Lattice QCD Solution to the U(1) Problem

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\textbf{Abstract}

It is shown in quenched lattice QCD that the mass splitting between $\eta'$ and pion arises from gauge configurations with non-zero topological charge $Q$, its magnitude increasing for larger values of $|Q|$; the contribution from disconnected quark loop is strongly hindered unless topological charge is excited. This demonstrates the explicit relation between the large $\eta'$ meson mass and gauge field topology, which is in line of the argument in the continuum of instantons and $1/N$ expansion.

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The U(1) problem has been one of the most outstanding problems in hadron physics. Glashow noted the inevitable existence of an isoscalar $\phi^0$ meson with mass of the order of pion mass in the presence of chiral SU(2) symmetry[1]. Weinberg has then shown that this isoscalar meson mass has to be smaller than $\sqrt{3}$ times $m_\pi$, which, however, conflicts with the experiment[2]. A naive idea to explain a large mass of $\eta$ (or $\eta'$ in the case with SU(3)) is that $\eta\eta$ annihilation into gluons contribute to boosting the flavur-singlet meson mass[3]: i.e., the OZI rule is largely violated for the pseudoscalar channels. An objection to this idea was that this process does not violate the conservation of axial U(1) current, and the argument of Weinberg remains intact. An important observation, however, is that the conservation is actually broken due to the presence of anomaly, when vacuum fluctuations of non-zero topological charge are excited, and this gives rise to a mass of the $\eta'$ meson[4]. This argument has been developed by Witten[5] and Veneziano[6]: they have derived a Ward identity in the presence of anomaly within the framework of 1/N expansion, and have predicted the mass of $\eta'$ which is close to the experiment. Whether this picture is valid in our world is still a remaining issue to be understood within QCD. Furthermore, the dynamical mechanism of topological fluctuations that boosts the $\eta'$ mass is not clear in their argument.

In the previous publication[7] we have calculated the $\eta'$ meson mass in the quenched approximation of lattice QCD, and have shown that $\eta'$ indeed has a mass much larger than the pseudoscalar octet as the chiral limit is approached. This successful prediction lends us hope that the origin of a large $\eta'$ mass could be clarified in a further study. A large $\eta'$-octet mass splitting arises if disconnected two quark loop amplitude, which contributes only to the $\eta'$ propagator, substantially cancels the single quark loop contribution at large time separations so that the $\eta'$ propagator decays faster than the octet propagator. It has been shown in earlier quenched lattice QCD studies that the two loop amplitude becomes significantly enhanced towards light quarks [8,9] and that the enhancement of the two loop amplitude is due to gauge configurations of non-zero topological charge [9]. In the present letter, we demonstrate that the $\eta'$-octet mass splitting calculated for gauge configurations having a given topological charge increases with the value of the topological charge, thereby establishing a direct connection between the mass of $\eta'$ meson and gauge field topology. This appears to be the first case where the role of instantons is compelling to understand the experiment.

The formalism and method of calculation is parallel to those given in Ref. [7]. We extract the splitting between $\eta'$ and pseudoscalar octet mass squared $m_\eta^{2} = m_{\eta'}^{2} - m_{\rho}^{2}$ from the ratio of the disconnected two quark loop amplitude to the single quark-loop amplitude, each projected onto the zero momentum state, defined by

$$R(t) = \frac{\langle \eta'(t)\eta(0)\rangle_{\text{disconnect}}}{\langle \eta(t)\eta(0)\rangle_{\text{connect}}}$$

The two-loop contribution is evaluated by the variant wall source technique developed in Ref. [10]. At the same time we calculate the topological charge,

$$Q = \frac{1}{32\pi^{2}} \sum_{n} \text{Re}Tr\left[U_{\mu\nu}(n)\bar{U}_{\mu\nu}(n)\right],$$

where $U_{\mu\nu}(n)$ is the plaquette in the $\mu\nu$ plane at site $n$, by applying the cooling method [11]. We then assemble gauge configurations with the same absolute magnitude of the topological charge $|Q|$ and calculate $R(t;|Q|)$, from which we extract $m_0$ as a function of $|Q|$.

