TOPOLOGICAL DILATONIC SUPERGRAVITY THEORIES

M.M.Leite and V.O.Rivelles*

Instituto de Física, Universidade de São Paulo
C. Postal 20516, CEP 01498-970, São Paulo, SP, Brazil

October, 1994

Abstract

We present a central extension of the \((m,n)\) super-Poincaré algebra in two dimensions. Besides the usual Poincaré generators and the \((m,n)\) supersymmetry generators we have \((m,n)\) Grassmann generators, a bosonic internal symmetry generator and a central charge. We then build up the topological gauge theory associated to this algebra. We can solve the classical field equations for the fields which do not belong to the supergravity multiplet and to a Lagrange multiplier multiplet. The resulting topological supergravity theory turns out to be non-local in the fermionic sector.

* Partially supported by CNPq
1. INTRODUCTION

There is a renewed interest in the study of two dimensional gravity theories. It stems from the fact that string theory predicts corrections to the usual gravity theories. To lowest order (Callan 1985) these corrections need the introduction of a massless dilaton field and a massless antisymmetric tensor. We can then ask what are the modifications introduced by these new fields with respect to the results obtained from Einstein theory. In fact we would like to know whether the conceptual and technical problems met in Einstein theory can be overcome in this new framework. Of particular importance is the process of black hole evaporation (Hawking 1975). In this case it is possible to throw away the technical difficulties of the four dimensional case while retaining the essential features responsible for the back reaction if we consider a simplified version of the model in two dimensions (Callan 1992). When we consider only the graviton and the dilaton fields in this simplified model it turns out that the resulting theory is a topological gravity theory (Verlinde 1991). This topological theory is obtained by gauging the Poincaré algebra in two dimensions. However in this formulation the fields have unusual transformation properties under Poincaré gauge transformations (Verlinde 1992). This can be overcome if we take as the gauge algebra a central extension of the Poincaré algebra where the usual commutator of the translation generators $P_a$ is replaced by $[P_a, P_b] = \epsilon_{ab}Z$ where $Z$ is a central charge (Cangemi 1992). This central extension of the Poincaré algebra has also been considered in the context of WZW models. It was used as a prototype for WZW models with non-semisimple groups. It is remarkable that the corresponding sigma model describes string propagation on a four dimensional gravitational plane wave background (Nappi 1993).

Supersymmetry was introduced in dilaton gravity theories in two dimensions with the purpose of studying the positive energy theorem (Park 1993) and black holes (Nojiri 1993). These formulations are the na"ive supersymmetric extensions of the bosonic theories to a superspace and are not topological. Supergravity was formulated as a topological theory by gauging the groups $OSp(2 | 1)$ and $OSp(2 | 2)$ and the resulting theory was respectively the $N = 1$ and $N = 2$ super de Sitter theories (Montano 1990, Li 1990). Only recently the $(0, 1)$ (Cangemi 1994) and $(1, 1)$ (Rivelles 1994) topological supergravity models were obtained. For the $(0, 1)$ case it was only needed the introduction of the supersymmetry generators for the centrally extended Poincaré algebra while for the $(1, 1)$ case besides the supersymmetry generators it was needed the introduction of another Grassmann generator (which does not generate supersymmetry) and a new bosonic generator (besides the central charge) (Rivelles 1994). Remarkably most of the resulting field equations can be solved. If we keep only the supergravity fields and a supermultiplet of Lagrange multipliers it can be shown (Rivelles 1994) that by eliminating the extra gauge and Lagrange multiplier fields the resulting theory is a supergravity theory non-local in the fermionic sector. Of course the non-local terms can be removed by the introduction of an auxiliary spinor field.

In this paper we present the full extended supersymmetric version of the dilaton gravity theories. We start by presenting the central extension of the $(m, n)$ super-Poincaré algebra. We then build up the quadratic Casimir operator from which a non-degenerated metric on the group can be obtained. The gauge theory for the $(m, n)$ algebra is then written and the proper field equations are solved to obtain the corresponding extended supergravity theory. As in the cases $(0, 1)$ and $(1, 1)$ the resulting supergravity theory

2
turns out to be non-local in the fermionic sector.

