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CALCULATING QCD CORRECTIONS TO SINGLE TOP PRODUCTION
IN HADRONIC INTERACTIONS

G. Bordes
Collège de France, IN2P3 - CNRS

B. van Eijk
NIKHEF-H, Amsterdam, Pays-Bas

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G. Bordes

Lab. de Physique Corpusculaire,
Collège de France, Paris, France

and

B. van Eijk*

NIKHEF, P.O. Box 41882
NL-1009 DB Amsterdam

Abstract

We give a detailed exposé on the calculation of $O(\alpha_s)$ corrections to the cross-section for semiweak production of single top quarks in hadronic interactions.

After introducing the different sub-processes and defining their kinematics, we briefly recall the computation of the $O(\alpha^2)$ $qb \rightarrow q't$ and $O(\alpha_s\alpha^2)$ $qg \rightarrow q't\bar{b}$ Born amplitudes squared. Next, the calculation of virtual and Bremsstrahlung corrections is presented. We adopt a massive quark and massive gluon regularisation scheme. We explicitly demonstrate cancellations among various radiative corrections and we describe the procedure of absorbing remaining singularities in the parton density distribution functions.

Finally, numerical results are given. We find a cross-section enhancement of roughly 30 % (for $m_{top} \approx 174 \text{ GeV}/c^2$) with respect to the Born contribution. We observe good agreement between the exact calculation and the approach where the mass singularity is subtracted from the sum of the $O(\alpha^2)$ and $O(\alpha_s\alpha^2)$ tree-level expressions.

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Résumé

Nous présentons un exposé détaillé du calcul des corrections $O(\alpha_s)$ à la section efficace de production semifaible des quarks top dans les interactions hadroniques.

Après avoir introduit les différents sous-processus et défini leur cinématique, nous rappelons le calcul des amplitudes de Born $O(\alpha^2)$ $qb \rightarrow q't$ et $O(\alpha_s\alpha^2)$ $qg \rightarrow q't\bar{b}$. Puis, le calcul des corrections virtuelles et de Bremsstrahlung est présenté. Nous adoptons un schéma de régularisation par l'introduction de masses des partons. Nous démontrons les compensations entre les diverses corrections radiatives et nous décrivons le procédé d'absorption des singularités restantes dans les fonctions de distributions des partons.

Finalement nous donnons des résultats numériques. Nous trouvons un accroissement de la section efficace d'environ 30 % (pour $m_{top} \approx 174 \text{ GeV} / c^2$) par rapport à la contribution de Born. Nous observons un bon accord entre le calcul exact et l'approche où la singularité de masse est soustraite de la somme des expressions $O(\alpha^2)$ et $O(\alpha_s\alpha^2)$ au niveau en arbre

1. Introduction

Top quark search with the CDF hadron collider experiment at Fermilab has lead to first direct evidence for the existence of the heavy member of the third family. The mass value quoted by the CDF collaboration $m_{top} = 174_{-10-12}^{+10+13} \text{ GeV} / c^2$ [1], is remarkably well in agreement with Standard Model predictions derived from electroweak precision data collected by experiments at the Cern Large Electron Positron (LEP) collider and Stanford linear accelerator $m_{top} = 174_{-12-18}^{+11+17} \text{ GeV} / c^2$ [2]. The second error in the latter reflects the uncertainty of the Higgs boson mass. Detailed analyses show that a Higgs mass as large as $1 \text{ TeV} / c^2$ leads to $m_t < 195 \text{ GeV} / c^2$ at 95 % confidence level. One may expect that the chance of detecting the top quark at the LEP collider will be extremely small (although with increasing beam energy the model independent lower mass limit, $m_t > 45 \text{ GeV} / c^2$, will significantly improve in the near future), whereas a possible luminosity and beam energy upgrade at the Fermilab $p\bar{p}$ collider will both provide significant increase in integrated luminosity and top production cross-sections.

Theoretical models for heavy flavour pair production at hadron colliders have been discussed in length in the literature. Several independent QCD calculations, leading to predictions for kinematical properties of the heavy quarks and absolute cross-section estimates, have shown to be rather successful in explaining the bottom quark rate [3]. Various detailed analyses of experimental data [4] have confirmed these theoretical predictions quite accurately. However, the mass of the top is large compared to the mass of its SU(2) partner, which leads beside rather different decay properties ($m_t > M_W + m_b$), to alternative production mechanisms.

In particular, it has been shown that semiweak production of top quarks dominates pure QCD pair production mechanisms when the mass of the top quark becomes sufficiently large ($\sim 250 \text{ GeV} / c^2$) [5]. This value is not much larger than the quoted CDF number and Standard Model expectation. The relative importance of the two processes varies rapidly for different top masses; the semiweak production rate being roughly ten times smaller than the QCD event rate for a top quark mass of $\sim 130 \text{ GeV} / c^2$ [5, 6, 7]. Since the mass of the top is in any case much larger than the mass of the bottom quark, semiweak production mechanisms tend to give rise to rather distinct event topologies. These event signatures [5], quite different from the ones appearing in heavy flavour pair production, justify precise and detailed studies on not only discovery potential, mass and width of the top quark [6, 7], but also on the structure of the top quark coupling to the charged weak current. It has been argued that the process may trigger new physics [7], CP-violation studies and the measurement of the number of degrees of freedom of the W [8] as well. In ref. [8] for instance, the authors argue that for a top quark with a mass of $\sim 140 \text{ GeV} / c^2$ such detailed studies could be carried out much easier at the Tevatron than at the projected LHC super collider. However, those claims should be taken with some caution since conclusions are based on rather optimistic assumptions for event rates and the capacity of isolating signal from background. A similar analysis leading to opposite conclusions has been published as well [9].

Several attempts have been made to estimate the semiweak cross-section as a function of the top quark mass and collider parameters, scale of the running strong coupling, structure function parametri-

sations and factorisation scale. Considering both bottom and top quarks as massive partons, the Born interaction for single top production is described by the $O(\alpha_s \alpha^2)$ process $qg \rightarrow q't\bar{b}$. For a relatively small value $\frac{m_b}{m_t}$, the quark-gluon scattering amplitude is dominated by the singularity arising from the b-quark and gluon becoming collinear. Resummation and absorption of this singularity in the bottom quark density distribution in the proton provides an alternative description. Our interaction can then be regarded as an $O(\alpha^2)$ Born process: $qb \rightarrow q't$. For a top quark mass up to a few hundred GeV/c^2 ($m_b \approx 5GeV/c^2$), the two pictures give comparable event descriptions and lead to quite similar cross-sections [6, 7]. Accurate determination of the absolute cross-section implies however, as in the analysis of b-collider data [3], precise evaluation of higher order terms. Contrary to the large $O(\alpha_s^3)$ contributions in QCD heavy flavour pair production (genuine higher order contributions which cannot be regarded as straightforward corrections to the lowest order process), we foresee an enhancement of the semiweak cross-sections, which will be of the same order as, or smaller than, the lowest order contribution itself.

In references [7] and [10], we have attempted to approximate the size of these higher order corrections, assuming that the two members of the third family are massive, by summing the $qb \rightarrow q't$ and $qg \rightarrow q't\bar{b}$ processes. Double counting is then avoided by subtracting the singularity from the quark-gluon scattering contribution [11]. In the same analysis, we have investigated in which regions the b-quark mass may be neglected and where the mass of the bottom still acts as a regulator for divergences. $O(\alpha_s^2 \alpha^2)$ tree level contributions have been calculated in ref. [10], enhancing the inclusive single top cross-section and giving rise to events in which the heavy flavours are accompanied by two additional jets.

We should stress that the method pursued so far, that is absorbing the collinear singularities in the structure functions and calculating the matrix elements at tree level only, does not guarantee a sufficiently precise estimate for the total cross-section at $O(\alpha_s \alpha^2)$ [9]. In order to verify our previous result, we calculate the complete set of diagrams up to $O(\alpha_s)$ for single top production:

$$\begin{aligned} qb &\rightarrow q't(g) \\ qg &\rightarrow q't\bar{b} \end{aligned} \tag{1.1}$$

guided by similar analyses for QCD pair production [3] as well as single heavy flavour production in electron-proton scattering via charged [12] or neutral currents [13].

In the next section we limit ourselves to the presentation of the formalism and ingredients for computing the various contributions to the cross-section. The lowest order tree level amplitudes for the processes of (1.1) are derived in section 3. In section 4 we describe the calculation of the hard Bremsstrahlung corrections to the $O(\alpha^2)$ processes. In order to regularise the collinear ($M = \text{mass}$) divergences, we apply the massive regularisation scheme, which was also used in ref. [12] to calculate the cross-section for semiweak top quark production in electron-proton scattering. Note that in ref. [12] no distinction between soft and hard Bremsstrahlung corrections are made. We prefer to discuss them separately (section 5). By introducing a small mass for the gluon, we explicitly study the area

separating hard and soft emission. This method implies that we introduce a small mass such that $m_g \ll m_q$, as discussed by Van Neerven and Vermaseren in ref. [14]. The treatment of the virtual corrections is presented in section 6, where ultraviolet (U.V.) divergences are treated in the massive gluon scheme [15]. In section 7, we pay special attention to the cancellations among the soft Bremsstrahlung and virtual contributions. Next, we emphasise the factorisation of the collinear singularity which leads to the absorption of remaining singularities in the parton density distribution functions in the deep inelastic scattering scheme (DIS) [16] (section 8). Finally, numerical results on the total cross-section are presented (section 9).

2. Description of the calculation

The hadronic cross-section σ^H for the semiweak process:

$$H(P_a) + H(P_b) \rightarrow q(k_1) + t(k_2) + X \quad (2.1)$$

reads in terms of the partonic interaction σ^P and the parton distribution functions $\Gamma(x_a)$, $\Gamma(x_b)$ (with x_a and x_b the momentum fractions carried by the partons):

$$\frac{d\sigma^H}{d\Omega_3} = \sum_{a,b} \int \frac{dx_a}{x_a} \Gamma(x_a) \frac{dx_b}{x_b} \Gamma(x_b) \frac{d\sigma^P}{d\Omega_3} \quad (2.2)$$

The computation of σ^P at first order in α_s is given by the square of the Born term, plus the interference of the Born term with the virtual graphs and the square of the sum of the real graphs with one single QCD coupling:

$$d\sigma = d\sigma^{Born} + d\sigma^{virtual} + d\sigma^{real} \quad (2.3)$$

The ensemble of virtual diagrams (self-energy and vertex corrections) contain U.V. and I.R. divergences, while the diagrams with gluon emission or gluon splitting have, besides I.R. divergences, collinear divergences due to vanishing quark masses. As emphasised in the introduction, we adopt the massive regularisation scheme. In this formalism, divergences are mastered by introducing four parameters which will not appear in the final result: a (small) mass for the gluon, m_g (in order to regularise the infrared divergence), a mass for the light quark, m_q (to regularise the collinear divergence) and a cut-off, ΔE (to separate soft and hard Bremsstrahlung contributions). The gluon and quark masses satisfy the relations $m_g \ll m_q$ and $m_q \ll \Delta E$. Finally, an U.V. cut-off is introduced: $\Lambda \gg \sqrt{s}$, with $\sqrt{s}$ the centre-of mass energy of the partonic sub-process. The massive regularisation scheme is only valid in absence of 3-gluon couplings and is well suited for Monte Carlo applications (see also ref. [12]), while only 1-loop corrections are computed [14].

