Heavy Meson Form Factors, Couplings and Exclusive Decays in QCD

A. Khodjamirian$^{a,b,1}$ and R. Rückl$^{a,c,2}$

$^a$ Sektion Physik der Universität München, D-80333 München, Germany
$^b$ Yerevan Physics Institute, 375036 Yerevan, Armenia
$^c$ Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, D-80805 München, Germany

Abstract

We discuss the form factors of the heavy-to-light transitions $B \rightarrow \pi$ and $D \rightarrow \pi$, the $B^* B \pi$ and $D^* D \pi$ coupling constants, and the nonfactorizable amplitude of the decay $B \rightarrow J/\psi K$ in the framework of QCD sum rules.

$^1$ Alexander von Humboldt Fellow
$^2$ supported in part by the German Federal Ministry for Research and Technology (BMFT) under contract No. 05 6MU93P

* presented by R. Rückl at the QCD-94 Workshop, Montpellier, July 7-13 1994
1 Sum Rules on the Light-Cone

The standard way to derive QCD sum rules for the transition form factors between two given ground state hadrons starts from the vacuum correlation function of three currents. Here we demonstrate an alternative method which may be used in cases where one of the hadrons is a light meson.

1.1 $B(D) \rightarrow \pi$ form factors

An important example is provided by the form factor $f_+^\pi(p^2)$ which enters the matrix element $<\pi(q)|\hat{n}\gamma_\mu b|B(p+q)>$, the momenta given in brackets. Consider the correlation function

$$<\pi(q)|i\{\hat{n}(x)\gamma_\mu b(x),\hat{b}(0)i\gamma_5 d(0)}|0>$$

between the vacuum and an on-shell pion state. It is possible to calculate this correlation function in the kinematical region of highly virtual $b$-quarks. After expanding the $b$-quark propagator near the light-cone, the correlation function (1) can be expressed in terms of light-cone wave functions of the pion, i.e. nonlocal matrix elements of the type $<\pi(q)|\hat{n}(x)\Gamma_\sigma d(0)|0>$ where $\Gamma_\sigma$ denotes a combination of Dirac matrices. Over the years a great deal has been learned about these wave functions (see e.g [1, 2]). All low-twist wave functions have been identified and their asymptotic form has been determined. Also the nonasymptotic corrections have been estimated.

In order to extract the desired form factor from the correlation function (1) we employ a QCD sum rule with respect to the $B$-meson channel. After Borel transformation one arrives at an expression of the form:

$$f_+^\pi(p^2) = \frac{f_+m_b^2}{2f_Bm_B^2}\int_\Delta^1 \frac{du}{u} (\phi_\pi(u) + \ldots) \exp\left(\frac{m_B^2 - m_b^2}{M^2} - \frac{m_b^2 - p^2(1 - u)}{uM^2}\right).$$

(2)

Here $\phi_\pi(u)$ represents the leading twist 2 pion wave function, while the ellipses denote contributions of higher twists. $M^2$ is the Borel parameter and $f_B$ is the $B$ meson decay constant as determined from an appropriate two-point sum rule. The integration limit $\Delta = (m_b^2 - p^2)/(s_0 - p^2)$ depends on the effective threshold $s_0$ above which the contribution from higher states in the $B$-channel is subtracted. In our calculation of $f_+^\pi$ we have included quark-antiquark wave functions up to twist four. In addition, we have also evaluated the contributions from quark-antiquark-gluon wave functions associated with the first-order correction to the free $b$-quark propagation. Further details can be found in [3].

In Fig. 1 numerical results are plotted for $f_+^\pi$. Making obvious replacements in (1) and (2), one can easily obtain the corresponding sum rule for the $D \rightarrow \pi$ (and also $B \rightarrow K$) form factor.

