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Current-current operator evolution in the chiral limit*

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Abstract

We consider the strong interaction corrections to current-current hadronic operators in the chiral limit. High-momentum contributions are computed in perturbative QCD with massless quarks while low-momentum contributions are estimated in a truncated non-linear σ -model for massless pseudoscalars. In the large- N approximation, the ambiguity on the momentum matching around the confining scale is removed by using the effective color-singlet gauge boson exchanged between the two currents as a universal probe.

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The effective impact of strong interactions on weak hadronic decays remains a subject of active research nowadays. In particular, the old, empirical $\Delta I = 1/2$ rule has always been at the center of controversial discussions [1], revived by the recent experimental results [2] on CP-violation in kaon decays. During the last couple of years, impressive calculations on short-distance, perturbative corrections to the strangeness-changing hadronic transitions have been performed [3]. The full next-to-leading order corrections for (electro) weak $\Delta S = 1$ Q_i operators have indeed been completed. One crucial message of that remarkable piece of work is the definite breakdown of perturbative QCD for typical loop-momenta below the confining scale of one GeV. In other words, non-perturbative physics should play a central role in explaining the $\Delta I = 1/2$ rule within the Standard Model. Unfortunately non-perturbative methods such as lattice simulation, QCD sum rules or $1/N$ expansion are far from reaching the same level of self-confidence. Combining the new short-distance results with potentially large strong interaction corrections below one GeV remains therefore a real challenge far beyond our present ability.

In this letter, we aim at improving the non-perturbative $\frac{1}{N}$ -approach [4] based on a truncated non-linear σ -model for pseudoscalars by

- a) keeping track of the loop-momentum flow through the critical one GeV scale;
- b) working at the operator level from the start;
- c) taking into account possible strong-anomaly contributions.

By doing so, we believe that the matching advocated in Ref.[4] and reviewed in length in Ref.[5] is better justified since we now consistently cut off the infrared short-distance and ultraviolet long-distance contributions. This leads to a rather

clean identification of the evolution-induced Q_6 penguin operator. For left-left current-current operator evolutions, virtual contributions from higher resonances (vector mesons, ...) around one GeV are required to improve the matching with perturbative QCD in the chiral limit. On the other hand, for left-right current-current operators such as Q_5 or Q_7 , the short-distance evolution alone provides us with scale independent amplitudes in the large- N limit, a remarkable property previously noticed [4] for the leading evolution of the penguin operators Q_6 and Q_8 .

1. Short-distance operator evolution with infrared cut-off.

For low-energy hadronic processes, the generic effective Hamiltonian contains both current-current [6] and density-density [7] four-quark operators. In the Standard Electroweak Model, each of them arises from some specific Fermi-type of interaction evolving down to a scale M at which the corresponding hadronic matrix element should be evaluated. This well-known operator evolution, mainly induced by strong interactions acting on the quark fields, can be treated within perturbative QCD (using the renormalization group equation techniques) as long as M is taken above the confining scale, say at the charm quark mass. However, for physical processes involving very light hadronic bound states (such as $K, \pi \dots$), a further short-distance evolution is necessary before using chiral perturbation truncated to the pseudoscalars.

So, let us first consider the following effective left-handed current-current operator

$$Q_{LL} \equiv L_\mu^{ab} L_{cd}^\mu \tag{1}$$

defined at the scale M with

$$L_\mu^{ab} \equiv \bar{q}^a \gamma_\mu (1 - \gamma_5) q^b \quad (2)$$

some color-singlet left-handed current for light quarks ($a, b = u, d, s$). If we now let this hadronic operator Q_{LL} evolve further down to the infrared cut-off Λ taken around one GeV, we obtain at the one-loop level

$$\begin{aligned} L_\mu^{ab} L_{cd}^\mu(M^2) &= L_\mu^{ab} L_{cd}^\mu(\Lambda^2) - 3 \frac{\alpha_s}{4\pi} \ln\left(\frac{M^2}{\Lambda^2}\right) \left\{ L_\mu^{ad} L_{cb}^\mu \right. \\ &\quad \left. + \frac{1}{9} [L_\mu L^\mu - 2RL]^{ad} \delta_{cb} + \frac{1}{9} [L_\mu L^\mu - 2RL]^{cb} \delta_{ad} + \mathcal{O}(1/N) \right\} (\Lambda^2) \quad (3) \end{aligned}$$

with α_s , the strong coupling constant. Notice that the $1/N$ -suppressed operators have been omitted to allow a consistent matching with the long-distance evolution considered below.