In the case where a gluon (k_3) is emitted off a light quark line, the kinematical invariants of the overall hadronic process can be related to the kinematics of the partonic sub-processes through the following definitions:

$$\begin{aligned}
S &= (P_a + P_b)^2 \\
s &= (p_a + p_b)^2 \\
T &= (P_b - k_2)^2 \\
t &= (p_b - k_2)^2 \\
U &= (P_a - k_2)^2 \\
u &= (p_a - k_2)^2
\end{aligned} \tag{2.4}$$

leading to the relations $s = x_a x_b S$, $t - m_t^2 = x_b (T - m_t^2)$ and $u - m_t^2 = x_a (U - m_t^2)$, where p_a and p_b denote the 4-momenta of the incoming partons. S , T , U and s , t , u are the Mandelstam variables for the hadronic and partonic processes respectively. Eq. (2.2) then becomes

$$\frac{S^2 d\sigma^H}{dTdU} = \sum_{a,b} \int \frac{dx_a}{x_a} f(x_a) \frac{dx_b}{x_b} f(x_b) S^2 \frac{d\sigma^P}{dtdu} \tag{2.5}$$

and we can write in the presence of gluon emission

$$s + t + u = s_g + m_t^2 \tag{2.6}$$

where $s_g = (k_1 + k_3)^2$. The transition between soft and hard Bremsstrahlung regions can then be expressed in terms of s_g , while for $s_g \rightarrow 0$ IR singularities are encountered. In combining (2.4) and (2.6), eq. (2.6) can be written in terms of x_a and x_b allowing for a change of integration variables. Consequently, the integral can now also be written in terms of s_g , while the soft and hard Bremsstrahlung regions can be integrated over separately:

$$\begin{aligned}
S^2 \frac{d^2\sigma}{dTdU} &= \int_{x_b^{\min}}^1 \frac{dx_b}{x_b} \left(\int_{m_g^2}^{-\epsilon t} ds_g + \int_{-\epsilon t}^{x_b(S+T-m_t^2)+U} ds_g \right) \times \\
&\quad f_a \left(\frac{s_g - m_t^2 - x_b(T - m_t^2)}{x_b S + U - m_t^2} \right) f_b(x_b) \frac{s^2}{s_g - m_t^2 - x_b(T - m_t^2)} \frac{d^2\sigma}{dtdu}
\end{aligned} \tag{2.7}$$

A similar expression can be obtained for gluons emitted off the heavy quark lines by removing the mass term in the integrand, replacing x_a by x_b and substituting the proper integration limits for the soft and hard part resp.:

$$\begin{aligned}
(m_t + m_g)^2 &< s_g < -\epsilon t + m_t^2 \\
-\epsilon t + m_t^2 &< s_g < x_a(S + T) + U
\end{aligned} \tag{2.8}$$

Note that s_g in the latter case reads $s_g = (k_2 + k_3)^2$.

The charged weak current between the light quark and heavy quark vertices separates the diagrams (gluon splitting or gluon emission from either line) into two parts which do not interfere. Therefore, the amplitude squared for each diagram can be obtained by contracting the expression for

the matrix element squared of its sub-diagrams.

In the following sections we present the amplitudes squared for the various sub-processes. The expressions have been obtained with the symbolic manipulation program FORM [17].

3. qq and qg lowest order tree level contributions

The expressions for the tree level amplitude squared for the processes (1.1) are quite similar to the ones obtained for weak heavy flavour production in e-p scattering. They are discussed in detail in ref. [7, 18]. Here we summarise the calculations, presenting the definition of the kinematics and recalling the final results. We distinguish three basic contributions. At order α^2 , there is the t-channel exchange of a charged W boson between a (anti) bottom quark and a light (anti) quark. The Feynman diagrams for this lowest order quark-bottom scattering process (ignoring charge conjugations) are depicted in fig. 3.1.

Fig. 3.1: Order α^2 quark-bottom scattering contributions to top production via W exchange in hadronic interactions. Although of minor importance, off-diagonal Cabibbo-Kobayashi-Maskawa mixings are to be included as well.

Including off-diagonal Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|U_{qq'}|$ [7] we obtain the following expressions for the differential cross-sections:

$$\frac{d\sigma^{Bom}(qb \rightarrow q't)}{dQ^2} = -\frac{\pi\alpha^2}{4s\sin^4\theta_W} \frac{s-m_t^2}{(Q^2+M_W^2)^2} |U_{qq'}| \quad (3.1)$$

and after crossing particle and antiparticle:

$$\frac{d\sigma^{Bom}(q\bar{b} \rightarrow q'\bar{t})}{dQ^2} = -\frac{\pi\alpha^2}{4s^2\sin^4\theta_W} \frac{u(u-m_t^2)}{(Q^2+M_W^2)^2} |U_{qq'}| \quad (3.2)$$

The usual kinematical invariants have been introduced in (2.4) while

$$Q^2 = -q^2 = -(p_a - k_1)^2 \quad (3.3)$$

and $\sin\theta_W$ denotes the Weinberg angle, m_t the mass of the top quark, M_W the mass and Γ_W the width of the W boson.

At order $\alpha_s\alpha^2$, quark-gluon scattering results in a light quark, bottom and top. Contributing diagrams are shown in fig. 3.2. The processes can be regarded as a gluon splitting into $b\bar{b}$ or $t\bar{t}$ pairs.

Fig. 3.2: Dominant $O(\alpha_s \alpha^2)$ quark-gluon scattering contributions to (anti) top production via W exchange in hadronic interactions. Although of minor importance, other CKM mixings should be included as well.

The mass of the heavy (anti) quark acts as a regulator for collinear divergences in the gluon splitting process. This divergent behaviour becomes apparent when the b quark mass becomes negligible compared to its momentum. In latter case one may keep the amplitude squared well behaved by absorbing the divergences into the quark parton distribution functions of the proton. Note that infrared divergences are absent and thus there is no need to introduce a massive gluon. The full set of Feynman graphs contributing to the process $qg \rightarrow q' t \bar{b}$ has already been detailed in ref. [7], where we computed the following expression for the differential cross-section:

$$\frac{d\sigma}{d\Omega_3} = \frac{\alpha_s \alpha^2}{512 s \pi^2} \sum |M^2| \quad (3.4)$$

The numerical factor 512 includes the averaging over the initial quark and gluon spins ($\frac{1}{2}$) and gluon colours ($\frac{1}{8} \text{Tr}[T_a T_a] = \frac{1}{2}$). Due to gauge invariance, the amplitude squared for the complete set of $O(\alpha_s \alpha^2)$ diagrams can be regarded as a Lorentz contraction of the amplitudes squared for a light quark (q) emitting a W boson ($q \rightarrow W q'$ in fig. 3.2) and the process $gW \rightarrow t \bar{b}$:

$$\sum |M|^2 = T_{\mu\nu}^q T_t^{\mu\nu} \frac{1}{(Q^2 + M_W^2)^2} \quad (3.5)$$

The three particle phase space can be parametrised in the centre-of-mass frame of the two incoming partons as [19]:

$$d\Omega_3 = -\frac{\pi ds_3 dt_3 dQ^2 d\phi}{8s(s_3 + Q^2)} \quad (3.6)$$

where ϕ is defined as the azimuthal angle of k_1 with respect to p_a and

$$\begin{aligned} s_3 &= (k_2 + k_3)^2 \\ t_3 &= (p_b - k_3)^2 \end{aligned} \quad (3.7)$$

The momentum transferred by the W boson is again defined as:

$$Q^2 = -q^2 = -(p_a - k_1)^2 \quad (3.8)$$

We are presently only interested in calculating the total cross-section for the top quark, thus ignoring the kinematical properties of the top and supplementary jets in the event. We will therefore integrate over k_3 and write the expression for the matrix element squared in the following general form:

$$T_t^{\mu\nu} = 16 \left[-F_1 p_b \cdot q g^{\mu\nu} + 2F_2 p_b^\mu p_b^\nu + iF_3 \varepsilon^{\mu\nu\rho\sigma} p_{b\rho} q_\sigma + F_4 q^\mu q^\nu + F_5 (q^\mu p_b^\nu + q^\nu p_b^\mu) \right] \quad (3.9)$$

where the factor 16 is introduced for convenience. The terms proportional to F_4 and F_5 may be dropped due to current conservation in the light quark vertex and eq. (3.9) reduces to

$$T_t^{\mu\nu} = 16 \left[-F_1 p_b \cdot q g^{\mu\nu} + 2F_2 p_b^\mu p_b^\nu + iF_3 \varepsilon^{\mu\nu\rho\sigma} p_{b\rho} q_\sigma \right] \quad (3.10)$$

The form factors F_i can be obtained through the relation $F_i = P_{\mu\nu}^i T_t^{\mu\nu}$. The projectors P^i read:

$$\begin{aligned} P_{\mu\nu}^1 &= -\frac{1}{2} \frac{\left[-(s_3 + Q^2)^2 \delta_{\mu\nu} + 4Q^2 p_{b\mu} p_{b\nu} + 2(s_3 + Q^2)(p_{b\mu} q_\nu + p_{b\nu} q_\mu) \right]}{(s_3 + Q^2)^2} \\ P_{\mu\nu}^2 &= 2 \frac{\left[-Q^2(s_3 + Q^2)^2 \delta_{\mu\nu} + 12Q^4 p_{b\mu} p_{b\nu} + 6Q^2(s_3 + Q^2)(p_{b\mu} q_\nu + p_{b\nu} q_\mu) \right]}{(s_3 + Q^2)^4} \\ P_{\mu\nu}^3 &= -\frac{2\varepsilon_{\mu\nu\rho\sigma} p_b^\rho q^\sigma}{(s_3 + Q^2)^2} \end{aligned} \quad (3.11)$$

as detailed by Gottschalk [20]. Since the amplitude squared for the process $g(p_b)W(q) \rightarrow t(k_2)\bar{b}(k_3)$ can be expressed in terms of the variables s_3 and t_3 , the angular integration over k_3 can be regarded as an integration over t_3 . For the integration limits of t_3 we obtain (neglecting the b-quark mass everywhere but in the singular logarithms):

$$\begin{aligned} t_3^{\min} &= -\frac{(s_3 - m_t^2)(s_3 + Q^2)}{s_3} \\ t_3^{\max} &= -m_b^2 \frac{(Q^2 + m_t^2)}{(s_3 - m_t^2)} \end{aligned} \quad (3.12)$$

In the limit where the b-quark becomes massless, the momentum fraction z_b the intermediate b takes from the gluon reduces to:

$$z_b = \frac{Q^2 + m_t^2}{Q^2 + s_3} \quad (3.13)$$

We write s_3 in terms of z_b and after integration, we arrive at results conform the calculations presented

in eqs. (9, 10, 11) of ref. [12]:

$$\begin{aligned}
 F_1 &= 2P_{qg}L_1 + (8x_0z_b^2 - 4z_b)(1-x_0)L_2 + 8x_0z_b(1-z_b) - 2\frac{1-z_b}{1-x_0z_b} \\
 F_2 &= z_b \left[2P_{qg}L_1 - 12x_0z_b(1-x_0)(1-2x_0z_b)L_2 \right. \\
 &\quad \left. - 12x_0z_b(1-z_b)(1-2x_0) + 4z_b(1-z_b) - \frac{2x_0(1-z_b)}{1-x_0z_b} \right] \\
 F_3 &= 2P_{qg}L_1 + (2-4x_0z_b(1-z_b))L_2 + 4z_b(1-z_b)(1-x_0)
 \end{aligned} \tag{3.14}$$

where the following shorthand notations are introduced:

$$\begin{aligned}
 x_0 &= \frac{Q^2}{Q^2 + m_t^2} \\
 L_1 &= \text{Log} \frac{Q^2(1-z_b)^2}{m_b^2 x_0 z_b^2 (1-x_0)} \\
 L_2 &= \text{Log} \frac{(1-x_0)z_b}{(1-x_0z_b)}
 \end{aligned} \tag{3.15}$$

while the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi splitting function (DGLAP) [21] appears in front of the singular logarithms L_1 : $P_{qg} \equiv P_{qg}(z_b) = \frac{1}{2} [z_b^2 + (1-z_b)^2]$.

