Covariant and heavy quark symmetric quark models

D. Tadić and S. Žganec

Physics Department, University of Zagreb, Bijenička c. 32, Zagreb, Croatia

Abstract

There exist relativistic quark models (potential or MIT-bag) which satisfy the heavy quark symmetry (HQS) relations among meson decay constants and form factors. Covariant construction of the momentum eigenstates, developed here, can correct for spurious center-of-mass motion contributions. Proton form factor and M1 transitions in quarkonia are calculated. Explicit expression for the Isgur-Wise function is found and model determined deviations from HQS are studied. All results depend on the model parameters only. No additional ad hoc assumptions are needed.
I Introduction

A simple, but covariant quark model[1-4], used previously to calculate meson form factors[5], possesses also the heavy quark symmetry (HQS)[6-14]. Actually this might be true for a whole class of quark models. This class contains models in which quarks are confined by a central potential. Their wave functions must be Lorentz boosted [2-5]. It is hoped that such models might serve as a useful semiempirical tool. They can be used to roughly estimate physical quantities and effects and to illustrate HQS relations.

Once the model confinement parameters [15-17], the quark masses and the interaction hypersurface[3,5] are selected, everything else follows from our formalism. No additional assumptions, as for example about $Q^2$-dependence of form factors [18] are needed. The HQS is intimately connected with the Lorentz-covariant character of the model.

Models hadron states, used previously [1-3], were not momentum eigenstates [19-24]. This can be remedied by a projection [19-25] of model states into momentum eigenstates. A Lorentz - covariant projection [25] is developed here. It is shown that this removal of the spurious center-of-mass motion improves the model description of proton electromagnetic form factors. Such corrections are not important if hadron contains heavy quarks c or b. In that case they are smaller than 5 %.

The model calculations give some corrections to the extreme HQS. Some of those, for example concerning meson decay constants $f_D$ and $f_{D_s}$, agree with QCD sum rule results [26]. Model predictions for meson form factors in the heavy quark limit (HQL) follow exactly the HQS requirements. One can extract model prediction for the Isgur-Wise function $\xi [7]$. 

2
II Relativistic model

Any static model in which quarks are confined by a central force can be relativized [1-5]. Earlier the MIT-bag model has been employed [3-5]. Here a harmonic oscillator confining potential [16-17] will be used.

In any of them one can envisage a hadron as located around $y$. The quark $q_i$ coordinate is

$$x_i = y + z_i$$  \hspace{1cm} (2.1)

The confining "ball" of mass $M$, can be boosted, acquiring the four-momentum $P$. Individual quark wave functions $\psi_n$ depend on $z$ and $P$

$$\psi_n(z^P) = S(P)\eta_n(Z^P)exp(-i\frac{Z^P}{\mu} \epsilon_n)$$  \hspace{1cm} (2.2)

Here

$$S(P) = (P\gamma_0 + M)/(2M(E + M))^{1/2}$$

$$E = (\vec{P}^2 + M^2)^{1/2}$$

$$z_\perp(P)_\mu = z_\mu - \beta^\mu(\beta \cdot z)$$

$$z_\parallel(P) = \beta_\mu z^\mu$$

$$\beta_\mu = P_\mu/M$$  \hspace{1cm} (2.3)

and $\epsilon_n$ is model energy. For $\beta_\mu = 0$ the Dirac spinor $\eta$ has a generic form

$$\eta_n(\vec{r}) = \begin{pmatrix} U_n(|\vec{r}|)\chi \\ i\vec{\sigma} \cdot \vec{r}V_n(|\vec{r}|)\chi \end{pmatrix}$$  \hspace{1cm} (2.4)

Here $\chi$ is the Pauli spinor.

One can introduce the quark field operator

$$\Psi(z^P) = \sum_n(a_n\psi_n(z^P) + b^*_n\overline{\psi}_n(z^P))$$  \hspace{1cm} (2.5)
and define model states for meson "m" for example:

\[ |m, M, P, s, y \rangle = \sum_{r', r''} C_{rr'}^{j} C_{rr''}^{i} a_{r''}^{i} b_{r'}^{j} |0 \rangle e^{-iP_y} \]  \hspace{1cm} (2.6)

Here \( m \) is the flavor (B,D etc.), \( M \) is the meson mass and \( s \) is the spin.

Using the configuration space operators \([27] \hspace{1cm} (2.5)\) one can obtain a model wave functions whose generic form is:

\[ \langle 0 | \Psi(z_1^P) \Psi(z_2^P) \Psi(z_3^P) | b, M, P, s, y \rangle = N_P e^{-iP_y} F_b(\psi_{j_1}(z_1^P) \psi_{j_2}(z_2^P) \psi_{j_3}(z_3^P)) \]

\[ = \ h_s^b(P, z_1, z_2, z_3) e^{-iP_y} \] \hspace{1cm} (2.7)

Here \( N_P \) is the norm and \( F_b \) symbolizes the symmetrized combination of quark flavors.

A quark line in the configuration space, in the non relativistic limit, corresponds to the normalization integral

\[ \int \psi^* \psi d^3 z = 1 \] \hspace{1cm} (2.8)

This can be generalized as

\[ Z = J(z) \bar{\psi}(z^P) \ F \psi(z^P) \] \hspace{1cm} (2.9)

Here

\[ J(z) = \int d^4 z \delta(Lz) \] \hspace{1cm} (2.10)

Among all possible hypersurfaces

\[ L \cdot z = 0 \] \hspace{1cm} (2.11)

only the one defined by

\[ L_{\mu} = (\beta^i_{\mu} + \beta^i_{\mu}) / \sqrt{(\beta^i_{\mu} + \beta^i_{\mu})^2} \] \hspace{1cm} (2.12)

leads to the proton electromagnetic formfactors \( f_i \) which satisfy the conserved current constraint \( f_3(Q^2) = 0 \) (5.2). A model defined on a hyperplane is connected \([2,3]\) with the quasipotential approximation \([28]\) of the Bethe-Salpeter equation.
The vertex spatial dependence follows from (2.9) by replacing

\[ \mathcal{F} \rightarrow \Gamma_\mu \]

\[ \Gamma_\mu = \gamma_\mu, \gamma_\mu \gamma_5, \text{ etc.} \]  

(2.13)

For mesons \( m_f, m_i \) (2.6) a current matrix element is

\[ V(\Gamma^\mu) = \int d^4y \langle y, s_f, P_f, M_f, m_f | J(z_1) \overline{\Psi}(z_1^{P_f}) \Gamma^\mu \Psi(z_1^{P_f}) \rangle \]

\[ \cdot J(z_2) \overline{\Psi}(z_2^{P_i}) \mathcal{L} \Psi(z_2^{P_i}) | m_i, P_i, s_i, y \rangle \]  