The plan of the paper is as follows: in Section 2 we present the centrally extended \((m,n)\) super-Poincaré algebra. In section 3 in order to have a self contained presentation we review the topological formulation of the central extension of the Poincaré algebra. Finally in Section 4 we present the \((m,n)\) topological supergravity theory based on the gauging of the algebra obtained in Section 1. We also solve some of the field equations and discuss the gauge fixing needed to obtain a supergravity theory involving only the supergravity fields and a multiplet of Lagrange multipliers. In Section 4 we comment on some possible applications of the \((m,n)\) algebra.

2. CENTRAL EXTENSION OF THE \((m,n)\) SUPER-POINCARÉ ALGEBRA

The central extension of the Poincaré algebra, which is peculiar to two dimensions, is given by (Cangemi 1992)

\[
[P_a, P_b] = \epsilon_{ab} Z
\]

\[
[J, P_a] = \epsilon_a^b P_b
\]

\[
[P_a, Z] = [J, Z] = 0
\]

where \(P_a\) is the translation generator, \(J\) is the Lorentz transformation generator and \(Z\) is a central charge. The flat Minkowski metric is \(h_{ab} = \text{diag}(-1,1)\) and \(e^{01} = 1\). This algebra can be obtained from the De Sitter algebra \(SO(2,1)\) by an unconventional contraction where \(J\) is replaced by \(J + \frac{K}{2}\) and then the limit \(\lambda \to \infty\) is taken (Cangemi 1992). It was also obtained earlier as a generalization of the Inönü-Wigner contraction (Saletan 1961).

The central extension of the \((m,n)\) super Poincaré algebra requires the introduction of the Grassmann generators \(Q^+i, U^-i (i = 1 \ldots m)\), and \(Q^-j, U^+j (j = 1 \ldots n)\), and the bosonic generators \(K^{ij}\). Here \(Q\) is the generator of supersymmetry. They close the following algebra together with Eq.(2.1)

\[
[J, Q_{\alpha}^{+i}] = -\frac{1}{2} Q_{\alpha}^{+i}, \quad [J, Q_{\alpha}^{-i}] = \frac{1}{2} Q_{\alpha}^{-i}
\]

\[
[J, U_{\alpha}^{+j}] = -\frac{1}{2} U_{\alpha}^{+j}, \quad [J, U_{\alpha}^{-j}] = \frac{1}{2} U_{\alpha}^{-j}
\]

\[
[P_a, Q_{\alpha}^{+i}] = \frac{1}{2} (\gamma_a U^{-i})_{\alpha}
\]

\[
[P_a, Q_{\alpha}^{-i}] = -\frac{1}{2} (\gamma_a U^{+j})_{\alpha}
\]

\[
[K_{\alpha}^{ij}, Q_{\alpha}^{+k}] = -\frac{1}{2} \delta^{ik} U_{\alpha}^{+j}
\]

\[
[K_{\alpha}^{ij}, Q_{\alpha}^{-k}] = \frac{1}{2} \delta^{jk} U_{\alpha}^{-i}
\]

3
\[
\{Q^{+i}_\alpha, Q^{+j}_\beta\} = (\gamma^+ \gamma^a C)_{\alpha\beta} \delta^{ij} p_a \\
\{Q^{-i}_\alpha, Q^{-j}_\beta\} = - (\gamma^- \gamma^a C)_{\alpha\beta} \delta^{ij} p_a \\
\{Q^{+i}_\alpha, Q^{-j}_\beta\} = (\gamma^+ C)_{\alpha\beta} k^{ij} \\
\{Q^{-i}_\alpha, U^{+j}_\beta\} = - (\gamma^+ C)_{\alpha\beta} \delta^{ij} Z \\
\{Q^{-i}_\alpha, U^{-j}_\beta\} = (\gamma^- C)_{\alpha\beta} \delta^{ij} Z \\
\{P_a, K^{i,j}\} = [J, K^{i,j}] = 0 \\
\{P_a, U^+_\alpha\} = [P_a, U^-_\alpha] = \{Q^+_\alpha, U^+_\beta\} = \{Q^-_\alpha, U^-_\beta\} = 0
\]

(2.2)

Our conventions are the following: the Dirac gamma matrices are \(\gamma^0 = i\sigma_2, \gamma^1 = \sigma_1, \gamma^2 = \sigma_3\) and \(C = i\sigma_2\) where \(\sigma_i\) are the Pauli matrices; the chiral projections are defined as \(\psi^\pm = \frac{1}{2}(1 \pm \gamma^0)\psi\). The algebra Eq.(2.2) satisfy the graded Jacobi identities.