The structure of the light quark tensor is much simpler since we only deal with the process $q \rightarrow q'W$ and thus do not need to carry out an integration over phase space. The matrix element squared reads:

$$T_q^{\mu\nu} = -8(2p_a^\mu p_a^\nu + g^{\mu\nu} p_a \cdot q - q^\mu p_a^\nu - q^\nu p_a^\mu - i\epsilon^{\mu\nu\rho\sigma} q_\rho p_{a\sigma}) \tag{3.16}$$

Contraction of (3.10) and (3.16) using

$$\begin{aligned}
 g_{\mu\nu} T_q^{\mu\nu} &= -8Q^2 \\
 p_{b\mu} p_{bv} T_q^{\mu\nu} &= 4s^2 - 4s(s_3 + Q^2) \\
 \epsilon_{\mu\nu\rho\sigma} p_b^\rho q^\sigma T_q^{\mu\nu} &= -8Q^2 s + 4Q^2(s_3 + Q^2)
 \end{aligned} \tag{3.17}$$

leads to the final expression for the differential cross-section:

$$\frac{d\sigma}{ds_3 dQ^2} = \frac{\alpha_s \alpha^2}{32 \sin^4 \theta_W} \frac{1}{(Q^2 + M_W^2)^2} \left[\frac{Q^2}{s^2} (F_1 - F_3) + \frac{2(s - Q^2 - s_3)F_2 + 2Q^2 F_3}{s(Q^2 + s_3)} \right] \tag{3.18}$$

Similar to the diagrams presented in figure 3.1, a top quark may be produced by scattering of a b-quark and a light quark, where the light quark now emerges from the gluon splitting into a $q\bar{q}$ pair (fig. 3.3). Hence these processes are of order $\alpha_s \alpha^2$ as well. The quark distribution functions in the proton are dominated by contributions from valence quarks and light sea quarks. The bottom quark flux is rel-

atively small for a wide range of x and Q^2 values. The formalism for calculating their contribution to the cross-section is almost identical to the one used in previous derivation.

Fig. 3.3: $O(\alpha_s \alpha^2)$ bottom-gluon scattering contributions to top production via W exchange. Also here, all possible CKM mixings should be included.

The heavy quark tensor can now be easily obtained from eq. (3.16) by assigning the proper kinematics (note that here $q = k_2 - p_b$):

$$T_t^{\mu\nu} = -8(2p_b^\mu p_b^\nu + g^{\mu\nu} p_b \cdot q - q^\mu p_b^\nu - q^\nu p_b^\mu - i\epsilon^{\mu\nu\rho\sigma} q_\rho p_{b\sigma}) \quad (3.19)$$

and we write the light quark tensor as

$$T_q^{\mu\nu} = 16 \left[F_1 p_a \cdot q g^{\mu\nu} + 2F_2 p_a^\mu p_a^\nu + iF_3 \epsilon^{\mu\nu\rho\sigma} p_{a\rho} q_\sigma + F_4 q^\mu q^\nu + F_5 (q^\mu p_a^\nu + q^\nu p_a^\mu) \right] \quad (3.20)$$

The form factors are defined in terms of the kinematical variables s_4 and t_4 :

$$s_4 = (k_1 + k_3)^2 \quad (3.21)$$

$$t_4 = (p_a - k_3)^2$$

The projectors read:

$$P_{\mu\nu}^1 = -\frac{1}{2} \frac{\left[-(s_4 + Q^2)^2 \delta_{\mu\nu} + 4Q^2 p_{a\mu} p_{a\nu} - 2(s_4 + Q^2)(p_{a\mu} q_\nu + p_{a\nu} q_\mu) \right]}{(s_4 + Q^2)^2}$$

$$P_{\mu\nu}^2 = 2 \frac{\left[-Q^2(s_4 + Q^2)^2 \delta_{\mu\nu} + 12Q^4 p_{a\mu} p_{a\nu} - 6Q^2(s_4 + Q^2)(p_{a\mu} q_\nu + p_{a\nu} q_\mu) + 2(s_4 + Q^2)^2 q_\mu q_\nu \right]}{(s_4 + Q^2)^4}$$

$$P_{\mu\nu}^3 = -\frac{2\epsilon_{\mu\nu\rho\sigma} p_a^\rho q^\sigma}{(s_4 + Q^2)^2} \quad (3.22)$$

$$P_{\mu\nu}^4 = \frac{4p_a^\mu p_a^\nu}{(s_4 + Q^2)^2}$$

$$P_{\mu\nu}^5 = -\frac{\left[-(s_4 + Q^2)^2 \delta_{\mu\nu} + 12Q^2 p_{a\mu} p_{a\nu} - 4(s_4 + Q^2)(p_{a\mu} q_\nu + p_{a\nu} q_\mu)\right]}{(s_4 + Q^2)^2}$$

Next, we have to integrate over t_4 for which the following integration limits apply:

$$t_4^{\min} = -(s_4 + Q^2)$$

$$t_4^{\max} = -m_q^2 \frac{Q^2}{s_4}$$
(3.23)

Contraction of (3.19) and (3.20) with

$$g_{\mu\nu} T_q^{\mu\nu} = -8(Q^2 + m_t^2)$$

$$p_{a\mu} p_{a\nu} T_q^{\mu\nu} = 4s^2 - 4s(s_4 + Q^2)$$

$$\varepsilon_{\mu\nu\rho\sigma} p_{a\rho} q_\sigma T_q^{\mu\nu} = 8Q^2 s - 4(Q^2 + m_t^2)(s_4 + Q^2)$$

$$q_\mu q_\nu T_q^{\mu\nu} = 4m_t^2(Q^2 + m_t^2)$$

$$(p_{a\mu} q_\nu + q_\mu p_{a\nu}) T_q^{\mu\nu} = 8m_t^2 s$$
(3.24)

leads to:

$$\frac{d\sigma}{ds_4 dQ^2} = \frac{\alpha_s \alpha^2}{32 \sin^4 \theta_W} \frac{1}{(Q^2 + M_W^2)^2} \times$$

$$\left[\frac{Q^2 + m_t^2}{s^2} (F_1 - F_3) + \frac{2(s - Q^2 - s_4)F_2 + 2Q^2 F_3}{s(Q^2 + s_4)} + \frac{4m_t^2(Q^2 + m_t^2)}{s^2(Q^2 + s_4)} F_4 + \frac{2m_t^2}{s(Q^2 + s_4)} F_5 \right]$$
(3.25)

Note that due to the mass difference between top and bottom quark, all five form factors survive. After integration over t_4 we obtain

$$F_1 = 2P_{qg}L_3 - 8z_a^2 + 8z_a - 2$$

$$F_2 = z_a(2P_{qg}L_3 - 16z_a^2 + 16z_a - 2)$$

$$F_3 = 2P_{qg}L_3 - (2 + 4z_a^2 - 4z_a)L_3 = 0$$

$$F_4 = 4(1 - z_a)$$

$$F_5 = -2P_{qg}L_3 + 16z_a^2 - 16z_a + 2$$
(3.26)

with

$$L_3 = \text{Log} \frac{Q^2(1-z_a)}{m_q^2 z_a} \quad (3.27)$$

where $P_{qg} \equiv P_{qg}(z_a)$, with $z_a = \frac{Q^2}{Q^2 + s_4}$ the fraction of the initial gluon momentum carried by the intermediate quark. In the limit $m_t \rightarrow 0$, eq. (3.25) reduces to the result for deep inelastic scattering [16]. We check also that when $x_0 \rightarrow 1$, the expressions for F_1 , F_2 and F_3 in eq. (3.14) become identical to those in eq. (3.26).

4. Hard Bremsstrahlung

The collinear divergences in the expressions for the matrix elements squared presented in previous section were shown to appear in the limit where the quark masses vanish. In this limit, Bremsstrahlung corrections to the Born process $qb \rightarrow q't$ turn out to give similar singular logarithms as we will see in the following. The relevant Feynman diagrams are depicted in figure 4.1:

Fig. 4.1: Bremsstrahlung corrections to the Born quark-bottom scattering contributions to top quark production through charged W exchange. Various CKM mixings are not explicitly shown.

In this section we assume $E^{\text{gluon}} > 0$, avoiding I.R. singularities. The differential cross-section for the Bremsstrahlung contributions is derived from eq. (3.4) by replacing the colour factor $\frac{4}{3}$ (incoming gluon) by $\frac{1}{2}$ (outgoing gluon):

$$\frac{d\sigma}{d\Omega_3} = \frac{\alpha_s \alpha^2}{192s\pi^2} \Sigma |M^2| \quad (4.1)$$

where $\Sigma |M|^2$ symbolises again the decomposition defined in eq. (3.5). Note that we are concerned with a massive top quark only; all other quarks have a physical mass equal to zero. Note also that in contrast with the diagrams discussed in the previous section, all graphs in fig. 4.1 lead to the same final state, while colour conservation forbids interference between the two upper and two lower diagrams.

First we consider emission off the bottom and top lines. The general expression for the matrix element squared for the sub-process $b(p_b)W(q) \rightarrow t(k_2)g(k_3)$, corresponding to the lower vertices, is

given in eq. (3.10), while the matrix element squared for the upper vertices, $q(p_a) \rightarrow q'(k_1)W(q)$, is written in eq. (3.16). The definitions for the phase space element (3.6), Mandelstam variables (3.7) and energy-momentum carried by the W (3.8) remain unchanged as well. Note however, that now k_3 represents the gluon energy-momentum, which implies that the integration limits on t_3 need to be modified:

$$t_3^{\min} = -\frac{(s_3 - m_t^2)(s_3 + Q^2)}{s_3} \quad (4.2)$$

$$t_3^{\max} = m_b^2 \frac{(Q^2 + m_t^2)}{(s_3 + Q^2)}$$

Integration, after introducing z_b (3.13), x_0 (3.15) and using the projectors defined in eq. (3.11), leads to the following expressions for the form factors:

$$F_1 = P_{qq}L_4 - 4\frac{z_b}{1-z_b} + \frac{z_b(1-z_b)}{1-x_0z_b} + \frac{1}{2}\frac{1-z_b}{(1-x_0z_b)^2}$$

$$F_2 = z_b \left[P_{qq}L_4 - \frac{2-z_b-2z_b^2+3z_b^3}{z_b(1-z_b)} + \frac{2-z_b-z_b^2}{z_b(1-x_0z_b)} + \frac{1}{2}\frac{1-z_b}{(1-x_0z_b)^2} - 2\frac{z_b}{1-z_b} \right] \quad (4.3)$$

$$F_3 = P_{qq}L_4 - 2\frac{1+z_b^2}{1-z_b} + \frac{1-z_b}{1-x_0z_b} + \frac{1}{2}\frac{1-z_b}{(1-x_0z_b)^2}$$

where the coefficient of the singular logarithm

$$L_4 = \text{Log} \frac{Q^2}{m_b^2 x_0 z_b (1-x_0 z_b)} \quad (4.4)$$

is the DGLAP probability distribution $P_{qq} \equiv P_{qq}(z_b) = \frac{1+z_b^2}{1-z_b}$ for a quark emitting a gluon with an energy fraction $1-z_b$ [21]. The contribution to the differential cross-section of the Bremsstrahlung off heavy quark lines takes, apart from a colour factor, the same form as the expression given in eq. (3.18):

$$\frac{d\sigma}{ds_3 dQ^2} = \frac{\alpha_s \alpha^2}{24 \sin^4 \theta_W} \frac{1}{(Q^2 + M_W^2)^2} \left[\frac{Q^2}{s^2} (F_1 - F_3) + \frac{2(s - Q^2 - s_3)F_2 + 2Q^2 F_3}{s(Q^2 + s_3)} \right] \quad (4.5)$$

with the form factors given in (4.3).