(2.14)
III  Momentum eigenstates

The factor $\exp(-iPy)$ (2.6) describes the motion of center-of-force (CF). The center-of-mass (CM) of centrally confined quarks oscillates about CF. As it is well known [1-6,25] the spurious center-of-mass-motion (CMM) persist even in the static ($\vec{P} = 0$) case. Thus the boosted centrally confined model (BCCM) states (2.6) are not the momentum eigenstates. This can be remedied by decomposing a BCCM state into momentum eigenstates $|l, s\rangle$ as follows [19-24]:

\[
|h, M, P, s, y\rangle = 2M \int d^4l \delta(l^2 - M^2) \theta(\omega) e^{-i\theta} \phi_P(l)|l, s\rangle = 2M \int \frac{d^3l}{2\omega} e^{-i\theta} \phi_P(l)|l, s\rangle
\]

(3.1)

Here $h$ denotes a hadron. The momentum eigenstates normalization is

\[
\langle l', s'|l, s\rangle = \delta_{ss'} \delta(l' - l)
\]

(3.2)

For a BCCM state one has

\[
\langle h, M, P, s, 0|0, s, P, M, h\rangle = 1 = \int \frac{d^4l}{\omega^2} |\phi_P(l)|^2
\]

(3.3)

This provides a normalization of the components $\phi$ of the momentum eigenstates.

The momentum eigenstates in (3.1) are not the exact physical hadron states but the model hadron states, i.e. some kind of "mock" hadron states [29].

In the occupation number space one finds for a baryon $b$, for example

\[
\langle y = 0, s, P, M, b|b, MP, s, y = \zeta_\perp(P)\rangle = M^2 \int \frac{d^3l}{\omega^2} |\phi_P(l, \omega)|^2 e^{-i\theta} \zeta_\perp(P)
\]

(3.4)
In the coordinate space this becomes

\[ \langle y = 0, s, P, M, h|b, M, P, s, y = \zeta_\perp(P) \rangle = \mathcal{M}(P, \zeta_\perp(P)) = \]
\[ = J(z_1)J(z_2)J(z_3)h_b^*(P, z_1, z_2, z_3) f_1 f_2 f_3 \]
\[ = h_b^*(P, z_1 - \zeta_\perp(P), z_2 - \zeta_\perp(P), z_3 - \zeta_\perp(P)) \]  (3.5)

For the proton, with all light quark masses equal \( m_u = m_d \), one finds

\[ \mathcal{M}(P, \zeta_\perp(P)) = [J(z)\bar{\psi}(z_\perp(P))S^{-1}(\frac{\not{P}}{E}) L\psi((z(P) - \zeta(P))_\perp)]^3 \]  (3.6)

Integrating (3.4) and (3.6) over \( \zeta \) one finds

\[ J(\zeta)\mathcal{M}(P, \zeta_\perp(P))e^{i\zeta_\perp(P)} = M^2 J(\zeta) \int \frac{d^3 k}{\omega_k^2} |\phi_P(k, \omega_k)|^2 e^{i(k \cdot -k \cdot \zeta_\perp(P))} \]  (3.7)

The end result is the Lorentz-covariant expression for the components of the momentum eigenstates:

\[ \frac{M}{\omega_l}|\phi_P(\vec{l}, \omega_l)|^2 = \frac{l \cdot P}{(2\pi)^3 M^2} \int d^4\zeta \delta(L \cdot \zeta) \mathcal{M}(P, \frac{\not{P}}{E}, \zeta) e^{i\zeta_\perp(P)} \]  (3.8)

Some explicit expressions for \( \mathcal{M}s \) are listed in Appendix.
IV Confinement

The Dirac equation for quarks can be solved for the potential

\[ V(r, P) = \frac{1}{2}(1 + \frac{P}{M})(V_0 - \frac{1}{2}Kz\perp(P)^2) \]  

which in the hadron rest frame has the harmonic oscillator (HO) shape [16]

\[ V(r) = \frac{1}{2}(1 + \gamma^0)(V_0 + \frac{1}{2}Kr^2) \]  

Here \( V_0 \) and \( K \) are model parameters. The rest frame solution has a general form (2.4), with:

\[
U_a = \exp(-r^2/2R_0^2) \\
V_a = r\beta_a U_a/R_{0a} \\
N_a = [R_0^3 \pi^{3/2}(1 + \frac{3}{2}\beta_a^2)]^{-1/2}
\]

The index \( a \) denotes the quark's flavor. The quantities \( R_{0a} \) and \( \beta_a \) depend on the constituent mass \( m_a \) and the energy \( E_a \).

\[
E_a = m_a + V_0 + 3[K/2(m_a + E_a)]^{1/2} \\
R_{0a}^4 = 2K(m_a + E_a) \\
\beta_a = R_{0a}^{-1}(m_a + E_a)^{-1}
\]

An approximate solution [6] for the linear potential \( V(r) = \frac{1}{2}(1 + \gamma^0)(V_0 + \lambda r) \) would also have the form (4.3), with accuracy of \( \sim 6\% \). All general HQS features, discussed bellow would, thus apply for that potential also.

In the heavy quark limit (HQL), where \( m_a \rightarrow \infty \) and \( E_a \rightarrow m_a \), one has

\[
\frac{\beta_a}{R_{0a}} \rightarrow \frac{\lambda}{2\sqrt{m_a}} \rightarrow 0
\]

Thus only the "large" component \( U \) survives in (4.3).

One can also show that in MIT - bag model [15] "small" component \( V \) vanishes in HQL. In the numerical evaluation MIT - bag model parameters employed previously by Ref.[5] will be used.
The HO model parameters are
\[ V_0 = -0.35 GeV \]
\[ K = 0.035 GeV^3 \] (4.6)
The constituent quark masses and related quantities \( \beta, E \) and \( R_0 \) are listed in Table I.