Previously only the (0, 1) and the (1, 1) cases were known. The minimal supersymmetric version of the extended Poincaré algebra consists in adding just one chiral supersymmetry generator. It is denoted by (0, 1) or (1, 0) algebra depending on the chirality chosen. In (Cangemi 1994) the fermionic generators are \(Q^-\) for supersymmetry and \(U^+\) for the extra fermionic charge. The fermionic generators satisfy the following (anti-)commutation relations besides the ones in Eq.(2.1) *

\[
\{J, Q^-_\alpha\} = \frac{1}{2} Q^-_\alpha, \quad \{J, U^+_\alpha\} = - \frac{1}{2} U^+_\alpha \\
\{P_a, Q^-_\alpha\} = - \frac{1}{2} (\gamma_a U^+_\alpha) \\
\{Q^-_\alpha, Q^-_\beta\} = - (\gamma^- \gamma^a C)_{\alpha\beta} P_a \\
\{Q^-_\alpha, U^+_\beta\} = (\gamma^- C)_{\alpha\beta} Z
\]

(2.3)

When both chiral supersymmetry generators \(Q^+\) and \(Q^-\) are present we can replace them by a single Majorana generator \(Q\). In this case the full algebra needs the introduction of one more bosonic generator \(K\). We also need one more Grassmann generator \(U^\pm\) besides \(U^+\) and like \(Q^\pm\) they are replaced by a single Majorana generator \(U\). They close the (1, 1) algebra (Rivelles 1994)

* The relation between the fermionic generators of (Cangemi 1994) and ours is just \(Q^-_{(ours)} = Q^-_{(Cangemi)}, U^+_{(ours)} = 16iQ^+_{(Cangemi)}, P_a_{(ours)} = 4iP_a_{(Cangemi)}\) and \(Z_{(ours)} = 16Z_{(Cangemi)}\) besides the opposite convention for the sign of \(\epsilon^{ab}\).
\[ [J, Q_\alpha] = -\frac{1}{2}(\gamma_2 Q)_\alpha, \quad [J, U_\alpha] = -\frac{1}{2}(\gamma_2 U)_\alpha \]

\[ [P_\alpha, Q_\alpha] = \frac{1}{2}(\gamma_\alpha U)_\alpha \]

\[ [K, Q_\alpha] = -\frac{1}{2}(\gamma_2 U)_\alpha \]

\[ \{Q_\alpha, Q_\beta\} = (\gamma^a C)_{\alpha\beta} P_a - (\gamma_2 C)_{\alpha\beta} K \]

\[ \{Q_\alpha, U_\beta\} = - (\gamma_2 C)_{\alpha\beta} Z \] (2.4)

Both algebras Eqs.(2.3) and (2.4) can be obtained by an unconventional contraction of the super De Sitter algebra in a very similar way as the algebra Eq.(2.1) was obtained from de De Sitter group (Rivelles 1994).

3. TOPOLOGICAL GRAVITY THEORIES

In this section we will formulate the topological gravity theory on the algebra Eq.(2.1). For simplicity we will consider only the bosonic case. The topological supergravity theory will be discussed in the next section.

In two dimensions a topological theory has the action

\[ S = \int Tr(\eta F) \] (3.1)

where \( F \) is the two-form field strength \( F = dA + A^2 \), \( A \) is the one-form gauge potential and \( \eta \) is a zero-form Lagrange multiplier in the coadjoint representation of the algebra. The action Eq.(3.1) is invariant under the usual gauge transformations

\[ \delta A^a = D\theta^a \equiv d\theta^a + f^a_{\ bc} A^b \theta^c \]

\[ \delta \eta_a = -f^c_{\ ab} \eta_b \theta^c \] (3.2)

where \( \theta \) is the gauge parameter. The field equations which follow from the action Eq.(3.1) are just \( F = 0 \) and \( D\eta = 0 \) where \( D \) is the appropriate covariant derivative on the algebra as defined in Eq.(3.2).