Let us now turn to the calculation of the amplitude squared for gluon emission off the light quark lines. This case is evidently quite similar to the calculation of gluon splitting vertices in fig. 3.3. We recall that due to a massive top, $T_{\mu\nu}^q$ does neither satisfy $q^\mu T_{\mu\nu}^q = 0$ nor $q^\nu T_{\mu\nu}^q = 0$ and we should use the full expression of eq. (3.20). The form factors are again defined in terms of the invariants s_4 and t_4 and are given in (3.21). The integration limits for t_4 become:

$$\begin{aligned}
t_4^{\min} &= -(s_4 + Q^2) \\
t_4^{\max} &= m_q^2 \frac{Q^2}{(s_4 + Q^2)}
\end{aligned}
\tag{4.6}$$

and we obtain (apart from a colour factor) the same expression as in eq. (3.25):

$$\begin{aligned}
\frac{d\sigma}{ds_4 dQ^2} &= \frac{\alpha_s \alpha^2}{24 \sin^4 \theta_W} \frac{1}{(Q^2 + M_W^2)^2} \times \\
&\left[\frac{Q^2 + m_t^2}{s^2} (F_1 - F_3) + \frac{2(s - Q^2 - s_4)F_2 + 2Q^2 F_3}{s(Q^2 + s_4)} + \frac{4m_t^2(Q^2 + m_t^2)}{s^2(Q^2 + s_4)} F_4 + \frac{2m_t^2}{s(Q^2 + s_4)} F_5 \right]
\end{aligned}
\tag{4.7}$$

Introducing z_a as the fraction of the quark momentum left after emitting a gluon:

$$z_a = \frac{Q^2}{Q^2 + s_4} \tag{4.8}$$

we derive the following expressions for the form factors:

$$\begin{aligned}
F_1 &= 2P_{qq}L_5 + \frac{1 - 6z_a - 2z_a^2}{1 - z_a} \\
F_2 &= z_a \left(2P_{qq}L_5 + \frac{1 - 2z_a - 6z_a^2}{1 - z_a} \right) \\
F_3 &= 2P_{qq}L_5 - \frac{1 + 2z_a + 4z_a^2}{1 - z_a} \\
F_4 &= \frac{1}{2} \\
F_5 &= -2P_{qq}L_5 - \frac{1 - 2z_a - 6z_a^2}{1 - z_a}
\end{aligned}
\tag{4.9}$$

with

$$L_5 = \text{Log} \frac{Q^2}{m_q^2 z_a (1 - z_a)} \tag{4.10}$$

and $P_{qq} \equiv P_{qq}(z_a)$. We indeed find that our expression for F_2 agrees with the result obtained in ref. [16] for deep inelastic scattering. Notice the divergent structure in the expressions (4.3) and (4.9) for $z_a, z_b = 1$ as a manifestation of the I.R. singularity, which obviously implies that expressions derived in this section are only valid when $z_a, z_b < 1 - \varepsilon$ (with ε proportional to the gluon energy, see also (2.7)).

5. Soft Bremsstrahlung

For the soft Bremsstrahlung corrections, we adopt the kinematical definitions given in figure 4.1. In the soft limit, we have chosen to control the I.R. singularity by introducing a (small) mass for the gluon. As in previous sections, the M-singularity is regulated by a light quark mass. In the gluon energy range $m_g \ll E_g \ll \omega_0$, where ω_0 is introduced to separate the soft and hard Bremsstrahlung regions, the largest contribution to the cross-section is arising from singular terms in k_3 [15]. These terms are obtained by calculating the matrix element squared for the two sets of diagrams in fig. 4.1, where all contributions proportional to k_3 in the numerator are dropped (eikonal approximation).

The matrix element squared (see eq. 4.1) for emission off the heavy quark lines then reads

$$|M|^2 = -\frac{64(s-m_b^2)(s-m_t^2)}{(Q^2+M_W^2)^2} \left(\frac{m_b^2}{(p_b \cdot k_3)^2} + \frac{m_t^2}{(k_2 \cdot k_3)^2} - \frac{Q^2 + m_b^2 + m_t^2}{p_b \cdot k_3 k_2 \cdot k_3} \right) \quad (5.1)$$

This can be regarded as the expression for the square of the Born amplitude (eq. 3.1) convoluted with the matrix element for the b quark or the t quark emitting a gluon. After integration over k_3 , logarithms proportional to the energy of the gluon will appear. The expression for three-body phase space in terms of k_3 becomes

$$d\Omega_3 = d\Omega_2 \frac{d^3 k_3}{2E_3} \quad (5.2)$$

and after substitution into (4.1), we obtain the following formula for the differential cross-section:

$$\frac{d\sigma}{d\Omega_2} = -\frac{d\sigma^{Born}}{d\Omega_2} \frac{2\alpha_s}{3\pi^2} \int \frac{d^3 k_3}{2E_3} \left(\frac{m_b^2}{(p_b \cdot k_3)^2} + \frac{m_t^2}{(k_2 \cdot k_3)^2} - \frac{Q^2 + m_b^2 + m_t^2}{p_b \cdot k_3 k_2 \cdot k_3} \right) \quad (5.3)$$

Integration of the first two terms in the integrand has been discussed in ref. [15]. By introducing an auxiliary vector [22], the integral of the third term can be solved (see also Appendix A) and we obtain

$$d\sigma = \frac{2\alpha_s}{3\pi} d\sigma^{Born} \left[-2 \log \frac{4\omega_0^2}{m_g^2} - L_b + 2 - \log \frac{4\omega_0^2}{m_g^2} L_b - \frac{1}{2} L_b^2 - \frac{\pi^2}{3} \right] \quad (5.4)$$

where L_b is defined according to:

$$L_i = \log \frac{m_i^2 x_0 (1-x_0)}{Q^2} \quad (5.5)$$

and x_0 in eq. (3.15).

Replacing the heavy flavour masses in eq. (5.1) by a light quark mass leads, after integration, to a similar expression. Taking the limit $m_q^2 \ll Q^2$ in eq. (5.4), replacing m_b by m_q and using the relation

$$x_0(1-x_0) = \frac{m_t^2}{Q^2 + m_t^2} = \frac{m_q^2}{Q^2} \quad (5.6)$$

in the substitution of m_t , then gives

$$d\sigma = \frac{2\alpha_s}{3\pi} d\sigma^{Born} \left[-2 \log \frac{4\omega_0^2}{m_g^2} \log \frac{m_q^2}{Q^2} - 2 \log^2 \frac{m_q^2}{Q^2} - 2 \log \frac{4\omega_0^2}{m_g^2} - 2 \log \frac{m_q^2}{Q^2} + 2 - \frac{\pi^2}{3} \right] \quad (5.7)$$

Next we derive the relation between ε - the cut-off parameter introduced in the previous section to separate hard and soft Bremsstrahlung - and ω_0 . The hard Bremsstrahlung region for heavy quarks is limited to $z_b = \frac{Q^2 + m_t^2}{Q^2 + s_3} < 1 - \varepsilon$. With s_3 defined in eq. (3.7) and $|\vec{k}_3| = \omega_0$ in the centre-of-mass of the $\vec{k}_2, \vec{k}_3$ frame, we obtain

$$s_3 = m_t^2 + 2m_t\omega_0 \quad (5.8)$$

which leads to

$$\omega_0 = \frac{\varepsilon Q^2}{2m_t x_0} \quad (5.9)$$

In case of light quarks $z_a = \frac{Q^2}{Q^2 + s_4} < 1 - \varepsilon$, with the definition for s_4 given in eq. (3.21), we find

$$\omega_0 = \frac{\varepsilon Q^2}{2m_q} \quad (5.10)$$

6. Virtual corrections

In this section we discuss in detail the self-energy and vertex corrections. The graphs in Fig 6.1 show the self-energy corrections to the diagrams in Fig 3.1.

Fig. 6.1: Self-energy corrections to the Born quark-bottom scattering contributions to top quark production through charged W exchange.

In case of the first diagram (the following derivation holds for the other diagrams as well), we write the relevant term in the amplitude:

$$\dots \bar{u}(k_1) \gamma_\mu (1 - \gamma_5) \frac{1}{\not{p}_a - m} \frac{4}{3} \frac{g_s^2}{(2\pi)^4} \int \frac{d^4 k g^{\sigma\rho} \gamma_\sigma (\not{p}_a - \not{k} + m) \gamma_\rho}{k^2 ((p_a - k)^2 - m^2)} u(p_a) \dots \quad (6.1)$$

where k is the four-momentum of the gluon, m the non-vanishing mass of the quark and g_s the quark-gluon coupling, while the other parameters are defined in fig. 6.1. This self-energy contribution can be regarded as a modification of the quark wave function. The integral gives rise to both U.V. and I.R. divergences. We regularise these U.V. and I.R. divergences by replacing the gluon propagator in the Feynman gauge:

$$D_{\mu\nu}(k) = -i \frac{g_{\mu\nu}}{k^2} \quad (6.2)$$

by the following expression [15]:

$$D_{\mu\nu}(k) = i g_{\mu\nu} \int_{m_g^2}^{\Lambda} \frac{dl}{(k^2 - l)^2} \quad (6.3)$$

where Λ regularises the U.V. divergence and m_g , the gluon mass, regularises the I.R. divergence. The integral expression in eq. (6.1) can then, after introduction of the proper Feynman parameter x , be written as [23] ($p = p_a$):

$$\Sigma(p) = \frac{4}{3} \frac{g_s^2}{(2\pi)^4} \int_0^1 dx \int_{m_g^2}^{\Lambda} dl \int d^4 k \frac{2(1-x)[4m + \gamma_\sigma (\not{p} - \not{k}) \gamma_\sigma]}{[(k^2 - l)(1-x) + x(p-k)^2 - m^2]^3} \quad (6.4)$$

By the substitution $K = k - px$, we get:

$$\Sigma(p) = \frac{4}{3} \frac{g_s^2}{(2\pi)^4} \int_0^1 dx \int_{m_g^2}^{\Lambda} dl \int d^4 K \frac{(1-x)[8m - 4(1-x)\not{p}]}{[K^2 - p^2 x^2 - l(1-x) + (p^2 - m^2)]^3} \quad (6.5)$$

After integration over K and l we are left with the following expression:

$$\Sigma(p) = -i \frac{4}{3} \frac{\alpha_s}{2\pi} \int_0^1 dx \log \left| \frac{p^2 x^2 + \Lambda^2 (1-x) - x(p^2 - m^2)}{p^2 x^2 + m_g^2 (1-x) - x(p^2 - m^2)} \right| [2m - (1-x)\not{p}] \quad (6.6)$$

In the on-shell renormalisation scheme, this expression can be written as a Taylor series expansion in the vicinity of $p = m$:

$$\Sigma(p, \Lambda) = \Sigma(p, \Lambda)|_{(p=m)} + (p-m) \frac{\partial \Sigma(p, \Lambda)}{\partial p} \Big|_{(p=m)} + \Sigma_R \quad (6.7)$$

The first term in the series is defined as the mass renormalisation counter term ($\delta m \equiv \Sigma(p, \Lambda)|_{(p=m)}$).

