\text{S(aclay)}}$ minima turn out to be $\chi^2_{\text{PYmin}} = 7.3$ ($P(\chi^2) = 2.6\%$) and $\chi^2_{\text{Smin}} = 2.9$ ($P(\chi^2) = 23.5\%$), indicating no compelling energy-dependence. The somewhat higher (lower) Cl^{37} (Ga^{71}) rates reported at this Conference [14-16] further support this conclusion.

Following the procedure normally used by the Particle Data Group [17], the errors on the F -factors have been multiplied by the appropriate scale-factors. With this procedure, the two F -factors turn out to be $F_{\text{PY}} = 0.39 \pm 0.11$ and $F_{\text{S}} = 0.56 \pm 0.11$, implying the existence of a solar neutrino deficit.

In order to quantify this deficit for our further analysis, we have calculated a new F -factor using in the χ^2 the arithmetic averages of the Princeton-Yale and Saclay rates and the larger Saclay errors. The function $\chi^2 = \chi^2(F)$ is also shown in fig. 1 and yields the result ($\chi^2_{\text{min}} = 4.2$, $P(\chi^2) = 12.2\%$),

$$F = 0.49 \pm 0.13, \quad (8)$$

including a scale-factor of 1.44.

Figure 1 The functions $\chi^2 = \chi^2(F)$ obtained by comparing the experimental data with the predictions of the Princeton-Yale (dotted line) and Saclay (dashed line) solar models (see Table I). F is the energy-independent factor by which all ν_e fluxes predicted by the same model are reduced. For each model the errors on the expected rates for the various detectors have been taken as uncorrelated and Gaussian-distributed. The full line is obtained by using in the χ^2 the arithmetic averages of the Princeton-Yale and Saclay rates and the larger Saclay errors.

4. MIXING MATRIX

From the three equations (3), (5) and (8), implementing the experimental constraints $P_{e\mu}^S < 1.5 \times 10^{-3}$ and $P_{\mu\tau}^S < 2 \times 10^{-3}$ [17], the three angles θ_{ij} are uniquely determined to be

$$\begin{aligned}\theta_{12} &= 0.51 \pm 0.07 \\ \theta_{13} &= 0.38 \pm 0.06 \\ \theta_{23} &< 0.03\end{aligned}\quad (9)$$

The U-matrix is then

$$U = \begin{bmatrix} 0.81 \pm 0.04 & 0.46 \pm 0.06 & 0.37 \pm 0.06 \\ -0.50 \pm 0.06 & 0.87 \pm 0.03 & \langle 0.03 \\ -0.31 \pm 0.05 & -0.20 \pm 0.05 & 0.93 \pm 0.03 \end{bmatrix} \quad (10)$$

The knowledge of the angles θ_{ij} allows also to calculate the short base

$$\begin{aligned}P_{ee}^S &= 0.76 \pm 0.06 & P_{e\mu}^S &< 2.7 \times 10^{-4} & P_{e\tau}^S &= 0.24 \pm 0.06 \\ & & P_{\mu\mu}^S &= 0.998 & P_{\mu\tau}^S &< 1.5 \times 10^{-3} \\ & & & & P_{\tau\tau}^S &= 0.76 \pm 0.06\end{aligned}$$

as well as the long base average transition probabilities

$$\begin{aligned}P_{ee}^L &= 0.49 \pm 0.06 & P_{e\mu}^L &= 0.32 \pm 0.05 & P_{e\tau}^L &= 0.19 \pm 0.05 \\ & & P_{\mu\mu}^L &= 0.62 \pm 0.06 & P_{\mu\tau}^L &= 0.06 \pm 0.03 \\ & & & & P_{\tau\tau}^L &= 0.75 \pm 0.06\end{aligned}$$

All limits are at the 90% C.L.

5. IMPLICATIONS AND OUTLOOK

The main implications of this scenario are the following.

- Short base experiments like Chorus and Nomad [18] are expected to see a ν_τ signal. If not from ν_μ - ν_τ ($P_{\mu\tau}^S$ is consistent with zero), at least from ν_e - ν_τ oscillations (owing to the about 1% ν_e component in the beam and the sizeable $P_{e\tau}^S$).

- Long base experiments [7,19] have a better chance to detect ν_μ - ν_τ oscillations ($P_{\mu\tau}^L$ is 2σ from zero), but the largest effect is anticipated in the ν_e - ν_μ channel.

- All β sources are expected to yield lower-than-canonically-calculated ν_e fluxes (both P_{ee}^S and P_{ee}^L are smaller than 1 and the wave-length of the fast oscillation is about 5mm/MeV). In passing, this implies that the "efficiencies" measured in the Ga^{71} "calibration" experiments [15,16] should turn out to be smaller than unity. This effect should also be present in experiments at nuclear reactors. In the short base experiments performed so far this has not been observed [20], but definite conclusions are made difficult by the large and poorly known systematic uncertainties on the calculated neutrino fluxes. A possible way out is represented by the Karmen experiment, which has reported a promising although not yet statistically powerful P_{ee}^S lower limit [21]. Long base reactor experiments, however, will have the advantage of a projected twice-as-large depletion-factor.

- The masses of the weak eigenstates $|\nu_\alpha\rangle$ are given by

$$m(\nu_\alpha) = \sum_i U_{\alpha i}^2 m_i$$

and from the U-matrix (10)

$$\begin{aligned}m(\nu_e) &= (2.7 \pm 0.9) \text{ eV} \\ m(\nu_\mu) &= (0.10 \pm 0.04) \text{ eV} \\ m(\nu_\tau) &= (16.8 \pm 1.2) \text{ eV}\end{aligned}\quad (11)$$

For Majorana neutrinos, such a massive ν_e could give rise to detectable neutrino-less double beta decay rates [22]. Furthermore, in about 13% of all β decays a heavy neutrino of mass m_3 (eq. (7)) is expected to be produced. Two-state analyses of the spectra end-points are needed. They could perhaps also solve the problem of the negative m_ν^2 in β decay [23].

• With masses as large as those of eq. (7), neutrinos' contribution to the total mass of the universe exceeds by far that of baryons. Assuming the universe to be critical (flat) and the only massive stable components of the universe to be baryons and neutrinos, the above results imply for the Hubble parameter H_0 and for the age of the universe T the results

$$\begin{aligned} H_0 &= (47.4 \pm 0.9) \text{ km s}^{-1} \text{ Mpc}^{-1} \\ T &= (13.8 \pm 0.3) \text{ Gyr.} \end{aligned} \quad (12)$$

The results of eqs. (12) are in excellent agreement with present observational evidence.

If, in addition to baryons and neutrinos, a third species of massive stable particles is assumed to contribute to the total (almost all dark) matter of a flat universe, from the known experimental information on H_0 and T , its fractional density Ω_x must satisfy the bound $\Omega_x < 0.49$. This is not inconsistent with any existing experimental evidence.

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