When we consider the algebra Eq.(2.1) we can expand the gauge potential in terms of the generators of the algebra

\[ A = e^a P_a + w J + AZ \] (3.3)

The fields \( e^a, w \) and \( A \) are going to be identified with the zweibein, the spin connection and an abelian gauge field, respectively. The Lagrange multiplier \( \eta \) can be expanded as

\[ \eta = \eta^a P_a + \eta J + \eta' Z \] (3.4)
where $\eta^a$ will turn out to be an auxiliary field, $\eta$ the dilaton and $\eta'$ the cosmological constant.

In order to write the action Eq.(3.1) we need to know the metric on the algebra. It can be read off from the quadratic Casimir operator. For the algebra Eq.(2.1) the quadratic Casimir operator is

$$C^{(2)} = P^a P_a + ZJ + JZ$$

from which we get the nondegenerated metric

$$<P_a, P_b> = h_{ab}, \quad <J, Z> = 1$$

Then the action Eq.(3.1) can be written as

$$S = \int [\eta_a F^a(P) + \eta F(J) + \eta' F(Z)]$$

The field equations which follow from $F = 0$ are just

$$De^a \equiv de^a + we^b e^a_{b} = 0$$

$$dw = 0$$

$$dA + \frac{1}{2} e^a e^b e_{ab} = 0$$

and those following from the variation of the gauge potential, i.e. $D\eta = 0$, are

$$D\eta_a + \eta' e_{a b} e^b = 0$$

$$d\eta + e^b e^a_{b} \eta_a = 0$$

$$d\eta' = 0$$

Notice that some field equations are just algebraic equations. Then Eq.(3.8a) can be solved for $w$ and gives the usual expression for the spin connection. Eq.(3.9b) can be solved for $\eta_a$. The field equation for $\eta'$ Eq.(3.9c) is a first order differential equation which can be solved locally and it states that $\eta'$ is a constant which we are going to identify with the cosmological constant $\Lambda$. After substituting these solutions of the field equations back into the action Eq.(3.7) we obtain the usual form for the dilaton gravity action

$$S = \int (\eta R + \frac{\Lambda}{2} e^a e^b e_{ab})$$

where $R$ is the curvature scalar. The action Eq.(3.10) should be complemented by the field equation Eq.(3.8c) for the abelian gauge field $A$. Eq.(3.8c) can be solved once we have a solution for the gravitational sector.
4. \((m, n)\) TOPOLOGICAL DILATONIC SUPERGRAVITY

We can now proceed in an analogous way to obtain the \((m, n)\) topological dilatonic supergravity theory using the central extension of the \((m, n)\) super-Poincaré algebra Eq.(2.2). We first find the quadratic Casimir operator

\[
C^{(2)} = P_a P^a - K^{iJ} K_{iJ} + JZ + ZJ + \frac{1}{2} C^{\alpha\beta}(Q^+_{\alpha} U^-_{\beta} + U^+_{\alpha} Q^-_{\beta} + U^+_{\alpha} Q^-_{\beta} + Q^+_{\alpha} U^-_{\beta})
\]

from which we can read off the nondegenerated metric

\[
< P_a, P_b > = h_{ab}, \ < K^{iJ}, K_{iJ} >= -\delta^{ij} \delta^{IJ}, \ < J, Z >= 1,
\]

\[
< Q^+_{\alpha}, U^-_{\beta} >= \frac{1}{2} C^{\alpha\beta} \delta^{ij}, \ < U^+_{\alpha}, Q^-_{\beta} >= \frac{1}{2} C^{\alpha\beta} \delta^{ij},
\]

\[
< U^+_{\alpha}, Q^-_{\beta} >= \frac{1}{2} C^{\alpha\beta} \delta^{IJ}, \ < Q^-_{\alpha}, U^+_{\beta} >= \frac{1}{2} C^{\alpha\beta} \delta^{IJ}
\]

(4.1)

The gauge potential \(A\) has the following expansion

\[
A = e^a P_a + w J + v^{iJ} K_{iJ} + AZ + \psi^{\alpha+i} Q^+_{\alpha} + \psi^{\alpha-i} Q^-_{\alpha} + \xi^{\alpha+i} U^+_{\alpha} + \xi^{\alpha-i} U^-_{\alpha}
\]

and the field strength

\[
F = F^a(P)P_a + F(J)J + F^{iJ}(K)K_{iJ} + F(Z)Z + \\
+ F^a(Q^+) Q^+_{\alpha} + F^{\alpha i}(Q^-) Q^-_{\alpha} + F^{\alpha i}(U^+) U^+_{\alpha} + F^{\alpha i}(U^-) U^-_{\alpha}
\]

has the following components

\[
F^a(P) = De^a + \frac{1}{2} \psi^{-I}_a \gamma^a \psi^{+I} - \frac{1}{2} \psi^{+I}_a \gamma^a \psi^{-I}
\]