The quark wave function renormalisation constant is defined as

$$Z_2 \equiv 1 + i \frac{\partial \Sigma(p, \Lambda)}{\partial p} \Big|_{(p=m)} \quad (6.8)$$

while Σ_R , the renormalised self-energy, contains all higher order terms and is U.V. convergent. The differentiation of eq. (6.6) with respect to p is straightforward as is the final integration over x . One finds:

$$Z_2 - 1 = -\frac{\alpha_s}{3\pi} \left(\log \frac{\Lambda^2}{m^2} + 2 \log \frac{m_g^2}{m^2} + \frac{9}{2} \right) \quad (6.9)$$

which contains U.V. and I.R. divergences and is the only contribution to the interference between the $O(\alpha_s \alpha)$ self-energy diagrams and $O(\alpha)$ Born diagrams since the quarks are on-shell. The self-energy correction for quark lines then becomes

$$d\sigma^{Self} = -\frac{2\alpha_s}{3\pi} d\sigma^{Born} \left(\log \frac{\Lambda^2}{m^2} + 2 \log \frac{m_g^2}{m^2} + \frac{9}{2} \right) \quad (6.10)$$

In figure 6.2 we present the diagrams for the vertex corrections on both light and heavy quark lines.

Fig. 6.2: Vertex corrections to the Born quark-bottom scattering contributions to top quark production through charged W exchange.

We start with the diagram in which the gluon connects the light quark lines. The relevant term in the expression for the matrix element reads:

$$\dots \bar{u}(k_1) \frac{4}{3} \frac{g_s^2}{(2\pi)^4} \int \frac{d^4 k g^{\sigma\rho} \gamma_\sigma (k_1 - k + m_q) \gamma_\mu (1 - \gamma_5) (\not{p}_a - k + m_q) \gamma_\rho}{k^2 ((k_1 - k)^2 - m_q^2) ((p_a - k)^2 - m_q^2)} u(p_a) \dots \quad (6.11)$$

with k again the gluon four-momentum, m_q a quark mass that regularises the collinear singularities and g_s the quark-gluon coupling. Four-momenta are defined in fig. 6.2. With FORM [17] we have verified that the amplitude squared for the interference between this vertex correction and the Born term can be written as

$$d\sigma^{Vertex} = V d\sigma^{Born} \quad (6.12)$$

such that:

$$V = \frac{4}{3} \frac{g_s^2}{(2\pi)^4} \int \frac{N d^4 k}{k^2 ((k_1 - k)^2 - m_q^2) ((p_a - k)^2 - m_q^2)} \quad (6.13)$$

N consists of a polynomial of second order in k . Thus V contains both U.V. and I.R divergences. We can now follow the same procedure as sketched above. However, instead of having a product of two terms in the denominator, we have to solve an integral with a triple product. This leads to an additional integration over the Feynman parameter 'y':

$$V = \frac{4}{3} \frac{g_s^2}{(2\pi)^4} \int_0^1 dx \int_0^1 dy \int_{m_g^2}^{\Lambda} dl \int d^4 k \frac{N}{\left[(k^2 - 2k(k_1 y + p_a(1-y)))x + (k^2 - l)(1-x) \right]^4} \quad (6.14)$$

By substituting $K = k - (k_1 y + p_a(1-y))x$ we get:

$$V = \frac{4}{3} \frac{g_s^2}{(2\pi)^4} \int_0^1 dx \int_0^1 dy \int_{m_g^2}^{\Lambda} dl \int d^4 K \frac{Q^2 [2(1-x) + 2x^2 y(1-y)] - K^2}{\left[K^2 - (k_1 y + p_a(1-y))^2 x^2 - l(1-x) \right]^4} \quad (6.15)$$

The integrals are quite tedious but well known [23] and eq. (6.12) becomes (Appendix B):

$$d\sigma^{Vertex} = \frac{2\alpha_s}{3\pi} d\sigma^{Bom} \left(\log \frac{\Lambda^2}{Q^2} - 4 \log \frac{m_q^2}{Q^2} - 2 \log \frac{m_q^2}{Q^2} \log \frac{m_g^2}{Q^2} + \log^2 \frac{m_q^2}{Q^2} + \frac{\pi^2}{3} + \frac{1}{2} \right) \quad (6.16)$$

The complete contribution from the virtual diagrams for the light quark lines is obtained by adding eqs. (6.10) and (6.16):

$$d\sigma^{Virtual} = \frac{2\alpha_s}{3\pi} d\sigma^{Bom} \left(-3 \log \frac{m_q^2}{Q^2} - 2 \log \frac{m_q^2}{Q^2} \log \frac{m_g^2}{m_q^2} - \log^2 \frac{m_q^2}{Q^2} - 2 \log \frac{m_g^2}{m_q^2} - 4 + \frac{\pi^2}{3} \right) \quad (6.17)$$

in agreement with for example the result quoted in ref. [24] for the on-shell regularisation scheme. Thus, by adding the self-energy and vertex contributions, compensation between the U.V. divergences occurs.

While the self-energy corrections are universal for light and heavy quarks, the vertex correction for the heavy quark lines turns out to be slightly more complicated than the one derived for light quarks. Taking the formula in eq. (6.11) as starting point, the heavy quark masses in the numerator can no longer be neglected, whereas different mass terms enter in the expression for the denominator as well. Nevertheless, the result looks quite similar to the one given in eq. (6.15):

$$V = \frac{4}{3} \frac{g_s^2}{(2\pi)^4} \int_0^1 dx \int_0^1 dy \int_{m_g^2}^1 dl \int d^4K \frac{K^2 + 2(Q^2 + m_t^2)(1-x) + 2x^2(1-y)Q^2y - T}{[K^2 - Q^2x^2y(1-y) - l(1-x) - m_t^2x^2y - m_b^2x^2(1-y)]^4} \quad (6.18)$$

with

$$T = \frac{m_t^2 s Q^2 xy(1-x)}{(s - m_t^2)(s - m_b^2)} \quad (6.19)$$

The integral may be split such that the heavy vertex contribution reads

$$d\sigma^{Vertex} = (V + I)d\sigma^{Bom} + d\sigma^R \quad (6.20)$$

with I the multi-dimensional integral over the term proportional to T . The non factorisable contribution, $d\sigma^R$, does not contain any divergences. Integration over K , x and y gives:

$$d\sigma^R = -\frac{2\alpha_s\alpha^2}{12\sin^4\theta_w} \frac{1}{(Q^2 + M_w^2)^2} \left(\frac{m_t^2}{s} \log(1-x_0) \right) \quad (6.21)$$

with x_0 defined in eq. (3.15). After performing the integral on the first term in eq. (6.20) (Appendix B), one obtains:

$$d\sigma^{Vertex} = \frac{2\alpha_s}{3\pi} d\sigma^{Bom} \times \left[-L_b L_g - 2L_b + \log \frac{\Lambda^2}{Q^2} + \frac{1}{2} + \frac{\pi^2}{3} - 2Li_2(1-x_0) + \frac{1}{2} L_b^2 - \log^2(1-x_0) + \log x_0 \right] + d\sigma^R \quad (6.22)$$

where Li_2 is defined as the dilogarithm (Spence function, see Appendix A, eq. (A.22)) and L_i is given in eq. (5.5). Combining the vertex and self-energy contributions leads to the expression for the full virtual corrections to the heavy quarks line:

$$d\sigma^{Virtual} = \frac{2\alpha_s}{3\pi} d\sigma^{Bom} \times \left[\frac{1}{2} L_b^2 - L_b L_g - \frac{3}{2} L_b - 4 + \frac{\pi^2}{3} - 2Li_2(1-x_0) - \log^2(1-x_0) - \log \frac{m_g^2 x_0}{Q^2(1-x_0)} - \log \frac{m_g^2}{m_b^2} \right] - \frac{2\alpha_s\alpha^2}{12\sin^4\theta_w} \frac{1}{(Q^2 + M_w^2)^2} \left(\frac{m_t^2}{s} \log(1-x_0) \right) \quad (6.23)$$

As expected, both formulae for the light quarks (6.17) and heavy quarks (6.23) contain singular logarithms in the limit $m_g \rightarrow 0$. In the next section we discuss the cancellation with the previously derived contributions from soft Bremsstrahlung.

7. Infrared cancellations

The soft Bremsstrahlung and virtual corrections, calculated in previous sections, may now be combined. For the case of heavy quarks, eqs. (5.4) and (6.23) give:

$$\begin{aligned} d\sigma &\equiv d\sigma^{\text{Soft}} + d\sigma^{\text{Virtual}} \\ &= \frac{2\alpha_s}{3\pi} d\sigma^{\text{Born}} \times \left[-2L_\omega - 2 - L_b \left(L_\omega + \frac{3}{2} \right) - 2Li_2(1-x_0) - \log^2(1-x_0) + 2\log(1-x_0) \right] \\ &\quad - \frac{2\alpha_s\alpha^2}{12\sin^4\theta_w} \frac{1}{(Q^2 + M_w^2)^2} \left(\frac{m_t^2}{s} \log(1-x_0) \right) \end{aligned} \quad (7.1)$$

with L_b defined according to (5.5) while L_ω reads:

$$L_\omega = \log \frac{4\omega_0^2 x_0 (1-x_0)}{Q^2} \quad (7.2)$$

Next, we substitute the expression for ω_0 (5.9) into (7.2), which then reduces to $L_\omega = 2\log\epsilon$ and we obtain

$$\begin{aligned} d\sigma &= \frac{2\alpha_s}{3\pi} d\sigma^{\text{Born}} \times \left[-4\log\epsilon - 2 - L_b \left(2\log\epsilon + \frac{3}{2} \right) - 2Li_2(1-x_0) - \log^2(1-x_0) + 2\log(1-x_0) \right] \\ &\quad - \frac{2\alpha_s\alpha^2}{12\sin^4\theta_w} \frac{1}{(Q^2 + M_w^2)^2} \left(\frac{m_t^2}{s} \log(1-x_0) \right) \end{aligned} \quad (7.3)$$

All terms proportional to the logarithm of the gluon mass have cancelled, which implies that no I.R. singularities survive.

