A muon source based on planar channelling radiation

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The usage of channelling radiation arising from a high energy electron beam traversing a crystal for the production of an intense low emittance muon beam is investigated. The optimal energy and divergence of the electron beam are computed for few crystal types based on analytical expressions of the radiation spectrum and of the muon pair production cross section. It is shown that the required beam properties seem within reach and that the source performance may be comparable to other muon sources currently considered.

I. INTRODUCTION

The radiation emitted by high energy particles trapped transversely in the potential generated by a crystalline lattice aligned with their direction of travel, so-called channelling radiation, have been extensively studied mostly aiming at the generation of high energy and high brightness photon beams for various applications, e.g. in \textsuperscript{11} and references therein. In the following, we investigate the potential of such a source of high energy photons for the generation of intense low emittance muon beams, via pair production on a target. The model for the channelling radiation is described based on existing literature in the first section, along with the interaction of the photon beam with the target generating the positive and negative muon beams. This allows to determine optimal parameters for the beam traversing the crystal. The zero order design of the electron accelerator meeting the requirement of this scheme is discussed in the second section, allowing for an estimation of the potential performance and its comparison to other types of sources.

II. CHAmmelling RADIATION

Following \textsuperscript{11}, we derive the radiation spectrum of channelled high energy electrons in a crystal potential by considering stable trajectories in the Pöschl-Teller (PT) potential and integrating the emitted spectrum of individual electrons over the incoming beam phase space. Typical PT potential parameters $a_{PT}$ and $b_{PT}$ as well as the inter-planar distance $d$ and the height of the channelling potential $U_c$ are listed in Tab. \textsuperscript{11}. Since we are targeting high energy electrons, the classical treatment is appropriate. The PT potential at a given distance from the mid-plane $x$ reads:

$$U_{PT}(x) = a_{PT} \tanh^2 \left( \frac{x}{b_{PT}} \right),$$

An incoming electron at $x$ with an energy $E_c$ and an angle $x'$ with respect to the channelling plane has an oscillatory trajectory in the potential characterised by an amplitude

$$A_c = b_{PT} \tanh^{-1} \sqrt{ \frac{E_c x'^2 + U_{PT}(x)}{a_{PT}} },$$

and a wave number

$$k_u = \sqrt{ \frac{2a_{PT}}{E_c} \frac{1}{b_{PT} \cosh \left( \frac{A_c}{b_{PT}} \right)} }.\tag{3}$$

The corresponding undulator parameter is given by:

$$K^2 = 4\gamma_c \frac{a_{PT}}{m_e c^2} \frac{\cosh \left( \frac{A_c}{b_{PT}} \right) - 1}{\cosh^2 \left( \frac{A_c}{b_{PT}} \right)},\tag{4}$$

with $m_e$ the mass of the electron and $c$ the speed of light. The amplitude and the corresponding undulator parameter for electrons with different coordinates at the entrance of the crystal are shown in Fig. \textsuperscript{1a}. The tungsten crystal, featuring a significantly higher potential, can channel particles with a higher amplitude, therefore larger undulator parameter. Nevertheless, the channelled particles remain mostly in the undulator regime, i.e. $K \approx 1$. In this condition we expect a narrow photon energy spectrum in the first harmonic of the undulator. In the following we will discard the energy lost in higher harmonics. Also, since we are aiming at the energy spectrum of the muons, for which the pair production spectrum is wide, we shall approximate the undulator spectrum of each electron by a delta function peaked at [2]:

$$E_\gamma = \frac{2\hbar c^2 k_u}{1 + K^2},\tag{5}$$

with $\hbar$ the reduced Plank constant. The number of photons emitted per electron and per passage through a crystal of length $L_c$ is given by:

$$n_\gamma = \frac{e^2 k_u}{24\pi \epsilon_0 \hbar (1 + K^2/2)^2} \frac{K^2}{c} \frac{L_c}{c}.$$\tag{6}

These derivations assume that the energy of the electrons traversing the crystal is constant. To remain within the validity of this approximation, we shall adjust the crystal...
length such that the maximum energy lost by an electron per passage does not exceed 10%:

\[ \frac{n_\gamma E_\gamma}{E_e} < 0.1 \quad \forall \ x, x' \in \mathbb{R}. \tag{7} \]

We note that the corresponding crystal lengths, listed in Tab. II, are lower than the dechannelling length given by [1]:

\[ L_{d-c} = 8.9 \cdot 10^{-6} u_c E_e L_r, \tag{8} \]

with the radiation length \( L_r \). The corresponding values for different designs are shown in Tab. II. Consequently the effect of dechannelling is neglected.