Table II shows model hadron masses calculated using either model states (2.6) or model dependent momentum eigenstates (3.2). The relevant formula for the valence quark contribution to the hadron mass \( \tilde{M}_Q \) is:
\[ \tilde{M}_Q^b = \langle h, M, 0, s, 0 | \int T^{00} \delta^3 x | h, M, 0, s, 0 \rangle \]
\[ = \langle h, M, 0, s, 0 | P^0 | h, M, 0, s, 0 \rangle \] (4.7)

Here \( T^{00} \) is the momentum energy tensor. One must add magnetic \( \Delta \tilde{M}_M \) and electric \( \Delta \tilde{M}_E \) effective one gluon exchange contributions [15,16] which for the HO potential model can be calculated explicitly. Finally one has BCCM based hadron mass without CMM corrections.
\[ \tilde{M} = \tilde{M}_Q + \Delta \tilde{M}_M + \Delta \tilde{M}_E \] (4.8)

Using momentum eigenstates one obtains the following identities for a meson \( m \) or a baryon \( b \):
\[ \tilde{M}^m = \int \frac{d^3 k}{4 \omega^2} |\varphi^m(k)|^2 \sqrt{M^2 + \omega^2} \] (4.9)
\[ \tilde{M}^b = \int \frac{d^3 k}{\omega^2} \frac{M^b}{\omega^2} |\phi^b(k)|^2 \sqrt{M^2 + \omega^2} \] (4.10)

Here \( \tilde{M}^m \) and \( \phi^b \) are determined by parameters from Table I. The CMM corrected masses \( M^{m,b}_C \) can be found numerically. Inspection of Table II reveals that CMM corrections improve the agreement with the experimental values [6]. The mass of the pion is quite wrong, as in all valence quark models which do not account for the Goldstone-boson nature of pion. Other theoretical masses are correct within 10% or better. CMM corrections increase mass difference in a SU(6) multiplet, \( (p, \Delta, etc.) \), bringing theory closer to experiment. Corrections decrease with the increase of the heavy quark mass. Thus for example \( (\tilde{M}_B - M_B)/M_B \simeq 1.6\% \).
Calculation of the proton formfactors is a useful test of any quark model. All calculational details have been discussed and described in Ref.'s [3] and [5]. It remains to be shown that the inclusion of CMM corrections improves upon earlier results.

These corrections are included by the equality

$$
\int d^4 y \prod_{i=1}^{3} J(z_i)(M, P_j, y) \sum_{i,j,k,\text{perm}} V^\mu(z_i) C(z_j) C(z_k) \cdot e^{-iQx_i} \langle M, P_i, y \rangle =
$$

$$
= (2\pi)^4 \delta(P_j - P_i - Q) J(z) \int \frac{d^3 l_0 d^3 l'}{\omega \omega'} M^2 \phi_P^* (\vec{l'}, \omega') \phi_P (\vec{l}, \omega) \langle \vec{l}' | V^\mu(z) e^{-iQz} | \vec{l} \rangle \tag{5.1}
$$

Here:

$$
V^\mu(z_i) = \overline{\Psi}(z_i) \gamma^\mu \Psi(z_i)
$$

$$
C(z_k) = \overline{\Psi}(z_k) \cdot \slashed{P} \Psi(z_k)
$$

$$
\langle \vec{l}' | V^\mu(0) | \vec{l} \rangle = \overline{u}(\vec{l}') [f_1(s^2) \gamma^\mu + f_2(s^2) i\sigma^{\mu\nu} s_\nu + f_3(s^2) s^\mu] u(\vec{l})
$$

$$
s = l' - l \quad ; \quad f_3(s^2) \equiv 0
$$

The l.h.s. of (5.1) is the expression used earlier [5] to calculate electromagnetic formfactors. Here it is written in the occupation number space.

In general one cannot invert the expression (5.1). However at the momentum transfer $Q^2 = 0$ one can determine [23] the Sach’s form factor $G_M(0)$.

The l.h.s. of (5.1) can be written as

$$
(5.1) (l.h.s.) = (2\pi)^4 \delta(P_j - P_i - Q) \chi^* [W^0 + \frac{i}{2M} \vec{\sigma} \times \vec{Q} W^2] \chi \tag{5.3}
$$

Here

$$
W^0 = I_0 \cdot Z^2
$$

$$
W^2 = I_2 \cdot Z^2
$$
\[ Z = \frac{M_f}{E_f} 4\pi \int drr^2 j_0(\rho)[U^2 + V^2] \]

\[ I_0 = 4\pi \frac{M_f}{E_f} \int drr^2 j_0(\tilde{\rho})[U^2 + V^2] \]

\[ I_1 = 4\pi \frac{M_f}{E_f} \int drr^2 [j_0(\tilde{\rho})U^2 - \left( \frac{1}{3} j_0(\tilde{\rho}) - \frac{2}{3} j_1(\tilde{\rho}) \right) V^2 + \frac{2E_f}{|P_f|} j_1(\tilde{\rho}) UV] \]

\[ \rho = \frac{|\tilde{P}_f|}{E_f} \sqrt{2|\tilde{\rho}|} \quad ; \quad \tilde{\rho} = \frac{|\tilde{P}_f|}{E_f} 2(M - \epsilon)|\tilde{\rho}| \]

The quantities \( W^a \) were identified \([3,5]\) as Sach’s formfactors

\[ W^0 \sim G_E \]
\[ W^2 \sim G_M \quad (5.4) \]

However (5.4) was obtained using BCCM states which are not momentum eigenstates.

More accurate approach is based on the equality

\[ (5.1)(r.h.s.) = (2\pi)^4 \delta(P_f - P_i - Q)D^\mu \]

\[ D^\mu = 2\pi \int l^2 d\sin \theta d\theta \frac{M^3}{(\omega \omega')^{3/2}} \phi_{P_f}(\tilde{\ell}, \omega') \phi_{P_i}(\tilde{\ell}, \omega) \frac{1}{\sqrt{4M^2(\omega + M)(\omega' + M)}} \]

\[ \frac{1}{(1 - \frac{q^2}{4M^2})} \left[ G_E(q^2)(\delta^\mu - \frac{\eta^\mu}{2M}) + G_M(q^2)(\eta^\mu - \frac{q^2}{4M^2} \delta^\mu) \right] \cdot \chi^+ \Gamma(\mu) \chi \quad (5.5) \]

\[ \chi^+ \Gamma(\mu) \chi = \chi^+[\delta_{\mu0} + \delta_{\mu3} + \delta_{\mu1} i\sigma_2 - \delta_{\mu2} i\sigma_1] \chi \]