For the light quarks, by combining (5.7) and (6.17), we obtain

$$d\sigma = \frac{2\alpha_s}{3\pi} d\sigma^{\text{Born}} \left(-3\log^2 \frac{m_q^2}{Q^2} - 2\log \frac{m_q^2}{Q^2} \log \frac{4\omega_0^2}{m_q^2} - 2\log \frac{4\omega_0^2}{m_q^2} - 5\log \frac{m_q^2}{Q^2} - 2 \right) \quad (7.4)$$

while the substitution for ω_0 gives

$$d\sigma = \frac{2\alpha_s}{3\pi} d\sigma^{\text{Born}} \left(\log^2 \frac{m_q^2}{Q^2} - 4\log \frac{m_q^2}{Q^2} \log\epsilon - 4\log\epsilon + \log \frac{m_q^2}{Q^2} - 2 \right) \quad (7.5)$$

After combining all Bremsstrahlung and virtual contributions and absorbing the relevant terms in the structure functions, it turns out that several divergent terms survive. The origin lies in the application of the eikonal approximation (eq. (5.1) and further) for which we have dropped all terms proportional to k_3 . A more careful analysis [24] of the soft Bremsstrahlung region leads to writing the soft contribution as an eikonal term ($m_g < E^{\text{gluon}} < \omega_0$) and a term for which $\omega_0 < E^{\text{gluon}} < \frac{\epsilon Q^2}{2m_q}$:

$$\int_{m_g}^{\omega_0} \frac{d\sigma}{dE^{gluon}} dE^{gluon} + \int_{\omega_0}^{\frac{\epsilon Q^2}{2m_q}} \frac{d\sigma}{dE^{gluon}} dE^{gluon} \quad (7.6)$$

The second integral is obtained by taking the exact expression for Bremsstrahlung in eq. (4.7), in the limit $z_a \rightarrow 1$, keeping all first order terms proportional to $\frac{m_q^2}{Q^2}$, but neglecting the gluon mass. The form factors F_i (4.9) are obtained after integration over t_4 . The modified integration limits now read (see eq. 4.6):

$$t_4^{\min} = m_q^2 - \frac{(s_4 - m_q^2)(s_4 + Q^2)}{s_4} \quad (7.7)$$

$$t_4^{\max} = m_q^2 \frac{Q^2}{s_4 + Q^2 - m_q^2}$$

The singular part of the form factors, F_i , becomes:

$$F_i = 2 \frac{1+z_a^2}{1-z_a} \log \left(\frac{Q^2}{m_q^2} \frac{1}{z_a(1-z_a+z_m)} \right) - \frac{8}{1-z_a} + \frac{1}{1-z_a+z_m} - \frac{z_m}{(1-z_a+z_m)^2} \quad (7.8)$$

where z_a is defined in (4.8) and we have introduced

$$z_m = \frac{m_q^2}{Q^2 + s_4 - m_q^2} \quad (7.9)$$

The two types of denominators, $1-z_a+z_m$ and $1-z_a$, stem from the outgoing quark propagator and angular integration respectively. Using (3.21) and (4.8), the integration on the gluon energy in (7.6) can now be written as an integral on the variable z_a :

$$\int_{1-\delta}^{1-\epsilon} F_i dz_a = \int_{1-\delta}^{1-\epsilon} \left[\frac{1+z_a^2}{1-z_a} \log \left(\frac{Q^2}{m_q^2} \frac{1}{z_a(1-z_a+z_m)} \right) - \frac{2}{1-z_a} + \frac{1}{1-z_a+z_m} - \frac{z_m}{(1-z_a+z_m)^2} \right] dz_a \quad (7.10)$$

where $\delta = \frac{2\omega_0 m}{Q^2}$. The integration over z_a is straightforward and we get:

$$\begin{aligned} \int F_i dz_a = & 3 \log^2 \frac{m_q^2}{Q^2} + \frac{7}{2} \log \frac{m_q^2}{Q^2} + 2 \log \frac{m_q^2}{Q^2} \log \frac{4\omega_0^2}{m_q^2} + 2 \log \frac{4\omega_0^2}{m_q^2} \\ & - 2 \log \frac{m_q^2}{Q^2} \log \epsilon - \log^2 \epsilon - \frac{7}{2} \log \epsilon - \frac{1}{2} - \frac{\pi^2}{3} \end{aligned} \quad (7.11)$$

Then, since the singular parts in the form factors are equal, eq. (4.7) becomes

$$\begin{aligned}
d\sigma &= \frac{\alpha_s \alpha^2}{24 \sin^4 \theta_W} \frac{1}{(Q^2 + M_W^2)^2} \frac{(s - m_l^2)}{s} \int F_i dz_a \\
&= \frac{2\alpha_s}{3\pi} d\sigma^{Born} \times \int F_i dz_a
\end{aligned} \tag{7.12}$$

Adding the soft contributions in eq. (7.12) to the sum of virtual and soft parts in (7.4) gives

$$d\sigma = \frac{2\alpha_s}{3\pi} d\sigma^{Born} \left(-\frac{3}{2} \log \frac{m_q^2}{Q^2} - 2 \log \frac{m_q^2}{Q^2} \log \varepsilon - \log^2 \varepsilon - \frac{7}{2} \log \varepsilon - \frac{5}{2} - \frac{\pi^2}{3} \right) \tag{7.13}$$

Both heavy quark (7.3) and light quark (7.13) contributions are free of I.R. divergences. However, they still contain M-singularities and depend on the value of the cut-off parameter ε .

8. Mass Factorisation in the DIS scheme

The mass singularities derived in previous section are due to initial state collinear divergences as becomes apparent in the analysis of the diagrams in figures 3.2, 3.3 and 4.1. According to the factorisation theorem [25], the total partonic cross-section can be expressed as a convolution of splitting functions and a M-singularity free term:

$$\frac{s^2 d\sigma_{ij}}{dt du} = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \Gamma_{il}(x_1, Q^2) \Gamma_{jm}(x_2, Q^2) \frac{\hat{s}^2 d\hat{\sigma}_{lm}}{d\hat{t} d\hat{u}} \tag{8.1}$$

where Γ_{il} , Γ_{jm} , denote the parton splitting functions among partons i , l and j , m resp., calculated at the factorisation scale Q^2 , x_1 and x_2 are the momentum fractions the partons carry after the 'splitting' of the initial partons, while the notation of the Mandelstam variables of the singularity-free contribution is extended by a ' $\hat{\cdot}$ '. Eq. (8.1) is valid up to all orders in α_s . The splitting functions, which contain the collinear divergences, have been computed in various schemes up to $O(\alpha_s^2)$ [26] and give rise to, after summation, the structure functions for the partons in the colliding hadrons (see also eq. (2.5)). All M-singularities are then removed from the expression for the matrix element squared for the partonic interaction and absorbed into the parton density distributions.

As we have seen before in the Bremsstrahlung case for light quarks (section 4), we find the same expression for the form factor F_2 as obtained in the DIS process. Therefore the DIS scheme appears to be a natural choice. At $O(\alpha_s)$, the processes we are considering only receive contributions from gluon splitting into quarks and gluon emission off quarks. The relevant splitting functions read [16]:

$$\begin{aligned}
\Gamma_{qq}(x, Q^2, m^2) &= \delta(1-x) + \frac{\alpha_s}{2\pi} \left[P_{qq}(x) \log \frac{Q^2}{m^2} + f_{qq}(x) \right] \\
\Gamma_{qg}(x, Q^2, m^2) &= \delta(1-x) + \frac{\alpha_s}{2\pi} \left[P_{qg}(x) \log \frac{Q^2}{m^2} + f_{qg}(x) \right]
\end{aligned} \tag{8.2}$$

They contain a non-singular part ($f(x)$), while the singular logarithms are multiplied by the DGLAP

splitting functions. The so-called non-reduced and reduced partonic cross-sections in eq. (8.1) can both be written as a power series in α_s . After combining (8.1) and (8.2) the Born cross-section becomes rather simple as no reduction occurs

$$\frac{d\hat{\sigma}_{qb}^{(0)}}{d\hat{t}d\hat{u}} = \frac{d\sigma_{qb}^{(0)}}{dtdu} \equiv \frac{d\sigma^{Born}}{dtdu} \quad (8.3)$$

since all other parton combinations are absent. Up to $O(\alpha_s)$ we then write

$$\begin{aligned} \frac{d\hat{\sigma}_{qb}^{(1)}}{d\hat{t}d\hat{u}} &= \frac{d\sigma_{qb}^{(1)}}{dtdu} - \frac{\alpha_s}{2\pi} \int \frac{dx_1}{x_1} \left[P_{qq} \log \frac{Q^2}{m_q^2} + f_{qq} \right] \frac{d\sigma_{qb}^{(0)}}{dQ^2} - \frac{\alpha_s}{2\pi} \int \frac{dx_2}{x_2} \left[P_{qq} \log \frac{Q^2}{m_q^2} + f_{qq} \right] \frac{d\sigma_{qb}^{(0)}}{dQ^2} \\ \frac{d\hat{\sigma}_{qg}^{(1)}}{d\hat{t}d\hat{u}} &= \frac{d\sigma_{qg}^{(1)}}{dtdu} - \frac{\alpha_s}{2\pi} \int \frac{dx_1}{x_1} \left[P_{gb} \log \frac{Q^2}{m_q^2} + f_{gb} \right] \frac{d\sigma_{qb}^{(0)}}{dQ^2} - \frac{\alpha_s}{2\pi} \int \frac{dx_2}{x_2} \left[P_{gb} \log \frac{Q^2}{m_q^2} + f_{gb} \right] \frac{d\sigma_{qb}^{(0)}}{dQ^2} \\ \frac{d\hat{\sigma}_{gq}^{(1)}}{d\hat{t}d\hat{u}} &= \frac{d\sigma_{gq}^{(1)}}{dtdu} - \frac{\alpha_s}{2\pi} \int \frac{dx_1}{x_1} \left[P_{gq} \log \frac{Q^2}{m_q^2} + f_{gq} \right] \frac{d\sigma_{qb}^{(0)}}{dQ^2} - \frac{\alpha_s}{2\pi} \int \frac{dx_2}{x_2} \left[P_{gq} \log \frac{Q^2}{m_q^2} + f_{gq} \right] \frac{d\sigma_{qb}^{(0)}}{dQ^2} \end{aligned} \quad (8.4)$$

corresponding to the relevant Feynman diagrams. The terms in the splitting functions have been calculated by Kubar-André and Paige [16]. For gluon splitting they obtain

$$\begin{aligned} P_{gq}(x) &= \frac{1}{2} (x^2 + (1-x)^2) \\ f_{gq}(x) &= -1 + 8x - 8x^2 + 2P_{gq} \log \frac{1-x}{x} \end{aligned} \quad (8.5)$$

while the expressions for gluon emission read

$$\begin{aligned} P_{qq}(x) &= \frac{1+x^2}{1-x} \theta(1-x-\varepsilon) + \left[\frac{3}{2} + 2\log \varepsilon \right] \delta(1-x) \\ f_{qq}(x) &= \left[\frac{1-2x-6x^2}{1-x} + 2 \frac{1+x^2}{1-x} \log \frac{1}{x(1-x)} \right] \theta(1-x-\varepsilon) + \delta(1-x) \left[-\log^2 \varepsilon - \frac{7}{2} \log \varepsilon - \frac{5}{2} - \frac{\pi^2}{3} \right] \end{aligned} \quad (8.6)$$

Both splitting functions (8.5) and (8.6) can be derived from the form factor expressions for the various processes. For gluon splitting, after substituting $x_0 = 1$, the expression for F_2 given in eq. (3.14) reduces to F_2 in (3.26). Similarly, for hard Bremsstrahlung, the expression for F_2 in (4.3) reduces to the one given in (4.9), while the terms proportional to the δ -function appear as a result of the summation of soft Bremsstrahlung and virtual corrections (7.13). x_1 and x_2 correspond to z_a and z_b which are the momentum fractions the parton carries after either gluon emission or gluon splitting.