The photon energy and rate expected for particles with different coordinates at the entrance of the crystal are shown in Fig. 1b. The photons with highest energy are produced by low amplitude particles, however due to the low undulator parameter the corresponding rates are low. On the other hand, large amplitude particles produce high rates of low energy photons. An optimisation of the electron beam divergence is therefore required to obtain the highest rates in a given range of interest.

Considering a Gaussian beam of r.m.s. divergence \( \sigma_e \) and r.m.s. beam size \( \sigma_e \) much larger than the distance between crystalline planes \( d \), we can write the total spectrum of the radiation by averaging over the beam distribution:

\[ N_{\gamma}(E') = \frac{1}{d} \int_{-d/2}^{d/2} dx \int_{-\infty}^{\infty} dx' \frac{n_\gamma}{\sqrt{2\pi}\sigma_e^2} e^{-\frac{x'^2}{2\sigma_e^2}} \delta(E' - E_\gamma) \tag{9} \]

We shall consider photon energies significantly higher than the muon pair production threshold, thus, given the target atomic number \( Z \) and mass number \( A \) the differential cross section is given by [3]:

\[ \frac{d\sigma}{dE_\mu} = \frac{4\alpha Z^2 \gamma^2}{E_\gamma} \left( 1 - \frac{4E_\mu}{3E_\gamma} \right) \left( 1 - \frac{E_\mu}{E_\gamma} \right) \log W \tag{10} \]

with \( r_\mu \) the classical radius of the muon, \( \alpha \) the fine structure constant and

\[ W = BZ^{-1/3} \frac{m_\mu}{m_e} \frac{1 + (D_n \sqrt{\epsilon} - 2) \frac{\delta}{m_e}}{1 + BZ^{-1/3} \sqrt{\epsilon} \frac{\delta}{m_e}}, \tag{11} \]

where \( \epsilon \) is the Euler number, \( m_\mu \) the muon mass, \( m_e \) the electron mass, \( B = 183 \) and \( D_n = 1.54A^{0.27} \) and finally:

\[ \delta = \frac{m_\mu^2}{2E_\mu} \left( 1 - \frac{E_\mu}{E_\gamma} \right). \tag{12} \]
We then find the muon energy spectrum by integrating over the photon energies $E'$:

$$N_\mu(E) = \int_0^\infty N_\gamma(E') \frac{d\sigma_\mu}{dE}(E, E') dE'$$  \hspace{1cm} (13)

This energy spectrum is very wide, we are interested in the fraction of muons that can be injected in a beam line with a finite relative energy acceptance given by $A_E$:

$$N_\mu(A_E) = \frac{\rho_L N_A}{M_t} \frac{E_{\mu,max}(1+A_E)}{E_{\mu,max}(1-A_E)} \int N_\mu(E) dE,$$  \hspace{1cm} (14)

with $E_{\mu,max}$ the muon energy corresponding to the peak of the spectrum, $L_t$, $\rho_L$ and $M_t$ the length, density and molar mass of the target and $N_A$ the Avogadro number. In the following, we shall consider a tungsten target at room temperature. Its length is chosen below the radiation length, such that the development of the electromagnetic shower is limited. A more accurate estimate of the optimal target length should be obtained with a detailed shower development model. Figure 2 shows the variation of the muon rate in a given energy acceptance for different electron beam divergence, allowing for an estimation of the optimal divergence in the various configuration. We note that, as the channelling potential of tungsten is higher, the optimal divergence is significantly higher than the other crystals. The maximum rate obtained based on such an optimisation as a function of the electron beam energy is shown in Fig. 3. We find that the carbon crystal does not feature any advantages in terms of both muon energy and rate with respect to the silicon crystal in this energy range. For electron beams below 20 GeV, the tungsten crystal outperforms the two others in terms of rate and energy. At higher energies, the silicon crystal features significantly higher rates. The initial muon energy is of lesser interest than the rate since the energy can be further increased afterwards with a dedicated accelerator. Nevertheless a design based on a lower energy electron beam might be more cost and energy efficient. Consequently, we focus on a low energy option for the design of the electron beam. Nevertheless a design based on a lower energy electron beam might be more cost and energy efficient. Consequently, we focus on a low energy option for the design of the electron beam.