(4.5a)

\[
F(J) = dw
\]

(4.5b)

\[
F^{\alpha i}(Q^+) = D \psi^{\alpha+i} \equiv d \psi^{\alpha+i} - \frac{1}{2} w \psi^{\alpha+i}
\]

(4.5c)

\[
F^{\alpha i}(Q^-) = D \psi^{\alpha-i} \equiv d \psi^{\alpha-i} + \frac{1}{2} w \psi^{\alpha-i}
\]

(4.5d)

\[
F^{\alpha i}(U^+) = D \xi^{\alpha+i} - \frac{1}{2} e^a (\psi^{-I}_a \gamma^a)^\alpha - \frac{1}{2} v^{iJ} \psi^{\alpha+i}
\]

(4.5e)

\[
F^{\alpha i}(U^-) = D \xi^{\alpha-i} + \frac{1}{2} e^a (\psi^{+I}_a \gamma^a)^\alpha + \frac{1}{2} v^{iJ} \psi^{\alpha-i}
\]

(4.5f)

\[
F^{iJ}(K) = dv^{iJ} + \psi^{+i} \psi^{+J}
\]

(4.5g)

\[
F(Z) = dA + \frac{1}{2} e^a e^b \epsilon_{ab} + \psi^{+i} \xi^{+i} - \psi^{-I} \xi^{-I}
\]

(4.5h)
The Lagrange multiplier $\eta$ can be expanded as

$$\eta = \eta^a P_a + \eta J + \eta^{ij} K^{ij} + \eta'' Z + \chi^{\alpha - i} Q_\alpha^{+ i} + \chi^{\alpha + i} Q^{- i}_\alpha + \zeta^{\beta - i} U^{+ i}_\beta + \zeta^{\beta + i} U^{- i}_\beta$$  \hspace{1cm} (4.6)

and the action takes the form

$$S = \int [\eta_a F^a (P) + \eta F (J) + \eta^{ij} F (K^{ij}) + \eta'' F (Z) +$$

$$+ \chi^{+ i}_\alpha F^{\alpha + i} (Q) + \chi^{- i}_\alpha F^{\alpha - i} (Q) + \zeta^{+ i}_\alpha F^{\alpha + i} (U) + \zeta^{- i}_\alpha F^{\alpha - i} (U)]$$  \hspace{1cm} (4.7)

As in the pure bosonic case we can solve some of the field equations and remain only with the supergravity fields $\epsilon^a$ and $\psi^\pm$ and a multiplet of Lagrange multipliers $\eta$ and $\chi^\pm$. By varying the action Eq.(4.7) with respect to the gauge fields we obtain

$$D\eta_a + \eta'' \epsilon_{a b} \epsilon^b + \frac{1}{2} \psi^{- i} \gamma_a \zeta^{+ i} - \frac{1}{2} \psi^{+ i} \gamma_a \zeta^{- i} = 0$$  \hspace{1cm} (4.8a)

$$d\eta + \epsilon^b e^a_b \eta_a + \frac{1}{2} \psi^{+ i} \chi^{+ i} - \frac{1}{2} \psi^{- i} \chi^{- i} + \frac{1}{2} \xi^{+ i} \xi^{+ i} - \frac{1}{2} \xi^{- i} \xi^{- i} = 0$$  \hspace{1cm} (4.8b)

$$d\eta^{ij} + \frac{1}{2} \psi^{+ i} \chi^{+ j} - \frac{1}{2} \psi^{- j} \zeta^{- i} = 0$$  \hspace{1cm} (4.8c)

$$d\eta'' = 0$$  \hspace{1cm} (4.8d)

$$D\chi^{+ i}_\alpha + \eta_a (\gamma^a \psi^{- i})_\alpha - \eta'' \xi^{+ i}_\alpha = \frac{1}{2} \epsilon^a (\gamma_a \zeta^{- i})_\alpha + \frac{1}{2} \psi^{+ i} \xi^{+ i}_\alpha + \eta^{ij} \psi^{- i}_\alpha = 0$$  \hspace{1cm} (4.8e)