For the case of gluon splitting into heavy quarks, the reduced form factors derived from eqs. (3.14) and (3.15) become:

$$\begin{aligned} \bar{F}_1 &= 2P_{qg} \bar{L}_1 + (8x_0 z_b^2 - 4z_b)(1-x_0)L_2 + 8x_0 z_b(1-z_b) - 2 \frac{1-z_b}{1-x_0 z_b} \\ &\quad + 2(8z_b^2 - 8z_b + 1) \end{aligned}$$

$$\begin{aligned}\tilde{F}_2 = z_b \left[2P_{qq}\tilde{L}_1 - 12x_0z_b(1-x_0)(1-2x_0z_b)L_2 \right. \\ \left. - 12x_0z_b(1-z_b)(1-2x_0) + 4z_b(1-z_b) - \frac{2x_0(1-z_b)}{1-x_0z_b} \right. \\ \left. + 2(8z_b^2 - 8z_b + 1) \right]\end{aligned}\quad (8.7)$$

$$\begin{aligned}\tilde{F}_3 = 2P_{qq}\tilde{L}_1 + (2 - 4x_0z_b(1-z_b))L_2 + 4z_b(1-z_b)(1-x_0) \\ + 2(8z_b^2 - 8z_b + 1)\end{aligned}$$

while the reduced singular logarithm $\tilde{L}_1$ (L_2 does not contain a mass singularity !) reads:

$$\tilde{L}_1 = \log \frac{1}{x_0(1-x_0)} \quad (8.8)$$

Gluon splitting into light quarks leads to:

$$\begin{aligned}\tilde{F}_1 &= 8z_a(z_a - 1) \\ \tilde{F}_2 &= \tilde{F}_3 = \tilde{F}_5 = 0 \\ \tilde{F}_4 &= 4(1 - z_a)\end{aligned}\quad (8.9)$$

For the soft and virtual contributions (subtracting (7.13) from (7.3)) we obtain:

$$\begin{aligned}d\sigma = \frac{2\alpha_s}{3\pi} d\sigma^{Bom} \times \left[\log^2 \varepsilon - \frac{\log \varepsilon}{2} + \frac{1}{2} - 2Li_2(1-x_0) - \log^2(1-x_0) + 2\log(1-x_0) + \frac{\pi^2}{3} \right] \\ - \frac{2\alpha_s\alpha^2}{12\sin^4 \theta_w} \frac{1}{(Q^2 + M_w^2)^2} \left(\frac{m_l^2}{s} \log(1-x_0) \right)\end{aligned}\quad (8.10)$$

which is free of M-singularities, but still contains logarithmic dependencies on the cut-off parameter ε .

For hard Bremsstrahlung, combining (4.3) and (4.9) gives the heavy quark reduced form factors:

$$\begin{aligned}\tilde{F}_1 &= P_{qq}\tilde{L}_4 - \frac{1}{2(1-z_b)} + \frac{z_b(1-z_b)}{1-x_0z_b} + \frac{1}{2} \frac{1-z_b}{(1-x_0z_b)^2} - 3z_b \\ \tilde{F}_2 &= z_b \left[P_{qq}\tilde{L}_4 - \frac{2}{z_b(1-z_b)} + \frac{1+2z_b}{2(1-z_b)} + \frac{1}{2} \frac{1-z_b}{(1-x_0z_b)^2} + \frac{2-z_b-z_b^2}{1-x_0z_b} \right] \\ \tilde{F}_3 &= P_{qq}\tilde{L}_4 - \frac{1}{2(1-z_b)} + \frac{1-z_b}{1-x_0z_b} + \frac{1}{2} \frac{1-z_b}{(1-x_0z_b)^2} - z_b - 2\end{aligned}\quad (8.11)$$

where $\tilde{L}_4$ reads:

$$\tilde{L}_4 = \log \frac{1-z_b}{x_0(1-x_0z_b)} \quad (8.12)$$

For light quarks we obtain:

$$\begin{aligned}
\tilde{F}_1 &= -4z_a \\
\tilde{F}_2 &= \tilde{F}_5 = 0 \\
\tilde{F}_3 &= -\frac{2+4z_a+10z_a^2}{1-z_a} \\
\tilde{F}_4 &= \frac{1}{2}
\end{aligned} \tag{8.13}$$

9. Numerical results

The compensation among the I.R. and U.V. divergences has been explicitly demonstrated in previous sections. In order to obtain the total $O(\alpha_s \alpha^2)$ cross-section, there remain a few integrations over kinematical variables (see for instance eq. (2.5)). For this purpose, we apply the numerical phase space integration algorithm VEGAS [27]. This algorithm has become an integral part of the Eurojet package [28], which also contains various structure function parametrisations.

The dominant contributions to the cross-section stem from the splitting of the gluon into heavy quarks as well as from the virtual and soft Bremsstrahlung corrections to the Born term at the heavy quark lines. It turns out that the contribution from the splitting of the gluon into a pair of light quarks is negligible, while the dominant part of the Bremsstrahlung from the light quark lines is absorbed into the parton distribution functions, as explained in section 8. We have varied the value of the cut-off parameter, defining the separation between soft and hard Bremsstrahlung, over a wide range. For $0.0001 < \varepsilon < 0.01$ full compensation occurs.

However, as common in absence of higher order calculations, there remain a few parameters for which an 'educated guess' has to be made: the parton densities in the colliding hadrons and the scale at which both strong and weak couplings are calculated (denoted as Q throughout this section). The same scale definition is applied in the determination of parton densities. For the structure function parametrisations we have used one of the recent density distributions derived in the DIS scheme by Tung et al. [29].

In figure 9.1, we present the individual contributions to the $O(\alpha_s \alpha^2)$ cross-section as a function of the top quark mass at Tevatron energies ($\sqrt{s} = 1.8 \text{ TeV}$) for $Q^2 = m_t^2$. To illustrate the cross-section enhancement, we have depicted the Born ($O(\alpha^2)$) cross-section as well.

We study the stability of the calculation and estimate the influence of higher order contributions by making different choices for the scale. The same curves as presented in fig. 9.1, now derived for $Q^2 = \frac{m_t^2}{4}$ are given below (fig. 9.2), while the effect of increasing the scale ($Q^2 = 4m_t^2$) results in fig. 9.3. We find that for a top quark mass of $175 \text{ GeV} / c^2$ the scale dependence in the total cross-section is rather weak: $\sigma \sim 2.36 - 2.54 \text{ pb}$, while for all choices the significant enhancement compared to the Born contribution remains: $\sigma^{\text{Born}} \sim 1.89 - 1.93 \text{ pb}$. Note that the scale variation of the Born term is small. Indeed we have taken for the renormalisation scale of the electroweak coupling the momentum transfer of the W boson. A more detailed study including various structure function parametrisations is in progress. The order of magnitude of the enhancement confirms the results of the subtraction pro-

cedure presented in refs. [7] and [8].

Fig. 9.1: Integrated cross-section [pb] contributions to single top production from Born, virtual, Bremsstrahlung and W-gluon fusion diagrams as a function of the top mass, calculated at $\sqrt{s} = 1.8\text{TeV}$ for $Q^2 = m_t^2$.

Fig. 9.2: Same curves as presented in fig. 9.1, however, now at the reduced scale: $Q^2 = \frac{m_t^2}{4}$.

Fig. 9.3: Various contributions to the total $O(\alpha_s \alpha^2)$ cross-section [pb] at $\sqrt{s} = 1.8\text{TeV}$ for $Q^2 = 4m_t^2$.

Next, we compare our results with the calculations on top pair production at the Tevatron by Laenen et al. [30]. As we have chosen the same scales and structure function parametrisation, a direct comparison can easily be made. In fig. 9.4 the total integrated cross-section for both processes is

Fig. 9.4 Comparison of the total cross-section [pb] for QCD top-pair production as presented in ref. [30] and $O(\alpha_s \alpha^2)$ single top production as a function of the top quark mass, at $\sqrt{s} = 1.8\text{TeV}$ and $Q^2 = \frac{m_t^2}{4}$, $Q^2 = m_t^2$, $Q^2 = 4m_t^2$ resp..

given as a function of the top mass for the different scale definitions. The $t\bar{t}$ cross-section for $m_t = 175 \text{ GeV}/c^2$ ranges $\sigma \sim 4.6 - 5.8 \text{ pb}$, roughly a factor 1.9–2.3 larger than the semiweak cross-section. However, no experimental acceptances, which are likely to be different for both processes, are folded into these numbers.

In conclusion, we have presented a complete $O(\alpha_s \alpha^2)$ calculation for the semiweak top production process at hadron colliders. In comparing total cross-sections, it becomes apparent that the single top process indeed may give a sizeable contribution to the total top production rate. However, confirmation will largely depend on experimental strategies used to tag top events and to disentangle them from backgrounds. For this, a detailed study on event topology and detector acceptance is indispensable.

We are now able to perform a realistic study of the signal to background ratio. In addition, due to the weak current in the single top production process, the polarisation of the top quark plays an important role in the determination of the lepton energy spectrum in the semileptonic decay of the top. Further work is in progress [32].

Appendix A

Integration over the first two terms in the integrand of eq. (5.3) can be reduced to solving the integral of the type [15]:

$$I = \int \frac{d^3k}{2k_0} \frac{1}{(p \cdot k)^2} \quad (\text{A.1})$$

where the energy of the gluon is bound by $m_g < k_0 < \omega_0$. After angular integrations, one may change variable, $\tan \Psi = \frac{|k|}{m_g}$, and obtains:

$$I = 2\pi \int_0^{\arctan\left(\frac{\omega_0}{m_g}\right)} d\Psi \frac{\sin \Psi \tan \Psi}{p_0^2 - |\vec{p}|^2 \sin^2 \Psi} \quad (\text{A.2})$$

Performing the integral and dropping terms proportional to $\frac{m_g}{\omega_0}$ in the final result, one finds

$$I = \frac{\pi}{p_0^2 - |\vec{p}|^2} \left(\log \frac{4\omega_0^2}{m_g^2} - \frac{p_0}{|\vec{p}|} \log \left| \frac{p_0 + |\vec{p}|}{p_0 - |\vec{p}|} \right| \right) \quad (\text{A.3})$$

The integration of the first term in the integrand in eq. (5.3), that is soft Bremsstrahlung off the incoming bottom quark line, can now easily be carried out by replacing p by p_b . The result reads

$$I_1 = \frac{\pi}{m_b^2} \left[\log \frac{4\omega_0^2}{m_g^2} - \frac{E_b}{|\vec{p}_b|} \log \left| \frac{E_b + |\vec{p}_b|}{E_b - |\vec{p}_b|} \right| \right] \quad (\text{A.4})$$

while for $m_b \ll m_t$

$$I_1 = \frac{\pi}{m_b^2} \left[\log \frac{4\omega_0^2}{m_g^2} - \log \left| \frac{(Q^2 + m_t^2)^2}{m_t^2 m_b^2} \right| \right] \quad (\text{A.5})$$

With $x_0 = \frac{Q^2}{Q^2 + m_t^2}$ defined in eq. (3.15) the following contribution to the expression in (5.4) is obtained:

$$I_1 = \frac{\pi}{m_b^2} \left[\log \frac{4\omega_0^2}{m_g^2} + \log \left| \frac{m_b^2 x_0 (1 - x_0)}{Q^2} \right| \right] \quad (\text{A.6})$$

For the second term in the integrand, the soft gluon emission off the top quark, $E_t \gg |\vec{p}_t|$ and expansion of the argument of the logarithm in terms of $\frac{|\vec{p}_t|}{E_t}$ leads to the simple expression

$$I_2 = \frac{\pi}{m_t^2} \left[\log \frac{4\omega_0^2}{m_g^2} - 2 \right] \quad (\text{A.7})$$

Integration over the third term may be carried out after the introduction of an auxiliary vector $P = \beta p_1$, satisfying $(P - p_2)^2 = 0$ as proposed by 't Hooft and Veltman [22]. The integral

$$I = \int \frac{d^3 k}{2k_0} \frac{1}{(p_1 \cdot k)(p_2 \cdot k)} \quad (\text{A.8})$$

($m_g < k_0 < \omega_0$) can be written, after the proper Feynman parameter transformation [23], as

$$I = \beta \int_0^1 dx \int \frac{d^3 k}{2k_0} \frac{1}{(p \cdot k)^2} \quad (\text{A.9})$$

where p is now a superposition of P and p_2 :

$$p = Px + p_2(1 - x) \quad (\text{A.10})$$

Thus, the integration over $d^3 k$ (using (A.4)) leads to:

$$I = \pi\beta \int_0^1 \frac{dx}{p_0^2 - |\vec{p}|^2} \left(\log \frac{4\omega_0^2}{m_g^2} - \frac{p_0}{|\vec{p}|} \log \left| \frac{p_0 + |\vec{p}|}{p_0 - |\vec{p}|} \right| \right) \quad (\text{A.11})$$

and after substitution of P

$$I \equiv I_3 + I_4 = \pi\beta \int_0^1 \frac{dx}{p_2^2 + 2p_2(P - p_2)x} \left(\log \frac{4\omega_0^2}{m_g^2} - \frac{p_0}{|\vec{p}|} \log \left| \frac{p_0 + |\vec{p}|}{p_0 - |\vec{p}|} \right| \right) \quad (\text{A.12})$$