1.2 $B^*B\pi$ ($D^*D\pi$) couplings

The same correlation function (1) considered above can also serve as a starting point for a calculation of the $B^*B\pi$ coupling [4] defined by the hadronic matrix element

$$\langle B^{*0}(p)\pi^+(q)|B^+(p+q)\rangle = -g_{B^*B\pi}q_\mu \epsilon^\mu.$$  

(3)
However, in contrast to 1.1 one now has to employ sum rules in both the \( B^- \) and \( B^*^- \) channels carrying the momenta \( p+q \) and \( p \), respectively. After double Borel transformation one obtains an expression for the product \( f_B f_{B^*} g_{B} g_{B^*} \) the leading twist term of which depends on the pion wave function \( \varphi_{\pi}(u) \) at a fixed momentum fraction \( u \simeq 1/2 \). Taking \( \varphi_{\pi}(1/2) = 1.2 \pm 0.2 \) together with the corresponding values of the higher twist wave functions \([2]\) and dividing by the values of \( f_B \) and \( f_{B^*} \) as given by appropriate two-point sum rules, we find, numerically, 
\[
g_{B} g_{B^*} = 29 \pm 3 \text{ and } g_{D^* D^*} = 12.5 \pm 1.
\]
From the latter value we predict the partial width \( \Gamma(B^+ \rightarrow D^0 \pi^+) = 32 \pm 5 \text{ keV} \). This estimate is consistent with the limit derived from recent measurements \([5, 6]\). Further details and a comparison with other estimates are given in \([4]\).

In conclusion, we emphasize that light-cone sum rules represent a useful alternative to the conventional QCD sum rule method. In this variant, the nonperturbative aspects are described by a set of wave functions on the light-cone with different twist and quark-gluon multiplicity. These universal functions can be studied in a variety of processes involving the \( \pi \) and \( K \) meson, or other light mesons.

2 \quad A Sum Rule for \( B \rightarrow J/\psi K \)

Nonleptonic two-body decays of heavy mesons are usually calculated in the factorization approximation for the appropriate matrix element of the weak Hamiltonian. However, as well known, naive factorization fails. In order to achieve agreement with experiment it is necessary to let the coefficients \( a_{1,2} \) multiplying the matrix elements deviate from their values \( a_{1,2} = c_{1,2} + c_{2,1}/3 \) predicted in short-distance QCD. Phenomenologically \([7]\), they are treated as free parameters to be determined from experiment.

The decay \( B \rightarrow J/\psi K \) provides an important example. In factorization approximation the amplitude is proportional to \( a_{2} f_K^2 f_\psi \) where \( f_K^2 \) is the \( B \rightarrow K \) form factor at \( p^2 = m_K^2 \) and \( f_\psi \) is the \( J/\psi \) decay constant. From the short-distance value for \( a_{2} \), the branching ratio is estimated to be almost an order of magnitude smaller than the experimental value \([9]\). On the other hand, dropping the term proportional to \( c_{1}/3 \) in \( a_{2} \) as suggested by \( 1/N_c \) expansion \([10]\) yields reasonable agreement. In fact, the factorizable term proportional to \( c_{1}/3 \) may be cancelled by nonfactorizable contributions being of the same order in \( 1/N_c \). Such a cancellation was first advocated in \([10]\) and then shown in \([11]\) to actually take place in two-body \( D \)-decays using QCD sum rule techniques in order to estimate the nonfactorizable amplitudes.