Here

\[ \tilde{\ell} = \tilde{\ell} + \tilde{Q} \quad ; \quad \omega^2 = \tilde{\ell}^2 + M^2 \]

\[ q = (q^0, \tilde{Q}) \quad ; \quad q^0 = \omega' - \omega \]

\[ \delta^0 = a_i a_f + \tilde{\ell}^2 + \tilde{Q} \cdot \tilde{\ell} \]

\[ \delta^1 = \delta^2 = (a_i - a_f)|\tilde{\ell}| \cos \theta + a_i |\tilde{Q}| \]

\[ \delta^3 = a_f |\tilde{\ell}| \cos \theta + a_i (|\tilde{\ell}| \cos \theta + |\tilde{Q}|) \]

\[ \eta^0 = (a_f - a_i) \tilde{Q} \cdot \tilde{\ell} - a_i \tilde{Q}^2 \quad (5.6) \]

\[ \eta^1 = \eta^2 = a_i a_f |\tilde{Q}| + |\tilde{Q}| (\tilde{\ell}^2 + \tilde{\ell} \tilde{Q}) - \tilde{\ell}^2 |\tilde{Q}| \sin^2 \theta + + (\omega' - \omega) (-a_i |\tilde{\ell}| \cos \theta - a_i |\tilde{\ell}| \cos \theta - a_i |\tilde{Q}|) \]
\[
\eta^3 = (\omega - \omega)[a_j |\vec{l}| \cos \theta - a_i (|\vec{l}| \cos \theta + |\vec{Q}|)]
\]

\[
a_i = \omega + M \quad ; \quad a_f = \omega' + M
\]

\[
\vec{Q} \cdot \vec{l} = |\vec{l}| |\vec{Q}| \cos \theta
\]

Four-momentum \(q\) is an average value of \(l - l'\) calculated between two wave-packets \(\phi_P\) which have speeds \(\beta_P^\mu\) and \(\beta_P'^\mu\) respectively.

For the Sach's form factors one can assume the well known dipole shapes

\[
\frac{G_E(q^2)}{G_E(0)} = \frac{G_M(q^2)}{G_M(0)} = (1 - \frac{q^2}{\eta^2})^{-1}
\]

(5.7)

The magnetic moment \(G_M(0) = \mu_P\) can be determined from the equalities (5.4) and (5.6) taken at \(Q^2 = 0\). One obtains

\[
\tilde{D}\big|_{Q=0} = 0
\]

\[
Q^2 \frac{\partial \tilde{D}}{\partial Q}\big|_{Q=0} = \frac{1}{2M}\chi^+ i\vec{\sigma} \times \tilde{Q} \chi \cdot \kappa = \frac{1}{2M}\chi^+ i\vec{\sigma} \times \tilde{Q} \chi \cdot W^2 (\tilde{Q} = 0)
\]

(5.8)

\[
\kappa = W^2 (\tilde{Q} = 0) = 4\pi \int l^2 dl \frac{M^2}{\omega^2} |\phi_P = 0(\vec{l}, \vec{\omega})|^2 \left[ \frac{G_E(0)}{3} \left( \frac{M}{\omega} - 1 \right) + \frac{G_M(0)}{3} \left( 1 + \frac{M}{\omega} + \frac{M^2}{\omega^2} \right) \right]
\]

With \(G_E(0) = 1\) one finds

\[
G_M(0) = 2.212
\]

(5.9)

which is about 20% to small. However without CMM corrections one would have obtained

\[
G_M(0) = 1.738
\]

(5.10)

which is much smaller than the experimental value [30] \(G_M(0) = 2.793\). The CMM corrections have resulted in 27 % improvement of the model value[23].

The equality (5.5) can be used to determine the parameter \(\eta\). For \(Q^2 < 1.17 GeV^2\) equality is, within 10% error, satisfied with

\[
\eta = 0.70 GeV^2
\]

(5.11)

which is very close to the experimental value [30] \(\eta_{exp} = 0.71 GeV^2\).
The fit (5.11) fails progressively as $Q^2$ increases above $1.17 GeV^2$. Qualitatively this agrees with other model based calculations, see for example Ref. [31].

An analogous formalism can be used for the nucleon axial vector coupling constant $g_A$. Without CMM corrections one finds $g_A = 1.14$. With CMM corrections the theoretical result $g_A = 1.22$ is surprisingly close to the experimental value [30].

A strong point in favor of BCCM with CMM corrections is that corrections are much larger for $G_M (27\%)$ than for $g_A (7\%)$, just as needed.
VI M1 transition in quarkonia

The M1 transitions

\[ V \rightarrow P + \gamma \]
\[ (^3S_1 \rightarrow ^1S_0 + \gamma) \]  \hspace{1cm} (6.1)

provide useful informations\cite{32,33} about CMM corrections for systems containing heavy quarks c and b. The decay amplitude is

\[ \langle m(P_f)|J_{\mu \lambda mg}(0)|v(P_i, \epsilon) \rangle = \frac{1}{(2\pi)^3 \sqrt{4E_i E_f}} \epsilon^{\mu \nu \sigma \rho} \epsilon_{\nu}(P_f - P_i)_{\sigma}(P_f + P_i)_{\rho} \]  \hspace{1cm} (6.2)

with the corresponding decay width

\[ \Gamma(v \rightarrow m\gamma) = \frac{4}{3} \alpha (\frac{\alpha}{2})^2 \omega_\gamma^3 \]
\[ \omega_\gamma = \frac{M^2 - M_f^2}{2M} \]  \hspace{1cm} (6.3)

Here \( \alpha \) is the fine structure constant.

In BCCM’s the form factor \( g \) can be calculated with \((g(s^2))\) and without \((\tilde{g}(s^2))\) CMM corrections. In the first case one starts with

\[ (2\pi)^4 \delta(P_f + k - P_i) \cdot \prod_{i=1}^{2} J(z_i) \sum_{l,n,perm} \kappa_{m=0}(P_f, z_1^P, z_2^P, y = 0) \]
\[ \langle \gamma_{\mu} \rangle e^{ikz_i} \kappa_{m=1}(P_i, z_1^P, z_2^P, y = 0) = (2\pi)^4 \delta(P_f + k - P_i) \mathcal{N}_i N_{ji} \]  \hspace{1cm} (6.4)

As \( \mathcal{N}_i \) must have the same form as (6.2) one can identify the form factor \( \tilde{g}(0) \). Here \( \kappa^i_m \) is the meson wave function analogous to (2.7). The calculation was carried out in the generalized Breit frame.
\[
\frac{E_i}{M_i} = \frac{E_f}{M_f} \quad ; \quad \frac{\vec{P}_i}{M_i} = - \frac{\vec{P}_f}{M_f} \quad ; \quad P_i = P_f + k \tag{6.5}
\]

\[
|\vec{k}| = \frac{M_i^2 - M_f^2}{2\sqrt{M_i M_f}} \quad ; \quad |\vec{P}| = \sqrt{\frac{M_i M_f - M_f^2}{2}}
\]

By expansion of \(N^\mu\) around \(Q^2 = 0\) one can find for smaller \(Q^2\)

\[
\tilde{g}(Q^2) = \frac{\tilde{g}(0)}{1 - Q^2/\Lambda_1 + Q^4/\Lambda_2 + \ldots} \approx \frac{\tilde{g}(0)}{1 - Q^2/\Lambda_1} \tag{6.6}
\]

