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EXOTIC PROBABILITY THEORY?

by

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Is Quantum Mechanics an Exotic Probability Theory?[†]

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Abstract

Bayesian complex probability theory is proposed as an alternative to quantum mechanics.

Quantum mechanics is often introduced with a discussion of the two slit experiment where the observed interference pattern forces us to conclude that the particle goes through both slits at once and is “both a particle and a wave.” This most basic argument has a loophole, however. The conclusion rests on probability theory, and, in particular, on the fact that probabilities are non-negative so that when the second slit is opened,

$$P(x) = P(x \text{ via slit 1}) + P(x \text{ via slit 2}) \geq P(x \text{ via slit 1}) \quad (1)$$

where $P(x)$ is the probability for the particle to arrive at position x on the screen. Perhaps, then, it is possible that the particle does go through either one slit or the other after all and the interference effects can be explained by modifying probability theory itself. Here we give a brief overview of this program which is put forward in references 1, 2 and 3. For other approaches to exotic probability theory, see references 4-7.

Probability theory is most often presented with the “frequentist” approach where probabilities are defined as limits of experimental frequencies. For example, if n successes occur in N trials of an experiment, the large N limit of n/N is called the “probability of success.” Such probabilities are then assumed to follow Kolmogorov’s axioms.⁸ From this point of view, probability theory is a theory of “random phenomena” which are well described by these axioms and, of course, probabilities are necessarily non-negative. For our purposes, then, it is necessary to adopt the more general “Bayesian” view where probabilities are not a priori defined as frequencies, but, instead, a frequency interpretation is derived as a consequence of the fundamental axioms.⁹ With the Bayesian view, probability is introduced as a measure of “likelihood” where if proposition a is known, the likelihood that proposition

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b is true is denoted¹⁰ " $(a \rightarrow b)$." For " $\rightarrow$ " to be a useful likelihood measure one expects it to have a few properties such as: if $(a \rightarrow b)$ is known, this should determine $(a \rightarrow \neg b)$ and the procedure to get from $(a \rightarrow b)$ to $(a \rightarrow \neg b)$ should be independent of a and b . As shown by Cox,¹¹ such considerations are enough to entirely fix probability theory which then takes the form

$$(a \rightarrow b \wedge c) = (a \rightarrow b)(a \wedge b \rightarrow c) \quad (\text{I.a})$$

$$(a \rightarrow b) + (a \rightarrow \neg b) = 1 \quad (\text{I.b})$$

$$(a \rightarrow \neg a) = 0 \quad (\text{I.c})$$

for all propositions a , b and c .

Adopting the Bayesian view, we introduce complex probabilities while keeping all of Cox's remaining conditions. Cox's analysis then leads to I.a-I.c once again except with complex $(a \rightarrow b \wedge c)$, $(a \rightarrow b)$ etc. In the Bayesian view of ordinary probabilities, the frequency interpretation can be derived from the axioms using an additional condition that if $(a \rightarrow b) = 0$, then if a is known, b never actually happens. In complex probability, however, a may be known and $(a \rightarrow b)$ may be zero but b may still be found to be true. If we suppose, however, that a special set of propositions U exists where for $x \in U$, $(a \rightarrow x) = 0$ does imply that x is never true if a is known, then a frequency interpretation can once again be derived. This motivates the introduction of the realistic "state space" axioms as described in references 1 and 2. The resulting complex probability theory is then shown to have a superposition principle, to include wavefunctions which are expansions in eigenfunctions of Hermitian operators, to have a path integral representation and to describe both pure and mixed systems. A scalar particle in R^d is shown to obey the Schrödinger equation with mass, vector potential and metric appearing as moments of a fundamental complex probability.

Although probability theory has historically been considered to be a branch of mathematics, its frequency meaning makes it a physical theory which may succeed or fail by experimental test. From this point of view, it is natural to attempt to explain quantum mechanical effects by modifying probability theory. We find that there is essentially one possible modification of probability theory which introduces complex probabilities while maintaining the Bayesian consistency conditions. Although this modified probability theory is both realistic and local,³ it nevertheless reproduces the predictions of standard quantum mechanics while coexisting with Bell's theorem and other limitations on local realistic theories.

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10. We use the standard notation " $\wedge$ " for *and* and " $\neg$ " for *not*. Probabilities which we denote by $(a \rightarrow b)$ might otherwise be written $P(b|a)$. The arrow notation $(a \rightarrow b)$ was chosen to suggest a numerical version of logical implication and is meant to be read "*a goes to b.*"
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