### III. Electron Beam

Let us consider an electron beam circulating in a synchrotron. Due to multiple scattering, the emittance of the circulating beam is significantly affected by the presence of a crystal. We can write the r.m.s. deflection angle $\theta_c$:

$$\theta_c = \frac{13.6 \cdot 10^6}{eE_e} \frac{L_c}{L_t}$$  \hspace{1cm} (16)

resulting in an emittance growth at each passage through the crystal and, since this contribution is significantly larger than other sources of emittance growth such as quantum excitations, we can write the equilibrium divergence:

$$\sigma_{\epsilon,\text{equ}}^2 = \frac{1}{2} \tau_c \theta_c^2$$  \hspace{1cm} (17)

with $\tau_c$ the damping time of the transverse emittances due to the emission of synchrotron radiation along the ring. Imposing that the equilibrium divergence matches the optimal divergence $\sigma_{\epsilon,\text{opt}}^2$ obtained above, we find the optimal damping time:

$$\tau_c,\text{opt} = \frac{\sigma_{\epsilon,\text{opt}}^2}{\theta_c^2}.$$  \hspace{1cm} (18)

For both designs in Tab. I, the required damping time is about half a turn, which is out of reach.

<table>
<thead>
<tr>
<th>Crystal (plane)</th>
<th>C(100)</th>
<th>Si(110)</th>
<th>W (110)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$ [Å]</td>
<td>0.89</td>
<td>1.92</td>
<td>2.24</td>
</tr>
<tr>
<td>$a_{\text{PT}}$ [eV]</td>
<td>10.1</td>
<td>23.0</td>
<td>138.6</td>
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<tr>
<td>$b_{\text{PT}}/d$ [Å]</td>
<td>0.19</td>
<td>0.145</td>
<td>0.096</td>
</tr>
<tr>
<td>$U_c$ [eV]</td>
<td>9.9</td>
<td>22.9</td>
<td>138.6</td>
</tr>
</tbody>
</table>

Table I. PT potential parameters and channelling potential height of carbon, silicon and tungsten crystals.
Figure 3. Muon output energy and rate per electron passage through the crystal as a function of the electron beam energy for carbon, silicon and tungsten crystals.

Figure 4. Spectral density of the photons (solid line) and muons (dashed line) generated by the passage of an electron through the crystal for the two designs described in Tab. II.

Table II. Muon source parameters. The symbols $\parallel$ and $\perp$ refer to the plane parallel and perpendicular to the channelling plane respectively.

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At high energies the main interaction of the electrons in the beam with the crystal is electron-positron pair production in the Coulombian field of the nuclei. However, the pair-production does not lead to a local deposition of energy, such that this mechanism does not threaten the integrity of the crystal. The electron beam will therefore deposit energy in the crystal mainly via collision with the electrons. This contribution is rather independent of the energy in the range of interest, corresponding to 1.6 and 2.1 MeV / cm²/g for silicon and tungsten [9]. The currents quoted in Tab. [1] are such that the power deposited in the crystal is about 50 W, based on the materials density at room temperature.

We observe that the silicon crystal is more performing in terms of muon rate not only thanks to the shape of the potential, as discussed in previous section, but also because its low density allows for a high electron beam current for the given energy deposition. A more accurate estimate of the maximum electron beam current should take into account the resistance of the material to thermal load.

IV. CONCLUSION

Considering the classical motion of high energy electrons in planar crystals and the muon pair production of the resulting photon beam impacting on a target, it is shown that a realistic electron LINAC or ERL of energy in the tens of GeV could be used to generate muon beams with a rate comparable to pion decay-based sources, yet featuring a smaller transverse emittance (e.g. [10, 11]). Such sources could be considered for various particle physics applications such as the generation of intense neutrino beams or multi-TeV lepton colliders.

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