The CMM corrections are introduced by using the equality

\[
N^\mu N_{ji} = J(z) \int \frac{d^3l d^3\phi}{4\omega \omega^\prime} \varphi_P(i\vec{l}, \omega, \omega^\prime) \varphi_P(i\vec{l}, \omega) \cdot \langle i\vec{l} | J^\mu_{cl,m} (z) e^{ik \cdot \vec{l}} | j \rangle \tag{6.7}
\]

\[
N^\alpha = \int \frac{d^3l}{(4\omega \omega^\prime)^{3/2}} \varphi_P(i\vec{l} - \vec{k}, \omega^\prime) \varphi_P(i\vec{l}, \omega) \frac{i}{\sqrt{2}} g(q^2) [2\omega(1 + \frac{M_f}{M_i}) |\vec{P}_i| - 2(\omega - \omega^\prime)\vec{l}] |\cos \theta|
\]

Here

\[
\omega^\prime = (\vec{l} - \vec{k})^2 + M_f^2 \quad ; \quad \cos \theta = \vec{P}_i \cdot \vec{l} / |\vec{P}_i| |\vec{l}| \quad ; \quad q = (\omega - \omega', \vec{k}) \tag{6.8}
\]

In (6.7) one must introduce the form (6.6) for \(g(q^2)\). Then using the equality (6.7), where \(N^\mu\) is determined by the integration over the model wave functions (6.4), one can determine \(g(0) \equiv g\).

Models predictions, based on the parameters listed in Table I are shown in Table III. The decay widths (6.3) are calculated using either \(\tilde{g}(0)\), (\(\Gamma\)) or \(g(0)\), (\(\Gamma\)). No attempts have been made to select model parameters in order to improve the agreement with the measured value \(\Gamma(\Psi \to \eta, \gamma) = (1.12 \pm 0.35) \times 10^{-6}\) GeV [34]. It is interesting that such, unadjusted, results are in a very reasonable agreement with the unadjusted results of Ref.[33] (Their Table II, columns 2,3), which were obtained in a quite different quark model.

The main aim here was to calculate the magnitude of CMM corrections. They turned out to be 4.4% or smaller, decreasing with the increase of the heavy quark mass. With \(b\) quark present CMM corrections are practically negligible. Indeed, when one of
the valence quarks is very heavy CF and CM almost coincide [6], so that the spurious CMM almost vanishes.
VII  Heavy quark symmetry limit and meson decay constants

The decay constant $f_m$, for a meson $m$, can be calculated in BCCM [22,24]. The Lorentz covariant CMM corrections are introduced through equality

$$\int d^4y J(z) \langle 0 | \bar{\Psi}(z^P) \gamma^\mu \gamma_5 \Psi(z^P) | m, P, M, s = 0, y \rangle e^{i\eta} = (2\pi)^4 \delta^{(4)}(q - P) Z^\mu(P) =$$

$$= \int d^4y J(z) \int \frac{d^3l}{2\omega} \varphi_P(\vec{l}, \omega) \langle 0 | J_5^p(z) | l, 0, m \rangle \cdot e^{i(k - P)\eta} e^{i\eta}$$

(7.1)

In the r.h.s. of (7.1) goes the meson decay constant $f_m$ defined for a momentum eigenstate $|P, 0, m\rangle$

$$< 0 | J_{\mu5}(x) | P, 0, m \rangle = \frac{i}{\sqrt{2E(2\pi)^3}} P_\mu f_m e^{-iP\eta}$$

(7.2)

In the HO potential version of BCCM integrations in (7.1) can be carried out explicitly. One finds

$$\sqrt{6} \frac{P^\mu}{M} K_{a,\beta} = \frac{(2\pi)^{3/2}}{2M\sqrt{2E}} \varphi_P(\vec{P}, E) P^\mu f_m$$

$$f_m = \sqrt{\frac{12}{M_m(2\pi)^{3/2} R_{a,\beta}^{(1)} C_{a,\beta}}} K_{a,\beta = m}$$

(7.3)

Here

$$K_{a,\beta} = 4\pi \int drr^2 (U_a U_\beta - V_a V_\beta)$$

$$c_{a,\beta}^{(1)} = 1 - 3 \left( \frac{c_\alpha}{R_{0\alpha}} + \frac{c_\beta}{R_{0\beta}} \right) R_{a,\beta}^2 + 15 \frac{c_\alpha c_\beta}{R_{0\alpha}^2 R_{0\beta}^2} R_{a,\beta}^4$$

(7.4)

$$R_{a,\beta}^2 = \frac{2 R_{0\alpha} R_{0\beta}}{R_{0\alpha} + R_{0\beta}} ; \quad c_\alpha = \frac{\beta_\alpha}{4 + 6\beta_\alpha^2}$$

In the HQL (4.5) one has

$$K_{a,\beta} \rightarrow 4\pi \int drr^2 U_a U_Q = K_{a,\beta}^{HQ}$$

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Here $Q$ is a heavy quark ($c, b$) while $\alpha$ denotes a light quark (u,d,s). With (7.5) a meson decay constant has $M_m^{-1/2}$ dependence as required by the heavy quark symmetry (HQS). One obtains for example

$$f_{B_{HQL}} = \sqrt{\frac{M_D}{M_B}} f_{D_{HQL}} = 0.6 f_{D_{HQL}}$$

(7.6)

With full expression (7.1), using parameters listed in Table I, one obtains

$$f_D = 130.6\, MeV \quad ; \quad f_B = 90.9\, MeV$$

$$f_B/f_D = 0.696$$

(7.7)

The ratio $f_B/f_D$ (7.7) is in a very good agreement with the result $f_B/f_D \approx 0.69$ obtained by the $1/m_Q$ expansion of the heavy-light currents [14,35]. However it is about 30% smaller than the results based on QCD sum rules, lattice calculations and semilocal parton-hadron duality[36].

The BCCM based calculation gives

$$f_{D_s} = 149, 2\, MeV$$

$$f_{D_s}/f_D = 1.14$$

(7.8)

The ratio $f_{D_s}/f_D$ is in reasonable agreement with previous results obtained from lattice QCD or potential models [36]. QCD sum rule analyses gave $f_{D_s}/f_D \approx 1.19$ [26] and $f_{D_s}/f_D \approx 1.1$ [37]. However absolute values (7.7,7.8) for heavy meson decay constants seem to be smaller than the QCD sum rule or lattice QCD based estimates [14,26,37-39].

The BCCM with CMM corrections predicts

$$f_{K^+} = 171\, MeV$$

which is in good agreement with the experimental value $f_{K^+} = (160.6 \pm 1.3)\, MeV$ [34].