The first operator is induced by the Feynman diagrams shown in Fig.1a and arises from a “Fierzing” of the original one since

$$[\bar{q}^a \gamma_\mu (1 - \gamma_5) q^b] [\bar{q}^c \gamma^\mu (1 - \gamma_5) q^d] = [\bar{q}^a \gamma^\mu (1 - \gamma_5) q^d] [\bar{q}^c \gamma_\mu (1 - \gamma_5) q^b]. \quad (4)$$

The other operators are induced by the diagrams displayed in Fig.1b and contain the product of color-singlet densities:

$$L^{ab} \equiv \bar{q}^a (1 - \gamma_5) q^b \quad (5)$$

$$R^{ab} \equiv \bar{q}^a (1 + \gamma_5) q^b.$$

The expression given in Eq.(3) turns out to be identical to the one obtained with standard perturbative QCD methods [6, 7] for massless quarks if we simply identify our physical infrared cut-off Λ with the usual renormalization scale μ . This result is

non-trivial in the sense that the naive quadratic cut-off dependence generated by the so-called penguin diagram (first diagram displayed in Fig.1b) is exactly cancelled by the second and third diagrams in Fig.1b such that we are left with logarithmic cut-off dependence only.

Similarly, if we consider the following effective left-right current-current operator

$$Q_{LR} \equiv L_\mu^{ab} R_{cd}^\mu \quad (6)$$

defined at the scale M , with

$$R_\mu^{ab} \equiv \bar{q}^a \gamma_\mu (1 + \gamma_5) q^b \quad (7)$$

some color-singlet right-handed current arising from the vector-like nature of gluonic (and photonic) interactions with quarks. The one-loop evolution between M and the physical infrared cut-off Λ gives

$$L_\mu^{ab} R_{cd}^\mu(M^2) = L_\mu^{ab} R_{cd}^\mu(\Lambda^2) + 3 \frac{\alpha_s}{4\pi} \ln \left(\frac{M^2}{\Lambda^2} \right) [-2R^{ad} L^{cb} + \mathcal{O}(1/N)](\Lambda^2). \quad (8)$$

The correction proportional to α_s is entirely induced by the Feynman diagrams shown in Fig.1a and leads, in the large- N limit, only to a density-density operator obtained from the following Fierz relation

$$[\bar{q}^a \gamma_\mu (1 - \gamma_5) q^b][\bar{q}^c \gamma^\mu (1 + \gamma_5) q^d] = -2[\bar{q}^a (1 + \gamma_5) q^d][\bar{q}^c (1 - \gamma_5) q^b]. \quad (9)$$

At this point we would like to emphasize that this short-distance evolution alone provides us with a scale-independent amplitude in the large- N limit. Using the Fierz relation given in Eq.(9) we conclude that the left-right current-current operator Q_{LR} should evolve the same way as the corresponding density-density operator

$$-\frac{2}{N}[\bar{q}^a (1 + \gamma_5) q^d][\bar{q}^c (1 - \gamma_5) q^b] \equiv \frac{1}{N}[-2R^{ad} L^{cb}]. \quad (10)$$

In Eq.(10) the factor $1/N$ stems from the projection on color-singlet density operators. Combining the $1/N$ leading part of Eq.(8) with Eq.(10), we find indeed

$$R^{ad}L_{cb}(M^2) = \left\{ 1 + 3N \frac{\alpha_s}{4\pi} \ln \left(\frac{M^2}{\Lambda^2} \right) \right\} R^{ad}L^{cb}(\Lambda^2) \quad (11)$$

which is the diagonal evolution of an arbitrary density-density operator in the large- N limit. In this limit, the scale dependence of this diagonal evolution is exactly cancelled by the scale dependence of the associated hadronic matrix element without having to invoke long-distance physics [4]. In fact we shall see that this nice feature solves an apparent paradox for the Q_7 electromagnetic penguin operator [8] in the framework of a chiral expansion.

To summarize, we have seen that the short-distance evolution of left-handed current-current operators has a logarithmic dependence on the cut-off which requires long-distance physics to obtain scale independent amplitudes. On the other hand, for left-right current-current ($Q_5, Q_7, \dots$) and density-density ($Q_6, Q_8, \dots$) operators a full short-distance calculation is feasible in the large- N approximation.