$$D\chi^{- i}_\alpha - \eta_a (\gamma^a \psi^{+ i})_\alpha + \eta'' \xi^{- i}_\alpha + \frac{1}{2} \epsilon^a (\gamma_a \zeta^{+ i})_\alpha - \frac{1}{2} \psi^{+ i} \xi^{- i}_\alpha + \eta^{ij} \psi^{- i}_\alpha = 0$$  \hspace{1cm} (4.8f)

$$D\zeta^{+ i}_\alpha - \eta'' \psi^{+ i}_\alpha = 0$$  \hspace{1cm} (4.8g)

$$D\zeta^{- i}_\alpha + \eta'' \psi^{- i}_\alpha = 0$$  \hspace{1cm} (4.8h)

Some of the field equations for the gauge fields can also be solved. Setting $F (P) = 0$ in Eq.(4.5a) we can solve algebraically for $w$ obtaining the usual supersymmetric spin connection

$$w = - (det \epsilon)^{-1} \epsilon^a \epsilon^{\mu \nu} (\partial_\nu \epsilon^b \epsilon_a b - \frac{1}{2} \psi_{\mu}^{+ i} \gamma_a \psi_{\nu}^{- i} + \frac{1}{2} \psi_{\mu}^{- i} \gamma_a \psi_{\nu}^{+ i})$$  \hspace{1cm} (4.9)

Then Eqs.(4.5e, f, g) can be solved for $\xi^{+ j}, \xi^{- i}$ and $v^{ij}$,

$$v^{ij} = - \frac{1}{2 \eta''} (\psi^{+ i} \xi^{+ j} + \psi^{- j} \xi^{- i})$$

$$\xi^{+ j} = - \frac{1}{2 \eta''} \epsilon^a (\gamma^a \psi^{- j})_\alpha - \frac{1}{(4 \eta'')} \xi^{- j} \eta'' \psi^{+ i}_\alpha$$

$$\xi^{- i} = - \frac{1}{2 \eta''} \epsilon^a (\gamma^a \psi^{+ i})_\alpha + \frac{1}{(4 \eta'')} \xi^{+ j} \eta'' \psi^{- i}_\alpha$$
\[ \zeta^{a-i} = -\frac{1}{2\eta''}e^{a}(\zeta^{+i}\gamma_{a})^{a} - \frac{1}{4(\eta'')^{2}}\zeta^{+i}\zeta^{-j}\psi^{a-j} \]  (4.10)

From the field equations for the Lagrange multipliers Eqs.(4.8a, c, d) we can solve for \( \eta_{a}, \eta^{iJ} \) and for \( \eta'' \). The solution for \( \eta'' \) is just \( \eta'' = \text{constant} \) which can be later identified with the cosmological constant, and

\[ \eta_{a} = \epsilon_{a}^{b}e_{b}^{\mu}(-\partial_{\mu}\eta - \frac{1}{2}\psi^{+i}\chi^{+i} + \frac{1}{2}\psi^{-j}\chi^{-j} - \frac{1}{2}\zeta^{+j}\zeta^{+j} + \frac{1}{2}\zeta^{-i}\zeta^{-i}) \]

\[ \eta^{iJ} = \frac{1}{2\eta''}\zeta^{+i}\zeta^{-j} \]  (4.11)

By substituting the solutions Eqs.(4.9-11) into the action Eq.(4.7) we obtain the effective action for the \((m, n)\) extended supergravity

\[ S = \int(\eta dw + \chi^{+i}_{a} D\psi^{a+i} + \chi^{-l}_{a} D\psi^{a-l} + \frac{1}{2}\eta''e^{a}e^{b}\epsilon_{ab} + \]

\[ + \frac{1}{2}\epsilon^{a}\psi^{-j}\gamma_{a}\zeta^{+j} - \frac{1}{2}\epsilon^{a}\psi^{+i}\gamma_{a}\zeta^{-i} + \frac{1}{2}\eta''e^{a}\zeta^{+j}\psi^{a-j}\psi^{a-j}) \]  (4.12)