The pion decay constant $f_{\pi} = 271\, MeV$ is to large ($f_{\pi,exp} = (131.73 \pm 0.15)\, MeV$ [34]), as it is usual in valence quark models.
The calculation of meson decay form factors has already been described [3,5] so only some examples need to be shown here. Matrix elements for $B \rightarrow D(D^*)$ transitions are:

\[ \langle P_f, s = 0, D | e^{-i\mu b} | B, s = 0, P_i \rangle = \frac{2\pi(-i)}{2\sqrt{E_i E_f}} [f_+(Q^2)(P_i + P_f)^\mu + f_-(Q^2)(P_i - P_f)^\mu] \]  

(8.1)

\[ \langle P_f, \epsilon, D^* | e^{-i\gamma b} | B, s = 0, P_i \rangle = \frac{2\pi(-i)}{2\sqrt{E_i E_f}} i\sigma(Q^2)\epsilon^{\mu\nu\sigma} \epsilon^*_\nu(P_i + P_f)(P_i - P_f)^\sigma \]  

(8.2)

\[ \langle P_f, \epsilon, D^* | e^{-i\gamma b} | B, s = 0, P_i \rangle = \frac{2\pi(-i)}{2\sqrt{E_i E_f}} [f(Q^2)\epsilon^\mu + a_+(Q^2)(\epsilon^* \cdot P_i)(P_i + P_f)^\mu + \epsilon_1(Q^2)(\epsilon^* \cdot P_i)(P_i - P_f)^\mu] \]  

(8.3)

Corresponding BCCM expressions in the generalised Breit frame (6.5) are

\[ f_+ = \frac{1}{\sqrt{4 M_i M_j}} [(M_i + M_j) M_f E_f f_{cb}^0 - (M_i + M_j) M_f E_f f_{cb}^3] Z_\pi \]  

\[ f_- = f_+[(M_i + M_j) \leftrightarrow (-)(M_i - M_j)] \]  

\[ g = M_f \sqrt{M_i M_j} \frac{V_{cb}^1(\lambda = +1)}{2 M_i E_f |P_f|} Z_\pi \]  

\[ f = \sqrt{4 M_i M_j} A_{cb}^1(\lambda = +1) Z_\pi \]  

(8.4)

\[ a_+ = \frac{1}{4 M_i M_j} \sqrt{\frac{M_f}{M_i}} [(M_i - M_j)(M_f E_f A_{cb}^0(\lambda = 0) - A_{cb}^1(\lambda = +1))] + \]  

\[ + (M_i + M_j)(\frac{M_f}{E_f})^2 [\frac{M_f}{|P_f|} A_{cb}^0(\lambda = 0) - A_{cb}^1(\lambda = +1)] \]  

\[ a_- = a_+[(M_i + M_j) \leftrightarrow (-)(M_i - M_j)] \]  

Here
CMM corrections have been neglected. For heavy-light quark combination they are always smaller than 5% (See Table III). In (8.5) one has introduced the spherical Bessel functions \( j_l(\tilde{\rho}) \) where:

\[
\tilde{\rho} = \frac{M_f}{E_f} \cdot B_{cb} \cdot |\vec{r}| \tag{8.6}
\]

\[
B_{cb} = \left[ (M_f + M_i) - (\epsilon_c + \epsilon_b) \right] \frac{[\vec{P}_1]}{M_f}
\]

The symbol \( \lambda \) labels the polarization of the vector meson \( D^* \). The expressions (8.4) contain also the overlap (free-line) (2.9) of the light spectator quark.

\[
Z_{\pi} = \frac{M_f}{E_f} 4 \pi \int d\hat{r} r^2 j_0(\rho)[U_u^2 + V_u^2] \tag{8.7}
\]

\[
\rho = 2 \epsilon_d \frac{[\vec{P}_1]}{E_f} |\vec{r}|
\]
Formulae (8.5) are a version of the more general formulae listed in Appendix ((A1) − (A7)) of Ref. [5]. Such formulae are valid for any BCCM, which includes BBM [5].

In the HQL (4.6):

\[ V_a = 0 \quad ; \quad \alpha = b, c \]
\[ U_b = U_c = U_{HQL} \]
\[ B_{cb} \to 0 \quad ; \quad \tilde{\rho} \to 0 \quad ; \quad j_0(0) = 1 \]

\[ r_{HQL}^0 = \frac{M_i}{E_f} A \pi \int d\tau r^2 U^2 = \frac{M_i}{E_f} K_{HQL} \]

\[ r_{HQL}^0 = 0 \]

\[ V_{HQL}^1 = \left| \frac{P_f}{M_f} \right| r_{HQL}^0 \quad ; \quad A_{HQL}^1 = \frac{E_f}{M_f} r_{HQL}^0 \]
\[ A_{HQL}^0 = 0 \quad ; \quad A_{HQL}^3 = r_{HQL}^0 \]

With

\[ \left| \frac{P_f}{M_f} \right|^2 / M_f^2 = \frac{(M_i - M_f)^2 - Q^2}{4M_i M_f} \]

\[ 4E_f E_i = (M_i + M_f)^2 \left[ 1 - \frac{Q^2}{(M_i + M_f)^2} \right] \]

one finds

\[ f_+ = 2 \sqrt{M_i M_f} \left[ 1 - \frac{Q^2}{(M_i + M_f)^2} \right]^{-1} (Z_{\pi K_{HQL}}) \]

\[ g = \frac{1}{(M_i + M_f)} f_+ \]

\[ a_+ = -g \]

\[ f = 2 \sqrt{M_i M_f} (Z_{\pi K_{HQL}}) \]

Very elegant relations among form factors can be found by using the formfactors from Ref.[18.], i.e.:

\[ F_1 = f_+ \quad ; \quad V = (M_i + M_f) g \]
\[ A_2 = -(M_i + M_f) a_+ \]

\[ A_1 = \frac{1}{(M_i + M_f)} f \]
As in HQL $M_{D^*} \equiv M_D = M_f$ one immediately obtains the well-known [14] HQS relations

$$F_1(Q^2) = V(Q^2) = A_2(Q^2) = \left[1 - \frac{Q^2}{(M_i + M_f)^2}\right]^{-1} A_1 = \frac{2\sqrt{M_i M_f}}{(M_i + M_f)} \frac{1}{1 - \frac{Q^2}{(M_i + M_f)^2}} Z_\pi K_{HQL}$$

(8.12)