Integration over the first term is trivial:

$$I_3 = \frac{\pi\beta}{(2p_2P - p_2^2)} \log \frac{4\omega_0^2}{m_g^2} \log \frac{2p_2P - p_2^2}{p_2^2} \quad (\text{A.13})$$

For the second term, we rewrite x in terms of p and change integration limits:

$$I_4 = -\pi\beta \int_{p_2^0}^{P_0} \frac{dp_0}{p^2(P_0 - p_2^0)} \frac{p_0}{|\vec{p}|} \log \left| \frac{p_0 + |\vec{p}|}{p_0 - |\vec{p}|} \right| \quad (\text{A.14})$$

Note that p_2^0 should be interpreted as the energy component of p_2 . The relation between p , $|\vec{p}|$ and x is derived from (A.10):

$$x = \frac{p_0 - p_2^0}{P_0 - p_2^0} \quad (\text{A.15})$$

$$p^2 = p_2^2 + 2p_2(P - p_2)x$$

Next, introduction of the variable

$$y = p_0 - |\vec{p}| \quad (\text{A.16})$$

gives

$$I_4 = -\pi\beta \int_{p_2^0 - |\vec{p}_2|}^{P_0 - |\vec{p}|} \frac{dy}{\left(\frac{p_2(P - p_2)}{P_0 - p_2^0} - y \right) (P_0 - p_2^0)} \frac{p_0}{p^2} \log \left| \frac{y^2}{p^2} \right| \quad (\text{A.17})$$

With the combination of expressions in (A.15) one derives

$$p^2 = \frac{p_2^2 + \frac{p_2(P - p_2)}{P_0 - p_2^0} (y - 2p_2^0)}{1 - \frac{p_2(P - p_2)}{P_0 - p_2^0} y} \quad (\text{A.18})$$

and the integral becomes

$$I_4 = -\frac{\pi\beta}{(P_0 - p_2^0)} \int_{p_2^0 - |\vec{p}_2|}^{P_0 - |\vec{p}|} \frac{dy}{2} \left[\frac{1}{vy} + \frac{1}{v(v - y)} - \frac{1}{p_2^2 + v(y - 2p_2^0)} \right] \log \left| \frac{(y - v)y}{p_2^2 + v(y - 2p_2^0)} \right| \quad (\text{A.19})$$

where we introduced

$$v = \frac{p_2(P - p_2)}{P_0 - p_2^0} \quad (\text{A.20})$$

This can now be written as

$$I_4 = -\frac{\pi\beta}{(P_0 - p_2^0)} \int_{p_2^0 - |\vec{p}_2|}^{p_0 - |\vec{p}|} \frac{dy}{2} \left[\frac{2}{v-y} + \frac{1}{v} \frac{d}{dy} \log \left| \frac{(y-v)y}{p_2^2 + v(y-2p_2^0)} \right| \right] \log \left| \frac{(y-v)y}{p_2^2 + v(y-2p_2^0)} \right| \quad (\text{A.21})$$

The second part of the integrand leads to the square of the logarithmic term, while the first part requires the introduction of the Spence function [31] defined as

$$Li_2(a) = -\int_0^a db \frac{\log(1-b)}{b} \quad (\text{A.22})$$

leading to

$$\begin{aligned} I_4 = & -\frac{\pi\beta}{(P_0 - p_2^0)} \cdot \left[\frac{1}{4v} \log^2 \left| \frac{(y-v)y}{p_2^2 + v(y-2p_2^0)} \right| \right. \\ & \left. - \frac{1}{2v} \log^2 \left| \frac{(y-v)v}{p_2^2 + v(y-2p_2^0)} \right| \right. \\ & \left. + \frac{1}{v} Li_2 \left(\frac{y-v}{v} \right) - \frac{1}{v} Li_2 \left(\frac{(v-y)v}{p_2^2 + v(y-2p_2^0)} \right) \right] \Bigg|_{y=p_2^0 - |\vec{p}_2|}^{y=p_0 - |\vec{p}|} \end{aligned} \quad (\text{A.23})$$

Using the identity [31]

$$Li_2(x) = -Li_2\left(\frac{1}{x}\right) - \frac{\pi^2}{6} - \frac{\log^2 x}{2} \quad (\text{A.24})$$

we finally obtain:

$$I_4 = -\frac{\pi\beta}{(P_0 - p_2^0)v} \left[\frac{1}{4} \log^2 \frac{p_0 - |\vec{p}|}{p_0 + |\vec{p}|} + Li_2 \left(\frac{v - p_0 + |\vec{p}|}{v} \right) + Li_2 \left(\frac{-v + p_0 + |\vec{p}|}{-v} \right) \right] \Bigg|_{p=p_2}^{p=P} \quad (\text{A.25})$$

which leads to

$$I_4 = -\frac{\pi\beta m_b^2}{(Q^2 + m_t^2)^2} \left[\frac{1}{2} \log^2 \frac{m_b^2 x_0 (1-x_0)}{Q^2} + \frac{\pi^2}{3} \right] \quad (\text{A.26})$$

I_1 , I_2 and $I_3 + I_4$ correspond to the three terms in the integrand of eq. (5.3). With $p_1 = p_b$, $p_2 = k_2$ in eq. (A8), β reads:

$$\beta = \frac{Q^2 + m_t^2}{m_b^2} \quad (\text{A.27})$$

and after substitution, one obtains the expression given in eq. (5.4).

Appendix B

In this Appendix, some details on the calculation of the integrals in eqs. (6.15) and (6.18) are presented. In solving the integral over the 4-momentum d^4K the following relation is used (see for instance ref. [23]):

$$\int \frac{d^4K}{(2\pi)^4} \frac{(K^2)^R}{(K^2 - C)^M} = \frac{i(-1)^{R-M}}{16\pi^2} C^{R-M+2} \frac{\Gamma(R+2)\Gamma(M-R-2)}{\Gamma(M)} \quad (\text{B.1})$$

After substitution, the result reads:

$$\begin{aligned} I &\equiv I_1 + I_2 + I_3 \\ &= \iint dx dy \left\{ 2x \frac{[(Q^2 + m_t^2)(1-x) + x^2 Q^2 y(1-y)]}{[x^2 p_y^2 + \Lambda^2(1-x)]} \right. \\ &\quad \left. - 2x \frac{[(Q^2 + m_t^2)(1-x) + x^2 Q^2 y(1-y)]}{[x^2 p_y^2 + m_g^2(1-x)]} + 2x \log \frac{x^2 p_y^2 + \Lambda^2(1-x)}{x^2 p_y^2 + m_g^2(1-x)} \right\} \end{aligned} \quad (\text{B.2})$$

where p_y is defined as:

$$p_y^2 = Q^2 y(1-y) + m_t^2 y + m_b^2(1-y) \quad (\text{B.3})$$

I_1 vanishes for a large value of Λ . I_2 reads:

$$\begin{aligned} I_2 &= -2 \frac{x^2}{2} \int_0^1 dy \int_0^1 dx \frac{x(1-x)(Q^2 + m_t^2) - x^3 m_t^2 y}{x^2 p_y^2 + \Lambda^2(1-x)} \\ &= -1 + 2 \int_0^1 dy \int_0^1 dx \frac{x^2(Q^2 + m_t^2) + x^3 m_t^2 y}{x^2 p_y^2 + m_g^2(1-x)} \\ &\quad - 2(Q^2 + m_t^2) \int_0^1 dy \int_0^1 dx \frac{x}{x^2 p_y^2 + m_g^2(1-x)} \end{aligned} \quad (\text{B.4})$$

We can safely neglect m_g^2 in the first integrand. The integral becomes:

$$I_2 = -1 - 2 \left[\frac{m_t^2}{4Q^2} \log \left| \frac{m_t^2}{m_b^2} \right| + \left(1 + \frac{m_t^2}{4Q^2} \right) \log \left(\frac{m_t^2 m_b^2}{(Q^2 + m_t^2)^2} \right) \right] - (Q^2 + m_t^2) \int_0^1 \frac{1}{p_y^2} \log \left(\frac{p_y^2}{m_g^2} \right) dy \quad (\text{B.5})$$

Which can be written as:

$$\begin{aligned}
I_2 = & -1 - 2 \left[\frac{m_t^2}{4Q^2} \log \left| \frac{m_t^2}{m_b^2} \right| + \left(1 + \frac{m_t^2}{4Q^2} \right) \log \left(\frac{m_t^2 m_b^2}{(Q^2 + m_t^2)^2} \right) \right] \\
& + (Q^2 + m_t^2) \log \frac{m_g^2}{Q^2} \int_0^1 \frac{1}{p_y^2} dy + (Q^2 + m_t^2) \int_0^1 \frac{\log \left(y^2 - y \left(1 + \frac{m_t^2}{Q^2} - \frac{m_b^2}{Q^2} \right) - \frac{m_b^2}{Q^2} \right)}{y^2 - y \left(1 + \frac{m_t^2}{Q^2} - \frac{m_b^2}{Q^2} \right) - \frac{m_b^2}{Q^2}} dy
\end{aligned} \tag{B.6}$$

Performing the integrals gives

$$\begin{aligned}
I_2 = & -1 - 2 \left[\frac{m_t^2}{4Q^2} \log \frac{m_t^2}{m_b^2} + \left(1 + \frac{m_t^2}{4Q^2} \right) \log \frac{m_t^2 m_b^2}{(Q^2 + m_t^2)^2} \right] \\
& - \left[\log \frac{m_t^2}{Q^2} - \log \left(\frac{Q^2 + m_t^2}{Q^2} \right) + \log \frac{m_b^2}{Q^2 + m_t^2} \right] \log \frac{m_b^2}{Q^2} \\
& + \frac{1}{2} \left[\log^2 \frac{m_t^2}{Q^2} + \log^2 \frac{m_b^2}{Q^2 + m_t^2} - 5 \log^2 \left(\frac{Q^2 + m_t^2}{Q^2} \right) \right] \\
& + \frac{\pi^2}{6} - Li_2 \left(\frac{m_t^2}{Q^2 + m_t^2} \right) + \log \frac{m_t^2}{Q^2} \log \left(\frac{Q^2 + m_t^2}{Q^2} \right) - Li_2 \left(\frac{m_b^2 Q^2}{(Q^2 + m_t^2)^2} \right) \\
& + Li_2 \left(\frac{Q^2}{Q^2 + m_t^2} \right) + \log \frac{m_b^2}{Q^2} \log \left(\frac{Q^2 + m_t^2}{Q^2} \right)
\end{aligned} \tag{B.7}$$

The computation of I_3 is rather straightforward and one finds

$$I_3 = \frac{m_t^2}{Q^2} \log \frac{m_t^2}{Q^2} - \frac{Q^2 + m_t^2}{Q^2} \log \left(\frac{Q^2 + m_t^2}{Q^2} \right) + \log \frac{\Lambda^2}{Q^2} + \frac{3}{2} \tag{B.8}$$

The combination of the three integrals leads, after introduction of the variable x_0 (see eq. (3.15) and 'Spence identity' [31]

$$Li_2(x) + Li_2(1-x) = \frac{\pi^2}{6} - \log x \log(1-x) \tag{B.9}$$

to the expression given in eq. (6.22).

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