In order to find the supergravity transformations which leave the action Eq.(4.12) invariant some care must be taken. Since we have solved the field equations for some of the gauge fields the gauge transformations Eq.(3.2) associated to these gauge fields must be fixed. Taking the solutions Eqs.(4.10) and (4.11) for \( v, \eta \) and \( \xi \) we can find that under a gauge transformation Eq.(3.2) generated by \( Q \) they transform with a supersymmetry transformation with parameter \( \epsilon \) plus a gauge transformation generated by \( \xi \) with parameter \( \epsilon' = 0 \) plus a gauge transformation generated by \( v \) with parameter \( \alpha^{iJ} = -\frac{1}{2\eta''}(\epsilon^{+i}\zeta^{+j} + \epsilon^{-j}\zeta^{-i}) \). Since \( \eta \) and \( \chi \) have a non-trivial gauge transformation generated by \( v \) and \( \xi \) the resulting supergravity transformation is a sum of a supergravity transformation with parameter \( \epsilon \) plus a gauge transformation generated by \( \xi \) with parameter \( \epsilon' \) plus a gauge transformation generated by \( v \) with a parameter \( \alpha \) with the values of \( \epsilon' \) and \( \alpha \) given above. The supergravity transformation on the remaining fields is the same as the ones generated by \( Q \). We then find

\[ \delta e^{a}_{\mu} = \epsilon^{+i}\gamma^{a}\psi^{-i}_{\mu} - \epsilon^{-l}\gamma^{a}\psi^{+l}_{\mu} \]

\[ \delta \psi^{a+i} = D\epsilon^{a+i} \]

\[ \delta \psi^{a-l} = D\epsilon^{a-l} \]

\[ \delta \chi^{a+i} = -\frac{1}{2\eta''}\zeta^{+i}\zeta^{+j}e^{+j}_{\alpha} - \eta_{a}\gamma^{a}\epsilon^{-i}_{\alpha} \]

\[ \delta \chi^{a-l} = -\frac{1}{2\eta''}\zeta^{+j}\zeta^{+j}e^{-i}_{\alpha} + \eta_{a}\gamma^{a}\epsilon^{+i}_{\alpha} \]

\[ \delta \zeta^{-i} = -\eta''\epsilon^{-i} \]
\[
\delta \zeta^+_\alpha = \eta'' \epsilon^+_\alpha \\
\delta \eta = -\frac{1}{2} \epsilon^+ \chi^+_i + \frac{1}{2} \epsilon^- \chi^-_i
\]  
(4.13)

where \( \eta_\alpha \) is given by Eq.(4.11). These transformations leave the action Eq.(4.12) invariant and close on-shell on a local translation, a local Lorentz transformation and a supergravity transformation.

Notice that in order to have just a set of supergravity fields we should eliminate \( \zeta^\pm \) from the action Eq.(4.12). By doing that the action would become non-local in the fermionic sector so we choose to keep the auxiliary field \( \zeta^\pm \). Then we have a new version for supergravity in two dimensions which differ, in the case (1,1), from the ones knew before (Nojiri 1993, Park 1993) as discussed in (Rivelles 1994).

5. CONCLUSIONS

We have presented the \((m,n)\) supersymmetric version of the extended Poincaré algebra. In order to obtain it we had to introduce a new Grassmann generator \( U \) and a new bosonic generator \( K \) besides the supersymmetry generator \( Q \). We then build up in the usual way the corresponding supergravity theory. When we eliminated the extra gauge and Lagrange multiplier fields the remaining supergravity fields have an action which is non-local in the fermionic sector.

As the extended Poincaré algebra has been used as a prototype for the study of non-semisimple WZW models the \((m,n)\) algebra presented in this paper can be used as a prototype for the supersymmetric case. It would be possible then to find out exact string backgrounds which are supersymmetric.

Also the non-local supergravity model presented in Section 4 should give rise to some new class of induced supergravity theories. Since the non-locality appears only in the fermionic sector it would be very interesting to find out the relation of this model to the know induced supergravity models. Other aspects, like the relation to supersymmetric matrix models, properties of black holes and the quantization of the model remain to be investigated.

Acknowledgments

MML would like to thank FAPESP for financial support.
REFERENCES

Verlinde H 1992 String Theory and Quantum Gravity ’91, Eds. Harvey J, Iengo R,
Narain K S and Verlinde H, p.178 (World Scientific, Singapore)