From (8.12) one easily extracts the Isgur-Wise function [7,14] which is actually determined by the overlap $Z_\pi$ (8.7). First one must realize that $K_{HQL}$ is actually the HQL of the normalization integral

$$N = \int dx x^2 (U^2 + V^2) \quad ; \quad N_{HQL} = K_{HQL} = 1$$

(8.13)

Then, with (8.12), (8.13) and the definition [14]

$$\xi(v \cdot v') = \lim_{m \to -\infty} RF_1(Q^2)$$

$$v \cdot v' = \frac{M_i^2 + M_f^2 - Q^2}{2 M_i M_f}$$

$$R = \frac{2\sqrt{M_i M_f}}{M_i + M_f}$$

(8.14)

one obtains

$$\xi(v \cdot v') = \frac{4 M_i M_f}{(M_i + M_f)^2} \left[1 - \frac{1}{(M_i + M_f)^2}\right] Z_\pi(Q^2)$$

(8.15)

It should be noted that both (8.12) and (8.15) include explicitly the kinematic factor $[1 - Q^2/(M_i + M_f)^2]$. Furthermore, at the maximum momentum transfer $Q_{\text{max}}^2 = (M_i - M_f)^2$ one finds [14,40]:

$$Z_\pi|_{P = P_f = 0} = 1$$

$$F_1 = V = A_2 = A_1^{-1} = R^{-1}$$

(8.16)

Thus in HQL the BCCM based relations coincide exactly with QCD based ones, what is only approximately true for other models [18,29,4].

It might be interesting to compare BCCM prediction for the $Q^2$-dependence of form factors, including the HQL limit, with other approaches. The results obtained for HO
model are shown in Fig. 2 using the same scale as in corresponding Fig.’s 1.3 and 5.8 in Ref.[14].

All results presented here can be obtained also in BCCM based on the MIT-bag model [15]. Fig.3 shows that both versions of BCCM produce quite simmilar results. Models prediction stay close to the HQS limit, which is, up to factor $R^{-1}$, given by Isgur-Wise function $\xi(v \cdot v')$. In BCCM one always obtains $V(Q^2) > A_2(Q^2) \cong [1 - \frac{Q^2}{(M_B + M_D)^2}]^{-1} A_1 > F_1$. This ordering differs from other quark models[14]. It does agree with QCD sum rule results (Fig. 5.8 in Ref. [14]). However quantitative agreement is not so good. The absolute values of QCD-sum rule formfactors are usaully larger than the corresponding BCCM values. The gaps separating $V, [1 - \frac{Q^2}{(M_B + M_D)^2}]^{-1} A_1, A_2$ and $F_1$ curves are also larger. BCCM, as used here, does not take into account the short distance corrections which are responsible [14] for 50% of the enhancement of $V$ relative to $F_1$ and $A_1$.

All BCCM based conclusions seem to be independent of the form of central confinement [3,5,15-17]. However the precise form of the $Q^2$-dependence might be influenced by the model details. Thus the selection of the particular version of BCCM could be some kind of fine tunning.
IX  Main characteristics

The main aim of this paper was to demonstrate how one can construct a whole class of quark models which are heavy quark \((b,c)\) symmetric. Such models are also Lorentz covariant, as it has been shown in Ref's [3] and [5]. The kinematic factor \((8.12), (8.15)\) which appears in HQS relations is a typical consequence of the Lorentz covariance.

The class of HQS models contains models [15-17] in which each quark is independently centrally confined. As it is well known [25] such models experience spurious CMM effects. It is demonstrated here that one can introduce CMM corrections in the manifestly covariant way. They notably improve \(\mu_p, g_A\), hadron masses and other quantities which involve "light" \((u,d,s)\) quarks. For "heavy-light" combinations CMM corrections diminish with the increase of the heavy quark mass (See Tables II,III). In the derivation of HQS relations \((8.12)\) they could have been neglected. However their presence, as in \((7.6)\), does not spoil HQS character of the model.

A BCCM is based on a static quark model (Examples in Ref.s 15-17) with specified boosts \((2.3)\), hyperplane projection \((2.12)\) and overlaps \((2.9)\). After BCCM is formulated all calculations depend only on the parameters of the underlaying static model. Basing BCCM on the MIT/-bag model one uses the usual bag-model parameters [15]. With a harmonic oscillator potential as a starting point one employs only parameters listed in Table I. All form factors \((7.3), (8.4)\), HQS relations \((7.6), (8.12)\), Isgur-Wise function \((8.15)\) etc are obtained by a straightforwad calculation, without any additional "add hoc" assumptions. The results in Table II, excellent \(g_A\) value and reasonable \(\mu_p\) \((5.9)\), are not due to any "fine tuning". Playing with parameters one could "improve" some of those outcomes, which would be pointless, as it does not lead to any new physical insights. Of more fundamental importance could be the selection of the type of central
confinement. It, obviously, pays to select the confinement which best mimicks the real physics. Some idea about the confinement dependence can be obtained by comparison of Fig.’s 2 and 3.

As it is usual with central valence quark models [15-17], BCCM also fails in the description of pion, by not being able to account for its Goldstone-boson character.

BCCM’s can be used to calculate corrections ((7,7), Fig.’s 2 and 3) to the extreme HQS results. However only those corrections which depend on the valence quark dynamics are included. Short distance QCD corrections [14] were not incorporated in BCCM. As the model is formulated in the quantum field formalism (2.5), which can be related to Furry bound state picture [27,42], some estimates of QCD effects might become feasible.

An important characteristic of the class of BCCM’s is that those models describe mesons and baryons within the same formalism. Here BCCM’s were mostly applied to mesons, but calculations of the electromagnetic [3-5] (5.4) and of the semileptonic [3] baryon form factors are equally feasible.
Appendix

The explicit expressions for the components $\phi$ of the momentum eigenstates can be found by using (3.8), (4.3) and Table I. For proton (nucleon) one finds:

\[
\frac{M}{\omega_l} |\phi_P(l, \omega_l)|^2 = \frac{\tilde{\omega}_l}{M} \frac{R_0}{\sqrt{3\pi}} \left\{ C_0 + C_1(\tilde{l})^2 + C_2(\tilde{l})^4 + C_3(\tilde{l})^6 \right\} \tag{A1}
\]

Here

\[
\tilde{\omega} = \frac{P^\mu}{M} ; \quad \tilde{l} = \tilde{\omega} + \frac{\vec{P}(\vec{P}^l)}{M(E + M) - \vec{P} / M \omega_l} \quad \tilde{l}^2 - \tilde{\omega}^2 = M^2
\]

\[
C_0 = 1 - 6c + 20c^2 - \frac{280}{9} c^3
\]

\[
C_1 = \frac{4}{3} [c - \frac{20}{9} c^2 + \frac{140}{9} c^3] R_0^2
\]

\[
C_2 = \frac{16}{27} [c^3 - \frac{14}{3} c^3] R_0^4
\]

\[
C_3 = \frac{64}{729} c^3 R_0^6
\]

\[
c = \frac{\beta^2}{4 + 6\beta^2}
\]