2. Long-distance operator evolution with ultraviolet cut-off.

At low-energy, perturbative QCD becomes useless and the non-linear σ -model is the relevant effective frame for strong interactions. If we truncate this model, keeping only the pseudoscalar nonet

$$\pi = \sum_{a=0}^8 \lambda_a \pi^a \quad (12)$$

with λ_a the Gell-Mann matrices normalized such that $Tr \lambda_a \lambda_b = 2\delta_{ab}$, the $\mathcal{O}(p^2)$ chiral invariant Lagrangian reads [9]

$$\mathcal{L}_s = \frac{f^2}{8} \left\{ Tr \partial_\mu U \partial^\mu U^\dagger + \frac{m_0^2}{12} [Tr(\ln U - \ln U^\dagger)]^2 \right\} \quad (13)$$

in the large- N limit. For massless up, down and strange quarks, only the singlet pseudoscalar ($a = 0$) gets a mass m_0 via the strong anomaly term which explicitly breaks the axial $U(1)$ symmetry.

We have seen that perturbative QCD corrections are controlled by the expansion parameter α_s , which behaves like $1/N$. In the truncated non-linear σ -model defined in Eq.(13), the color factor N appears through the decay constant $f \sim \sqrt{N}$ and the singlet pseudoscalar mass $m_0 \sim 1/N$. A consistent analysis of the operator evolution can therefore be performed in the $1/N$ expansion frame [4]. We simply have to compute the leading loop corrections induced by the pseudoscalars. In order to get evolutions at the operator level like in perturbative QCD, we consider the powerful background field method.

Let us adopt the following decomposition

$$U \equiv e^{i\sqrt{2}\xi/f} \bar{U} \quad (14)$$

with

$$\xi = \sum_{a=0}^8 \lambda_a \xi^a. \quad (15)$$

The classical field

$$\bar{U} = e^{i\sqrt{2}\pi/f} \quad (16)$$

satisfies the equation of motion

$$\bar{U} \square \bar{U}^\dagger - \square \bar{U} \bar{U}^\dagger = \frac{m_0^2}{N} \text{Tr}(\ln \bar{U} - \ln \bar{U}^\dagger). \quad (17)$$

Expanding around this classical field $\bar{U}$, the linear dependence on ξ disappears in $\mathcal{L}_s$ and only the quadratic terms in ξ contribute to the one-loop Feynman diagrams displayed in Fig.2:

$$\begin{aligned}
\mathcal{L}_s(\xi) &= \frac{1}{2}\partial_\mu \xi^a \partial^\mu \xi_a - \frac{1}{2}m_0^2 \xi^0 \xi_0 \\
&\quad + \frac{1}{4}Tr([\partial_\mu \xi, \xi] \partial^\mu \bar{U} \bar{U}^\dagger) \\
&\quad + \mathcal{O}(\xi^3).
\end{aligned} \tag{18}$$

We notice that the strong anomaly only shows up as a mass term for the singlet ξ^0 .

Once again, let us first consider the evolution of left-handed current-current operators Q_{LL} defined in Eq.(1). The bosonized left-handed current

$$L_\mu^{ab} = i \frac{f^2}{2} (\partial_\mu U U^\dagger)^{ba} \tag{19}$$

has a linear dependence in ξ given by

$$L_\mu^{ab}(\xi) = -\frac{f}{\sqrt{2}} \{ \partial_\mu \xi + [\xi, \partial_\mu \bar{U} \bar{U}^\dagger] \}^{ba} + \mathcal{O}(\xi^2). \tag{20}$$

In the chiral and large- N limits, the non-factorizable meson evolution of Q_{LL} is then computed from the Feynman diagrams shown in Fig.2 and we obtain the following classical induced operators

$$\begin{aligned}
L_\mu^{ab} L_{cd}^\mu(\Lambda^2) &= \bar{L}_\mu^{ab} \bar{L}_{cd}^\mu(0) \\
&\quad - 6 \frac{\Lambda^2}{(4\pi f)^2} \left\{ \bar{L}_\mu^{ad} \bar{L}_{cb}^\mu - \frac{1}{2} (\bar{L}_\mu \bar{L}^\mu)^{ad} \delta_{cb} - \frac{1}{2} (\bar{L}_\mu \bar{L}^\mu)^{cb} \delta_{ad} \right\} (0).
\end{aligned} \tag{21}$$

The presence of a commutator in Eqs.(18) and (20) implies that the strong anomaly does not contribute to the evolution of Q_{LL} .