Recently, we have investigated the problem of factorization in \( B \) decays using \( B \rightarrow J/\psi K \) as a study case \([12]\). Following the general idea put forward in \([11]\), we calculate the four-point correlation function
\[
<0 \mid T \{ j_{\mu}^K(x) j_{\nu}^\psi(y) H_W(z) j_{5}^B(0) \} \mid 0>
\]
by means of the short-distance OPE. Here \( j_{\mu}^K = \bar{u} \gamma_{\mu} \gamma_5 s \), \( j_{\nu}^\psi = \bar{c} \gamma_{\nu} \gamma_5 c \) and \( j_{5}^B = \bar{b} \gamma_5 u \) are the generating currents of the mesons involved and \( H_W \) is the effective weak Hamiltonian. To the lowest nonvanishing order in \( a_s \) the nonfactorizable contributions to (4) only arise from the operator \( \bar{O}_2 = (\bar{c} \gamma_{\rho} \gamma_5 c)(\bar{s} \gamma_{\rho} \lambda^3 b)/4 \), where \( \Gamma_{\rho} = \gamma_{\rho}(1 - \gamma_5) \). Obviously, the contribution of this operator to the \( B \rightarrow J/\psi K \) amplitude vanishes by factorization. Parametrizing the nonfactorizable matrix element by \( \langle J/\psi K \mid \bar{O}_2 \mid B \rangle = \frac{2}{3} \langle f_\psi \rangle (\epsilon^\psi \cdot q) \) we construct a sum rule for \( f \) which enters the correlation function (4) through the ground state contribution. In the QCD part of this sum rule all nonperturbative contributions to (4) from vacuum condensates up to dimension 6 are included. In the hadronic part a complication arises from intermediate states in the \( B^- \) channel carrying the quantum numbers of a \( \bar{D} D^* \) pair. These virtual states
are created by weak interaction and converted into the $J/\psi K$ final state by strong interaction. By examination of the quark-gluon representation of (4) one can identify corresponding four-quark $\bar{u}s\bar{c}$ intermediate states. Invoking quark-hadron duality we cancel this piece of the QCD part against the unwanted hadronic contribution.

We then perform, as usual, a Borel transformation in the $B-$meson channel and take moments in the charmonium channel. The spacelike momentum squared in the $K$-meson channel is kept fixed. Since, as explained in [12], subtraction of higher state contributions using the local quark-hadron duality is not possible in this case, we take them into account by using a simple two-resonance description in each of the three channels. This approximation, finally, yields $\tilde{f} = -(0.045 \div 0.075)$. The full $B \to J/\psi K$ decay amplitude is proportional to

$$a_2 = c_2 + \frac{c_1}{3} + 2c_1 \tilde{f}/f^+_K$$

where the first two coefficients are associated with the factorizable matrix element, while the third term represents the leading nonfactorizable contribution. We find that the factorizable nonleading in $1/N_c$ term and the nonfactorizable term in (5) are opposite in sign. Although the nonfactorizable matrix element is considerably smaller than the factorizable one, $|\tilde{f}/f^+_K| \simeq 0.1$, it has a strong quantitative impact due to its large coefficient, $|2c_1/(c_2 + c_1/3)| \simeq 20 \div 30$. In fact, if $|\tilde{f}|$ is close to the upper end of the predicted range, the nonfactorizable contribution almost cancels the factorizable one proportional to $c_1/3$. This is exactly the scenario anticipated by $1/N_c$-rule [10].

It is very interesting to note that our theoretical estimate yields a negative overall sign for $a_2$ in contradiction to a global fit to data [9]. Furthermore, there is no theoretical reason in our approach to expect universal values or even universal signs for the coefficients $a_{1,2}$ in different channels. Universality can at most be expected for certain classes of decay modes, such as $B \to D\pi$ or $B \to D\bar{D}$, etc. Also, there is no simple relation between $B$ and $D$ decays in our approach since the OPE for the corresponding correlation functions significantly differ in the relevant diagrams and in the hierarchy of mass scales. We hope to be able to clarify these issues further.

Concluding we would like to stress that QCD seems to predict a much richer pattern in two-body weak decays than what is revealed by the current phenomenological analysis of the data.

**Acknowledgements.**

R.R. thanks Stephan Narison for organizing this fruitful workshop. This work is supported by the German Federal Ministry for Research and Technology (BMFT) under contract No. 05 6MU93P.
References

Figure 1: $B \to \pi$ form factor obtained from the light-cone sum rule (solid curve). The quark model prediction [7] (dash-dotted curve) and the result of a conventional three-point sum rule [8] (dashed curve) are shown for comparison.