The normalization of the proton component (A1) is

\[
\int d^3l \frac{M^2}{\omega_l^2} |\phi_P(l, \omega_l)|^2 = 1 \tag{A3}
\]

The meson component is

\[
\frac{|\varphi_P(l, \omega_l)|^2}{2\omega_l} = \frac{2(l \cdot P)}{(2\pi)^{3/2} M} R_{ab}^3 \frac{3}{\beta^2 R_{ab}^{1/2}} \left( C_{ab}^{(1)} + C_{ab}^{(2)} \tilde{l} + C_{ab}^{(3)} \tilde{l}^2 \right) \tag{A4}
\]

Here with flavors $a,b$:

\[
\tilde{\omega} = \frac{P^\mu}{M} ; \quad \tilde{l} = \tilde{\omega} + \frac{\vec{P} \cdot \vec{l}}{M(E + M)} - \frac{\vec{P}}{M \omega_l}
\]
\[ C_{ab}^{(1)} = 1 - 3 \left( \frac{c_a}{R_{0a}} + \frac{c_b}{R_{0b}} \right) R_{ab}^2 + 15 \frac{c_a c_b}{R_{0a} R_{0b}} R_{ab}^4 \]

\[ C_{ab}^{(2)} = \left( \frac{c_a}{R_{0a}^2} + \frac{c_b}{R_{0b}^2} \right) R_{ab}^4 - 10 \frac{c_a c_b}{R_{0a} R_{0b}} R_{ab}^6 \]  \hspace{1cm} (A5)

\[ C_{ab}^{(3)} = \frac{c_a c_b}{R_{0a}^2 R_{0b}^2} R_{ab}^8 \]

\[ R_{ab}^2 = \frac{2 R_{0a}^2 R_{0b}^2}{R_{0a}^2 + R_{0b}^2} \]

\[ c_a = \frac{\beta^2_a}{4 + 6 \beta^2_a} \]

The normalization is

\[ \int d^3l \frac{1}{4\omega_l^2} |\varphi_{p(\vec{l}, \omega_l)}|^2 = 1 \]  \hspace{1cm} (A6)
References


Table I. HO model parameters

<table>
<thead>
<tr>
<th>Flavor</th>
<th>m(GeV)</th>
<th>E(GeV)</th>
<th>β</th>
<th>$R_b$ (GeV$^{-1}$)</th>
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<td>0.426</td>
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<td>s</td>
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<td>c</td>
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<td>2.00</td>
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<td>b</td>
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<td>5.221</td>
<td>0.062</td>
<td>1.52</td>
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Table II. Hadron masses in HO model

| Hadron | $\tilde{M}^*$ | $M^*$ | $M_{Exp}^*$ | $M/\tilde{M}$ | $|M-M_{Exp}|/M_{Exp}$ | % |
|--------|--------------|-------|-------------|---------------|---------------------|---|
| $p$    | 1.191        | 0.928 | 0.938       | 0.78          | 1.1                 |   |
| $\Delta$ | 1.365        | 1.138 | 1.236       | 0.83          | 8.0                 |   |
| $\pi$  | 0.679        | 0.329 | 0.139       | 0.48          | –                   |   |
| $\rho$ | 0.910        | 0.677 | 0.770       | 0.74          | 12.1                |   |
| $K$    | 0.817        | 0.528 | 0.498       | 0.65          | 6.0                 |   |
| $K^*$  | 1.019        | 0.798 | 0.892       | 0.78          | 10.5                |   |
| $\eta_c$ | 3.381        | 3.267 | 2.979       | 0.97          | 9.7                 |   |
| $\Psi$ | 3.433        | 3.322 | 3.097       | 0.97          | 7.3                 |   |
| $D^+$  | 1.906        | 1.752 | 1.869       | 0.92          | 6.3                 |   |
| $D^{*+}$ | 2.005        | 1.858 | 2.010       | 0.93          | 7.6                 |   |
| $D_s$  | 2.138        | 1.994 | 1.969       | 0.93          | 1.2                 |   |
| $D_s^*$ | 2.229        | 2.091 | 2.110       | 0.94          | 0.9                 |   |
| $B^+$  | 5.207        | 5.125 | 5.279       | 0.98          | 2.9                 |   |
| $B^{*+}$ | 5.249        | 5.168 | 5.325       | 0.98          | 2.9                 |   |
| $B_s$  | 5.580        | 5.501 | 5.384       | 0.99          | 2.1                 |   |
| $B_s^*$ | 5.620        | 5.541 | 5.431       | 0.99          | 2.0                 |   |

* All masses are in GeV.
Table III. The M1 transition decay widths without ($\tilde{\Gamma}$) and with ($\Gamma$) CMM corrections

<table>
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<tr>
<th>MODE</th>
<th>$\bar{g}(0)(GeV^{-1})$</th>
<th>$\bar{\Gamma}_d(10^{-6}GeV)$</th>
<th>$g(0)(GeV^{-1})$</th>
<th>$\Gamma(10^{-6}GeV)$</th>
<th>$\frac{g(0)-\bar{g}(0)}{\bar{g}(0)}%$</th>
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<td>$\Psi \rightarrow \eta_c \gamma$</td>
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<td>2.041</td>
<td>0.377</td>
<td>2.148</td>
<td>2.6</td>
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<td>$D^{*+} \rightarrow D^+ \gamma$</td>
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<td>0.393</td>
<td>-0.132</td>
<td>0.429</td>
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<td>$D^{0*} \rightarrow D^0 \gamma$</td>
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<td>16.656</td>
<td>0.847</td>
<td>18.164</td>
<td>4.4</td>
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<td>-0.064</td>
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<td>-0.371</td>
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<td>0.293</td>
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Figure captions

**Fig. 1** Vertex for the semileptonic $B \to D$ transitions. Quark lines ($b, c, \bar{d}$), momenta $P_i, j$ and the overlap integral $Z$ are indicated.

**Fig. 2** Predictions for the weak decay form factors in HO based BCCM. Dot-dashed line corresponds to $V$, full line to $A_2$ and $[1 - Q^2/(M_B + M_D)^2]^{-1} A_1$ and the dashed line to $F_1$. HQS limit coincides with the full line.

**Fig. 3** Predictions for the weak decay form factors in MIT-bag based BCCM. Line identification is the same as in Fig. 2.