For completeness, we also compute the low-momentum evolution of left-right operators Q_{LR} defined in Eq.(6). The bosonized right-handed current

$$R_\mu^{ab} = i \frac{f^2}{2} (\partial_\mu U^\dagger U)^{ba} \tag{22}$$

contains only one linear term in ξ

$$R_\mu^{ab}(\xi) = \frac{f}{\sqrt{2}}(\bar{U}^\dagger \partial_\mu \xi \bar{U})^{ba} + \mathcal{O}(\xi^2) \quad (23)$$

and the Feynman diagrams displayed in Fig.2 lead to

$$\begin{aligned} L_\mu^{ab} R_{cd}^\mu(\Lambda^2) &= \bar{L}_\mu^{ab} \bar{R}_{cd}^\mu(0) + \frac{f^2}{2} \frac{\Lambda^4}{(4\pi)^2} \bar{U}^{bc} \bar{U}_{da}^\dagger(0) \\ &- \frac{\Lambda^2}{(4\pi f)^2} \left(\frac{if^2}{2} \right)^2 \left\{ \bar{U}^{bc} (\partial^2 \bar{U}^\dagger + \partial_\mu \bar{U}^\dagger \partial^\mu \bar{U} \bar{U}^\dagger)_{da} \right. \\ &+ \left. (\partial^2 \bar{U} + \partial_\mu \bar{U} \partial^\mu \bar{U}^\dagger \bar{U})^{bc} (\bar{U}^\dagger)_{da} \right\} (0). \end{aligned} \quad (24)$$

Again it is easy to check that the strong anomaly has no effect.

The Λ^4 contribution in Eq.(24) clearly emerges from the first diagram of Fig.2 but is unphysical. It is indeed well-known that quartic cut-off dependence violates chiral symmetry. In fact it is always cancelled once we require the measure to be chiral invariant [10]. In our case, the choice for the background field decomposition given in Eq.(14) requires the following modification of the strong Lagrangian (13)

$$\mathcal{L}_s \rightarrow \mathcal{L}_s - \frac{f^2}{2} \frac{\Lambda^4}{(4\pi)^2} \bar{U}^{bc} \bar{U}_{da}^\dagger \quad (25)$$

in the presence of the Q_{LR} operator. This modification exactly cancel the Λ^4 term in Eq.(24), as it should be. In particular, we notice that the Q_5 operator

$$Q_5 \equiv L_\mu^{sd} (R_{uu}^\mu + R_{dd}^\mu + R_{ss}^\mu) \quad (26)$$

appearing in the short-distance analysis of the $\Delta I = 1/2$ rule has no further evolution below the cut-off Λ , while the Q_7 operator

$$Q_7 \equiv L_\mu^{sd} (R_{uu}^\mu - \frac{1}{2} R_{dd}^\mu - \frac{1}{2} R_{ss}^\mu) \quad (27)$$

most relevant for the ε'_k CP-violating parameter evolves into $\mathcal{O}(p^2)$ operators, if we restrict ourselves to the pseudoscalar mesons.

3. Loop-momentum matching.

We have estimated both short-distance and long-distance operator evolutions on the basis of the Feynman diagrams displayed in Figs.1 and 2, respectively. If we systematically associate the cut-off Λ to the effective color-singlet gauge boson exchanged between the two currents and represented by dashed lines, we are then allowed to compare these evolutions and to sum them up. If the truncated non-linear σ -model provides a reasonable frame for non-perturbative operator evolution, we should then get a good matching such that the physical amplitudes are (almost) cut-off independent around one GeV.

We first notice that our systematic identification of the short-distance infrared cut-off with the long-distance ultraviolet one could not be carried out in Ref.[4] where the effective gauge boson is integrated out from the start. As a consequence, the coefficient appearing in front of the quadratic cut-off dependent Q_{LL} evolution in Ref.[4] has to be rescaled by a factor of 3/2 (see Eq.(21) where our normalization of $f \simeq f_\pi = 132 \text{ MeV}$ differs from the one used in Ref.[4] by a factor of $\sqrt{2}$). Such a modification of the quadratic cut-off dependence once the momentum flow is correctly identified was already noticed [11] in the case of an estimate for the long-distance contributions to the $K_L - K_S$ mass difference.

Let us now compare the short-distance evolution (Eq.(3)) with the long-distance evolution (Eq.(21)) for left-handed operators. We immediately realize that the density-density operator generated from the perturbative QCD regime does not

seem to show up in the non-perturbative regime considered. However, an ambiguity inherent to the non-linear realization of nonet operators appears at $\mathcal{O}(p^2)$: there is only one pseudoscalar meson operator $(\partial_\mu U \partial^\mu U^\dagger)^{ab}$ to describe both the corresponding current-current and density-density four-quark operators. We therefore use Eqs.(3) and (21) to relate these operators *at the cut-off* Λ . A straightforward identification gives the relation

$$(RL)^{ab}(\Lambda^2) = \frac{11}{4}(L_\mu L^\mu)^{ab}(\Lambda^2) + \mathcal{O}\left(\frac{1}{N}\right) \quad (28)$$

which turns out to remain valid after the inclusion of vector mesons in loops [12]. Applied to the most famous density-density penguin operator $Q_6 \equiv -2(RL)^{sd}$, Eq.(28) together with Eq.(21) imply the interesting relation

$$Q_6(\Lambda^2) = -\frac{11}{2}Q_4(0) + \mathcal{O}\left(\frac{1}{N}\right) \quad (29)$$

with $Q_4 = (L_\mu L^\mu)^{sd} = \frac{f_\pi^4}{4}(\partial_\mu U \partial^\mu U^\dagger)^{sd}$ the standard $\Delta I = 1/2$ operator for hadronic K -decay. We can compare this “QCD-inspired” relation between Q_6 and Q_4 with the one obtained in the large- N limit [13]

$$Q_6(\mu^2) \cong -\frac{1}{4} \left[\frac{1\text{GeV}}{m_s(\mu^2)} \right]^2 Q_4(0) \quad (30)$$

if the leading $\mathcal{O}(p^4)$ terms are introduced in the effective strong interaction Lagrangian $\mathcal{L}_s$. Notice the quadratic dependence on the strange quark mass m_s which gives rise to the main uncertainty in the theoretical estimates of the CP-violating ε'_K parameter. The expressions (29) and (30) agree in sign, a crucial property to favor a positive ε'_K . They also agree in magnitude if the strange quark mass m_s is about 200 MeV at the cut-off scale Λ .

For left-right operators, the comparison of Eq.(8) with Eqs.(24) and (25) suggests the necessity to introduce higher resonances in the framework of the non-linear σ -

model. Let us indeed consider the evolution of the $\mathcal{O}(p^2)$ Q_7 electromagnetic penguin operator defined in Eq.(27). As already noticed, its long-distance evolution due to pseudoscalar mesons gives rise to $\mathcal{O}(p^2)$ operators. This is clearly at variance with its short-distance evolution into the $\mathcal{O}(p^0)$ Q_8 operator:

$$Q_8 \equiv -2(R^{su}L^{ud} - \frac{1}{2}R^{sd}L^{dd} - \frac{1}{2}R^{ss}L^{sd}). \quad (31)$$

However, we have seen that both left-right and density-density operator evolutions can in principle be fully treated in the large- N perturbative QCD framework.

In conclusion, we have considered the strong interaction corrections to current-current hadronic operators in the chiral limit. The ambiguity on the momentum matching around one GeV has been removed by using the effective, unconfined, gauge boson exchange as a bridge between the meson and the quark-gluon pictures.

For the left-handed current-current operators, the logarithmic evolution derived within perturbative QCD is turned into a physical, quadratic one for a low loop-momentum. This quadratic dependence on the cut-off is easy to understand from the $1/N$ expansion point of view. Perturbative QCD corrections are controlled by the dimensionless expansion parameter $\alpha_s \sim \frac{1}{N}$. On the other hand, in the non-linear σ model restricted to the pseudoscalar octet, the color factor N only appears through the dimensionful decay constant $f \sim \sqrt{N}$. Consequently, the long-distance expansion parameter must scale like $\Lambda^2/f_\pi^2 \sim 1/N$ in the chiral limit. In this limit, we have to include higher resonances before doing any serious numerics. Vector mesons around one GeV introduce a new (N -independent) mass parameter m_V which does not vanish in the chiral limit. A logarithmic dependence on the cut-off is then generated below the confining scale and improves the matching. This has been explicitly checked for the amplitudes associated with the B_K -parameter [14]

and the $\Delta I = 1/2$ rule in K -physics [12].

For left-right operators we have argued that a short-distance analysis in the large- N approximation provides us with scale-independent amplitudes.

Finally we have shown that the strong anomaly does not contribute to the current-current operator evolution.

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Figure Captions

Fig. 1 The one-loop non-factorizable diagrams contributing to the short-distance evolution of current-current operators for massless quarks. The full lines are quark lines and the curled lines represent the gluons. The crosses denote the quark currents defined in Eqs.(2) and (7). The effective color-singlet gauge boson exchanged between the two currents is displayed by a dashed line.

Fig. 2 The one-loop non-factorizable diagrams contributing to the long-distance evolution of current-current operators in the chiral limit. The full lines are meson lines and the dots represent the strong vertices defined in Eq.(18). The crosses denote the pseudoscalar currents defined in Eqs.(20) and (23). The effective color-singlet gauge boson exchanged between the two currents is displayed by a dashed line.

Fig.1a

Fig.1b

Fig.2