Our calculations are made at $\beta = 6/(g^2) = 5.7$($a = 0.14$fm) on a $12^3 \times 20$ lattice in quenched lattice QCD, employing the Wilson quark action with $K = 0.1665(m_{\pi}/m_{\rho} = 0.59)$. We analyzed 240 gauge configurations, each 1000 pseudo heat-bath sweeps apart. Twenty-five sweeps are made to cool down the configurations in order to calculate topological charge.

Figure 1 shows the distribution of topological charge. The values actually measured on the lattice are not necessarily integers, since we are not working at a coupling sufficiently close to the continuum limit and hence $Q$ defined in
(2) receives a finite lattice correction of O(a^3). Defining \( \bar{Q} \) to be the value of \( Q \) rounded off to the nearest integer, we find that \( (Q - \bar{Q})/\bar{Q} \) shows a Gaussian-like distribution with a full width half maximum of 0.11 (see Fig. 2; we have excluded configurations with \( \bar{Q} = 0 \) for this figure).

In Fig. 3 we present \( R(t; \bar{Q}) \) for typical values of \( \bar{Q} \): \( \bar{Q} = 0, 2, 5 \). For the one quark-loop amplitude in the denominator we used the average over the whole ensemble since it shows little variation depending on \( \bar{Q} \). We see that the function \( R(t; \bar{Q}) \) is consistent with zero for \( \bar{Q} = 0 \), and it shows a clear increase as \( \bar{Q} \) increases. This demonstrates that the effect of instantons generates the two-loop contribution; the contribution from the two-loop diagram is strongly suppressed unless topological charge is excited. A large mass of \( \eta' \) does not arise merely from \( \bar{q}q \) annihilation into gluons; this suggests that the prototype OZI rule holds well even for the \( 0^- \) channel if the topological effect could be switched off. We note in this context that the validity of the OZI rule has been demonstrated for \( n^- \) scattering where the quark annihilation amplitude in the \( f = 0 \) channel with purely gluonic intermediate states has been seen to be very small compared to other contributions[10].

The dependence of the \( \eta' \)-octet mass splitting \( m_{\eta'} \) on \( \bar{Q} \) is presented in Fig. 4 in lattice units [12]. We have extracted \( m_{\eta'} \) by fitting the data to the form,

\[
R(t; \bar{Q}) = \frac{m_{\eta'}}{2m_0} \left( t + \text{const.} \right).
\]

where the value of \( m_0 \) is taken from a single-exponential fitting of the pion propagator calculated for the entire ensemble. The fitting range is chosen to be \( 4 \leq t \leq 8 \) where the cutoff at a small \( t \) is made in order to avoid a possible contamination from higher excited states in agreement with the analysis for the pion propagator that indeed exhibits such a contribution. We set the constant term to be zero following Ref. [7]. As Fig. 4 shows, \( m_{\eta'} \) increases with \( \bar{Q} \). The mass splitting \( m_{\eta'} \) calculated from the entire ensemble is 0.379(13), or, using the physical scale, 550 MeV at \( m_{\eta'}/m_{\eta} = 0.39 \) for this figure.

We have repeated the analysis at two other values of the hopping parameter \( K = 0.164(m_{\eta'}/m_{\eta} = 0.74), 0.165(0.79) \) on the \( 12^3 \times 20 \) lattice, and also at \( K = 0.160(m_{\eta'}/m_{\eta} = 0.42) \) on an \( 10^3 \times 20 \) lattice with 49 configurations. The connection between topological charge and the \( \eta' \)-octet mass splitting as observed in Figs. 3 and 4 is also seen at these values of \( K \). An increasingly rapid increase of \( m_{\eta'}(\bar{Q}) \) for non-zero \( \bar{Q} \) toward light quark masses, expected from the existence of the associated fermion zero modes, was not clearly observed, however, probably due to still heavy quarks \( (m_{\eta'}/m_{\eta} \geq 0.42) \) employed in the analysis.

In sum we clarified the issue concerning the U(1) problem: it is the dynamical role of instantons that boosts the \( \eta' \) mass, which leads to an apparent large violation of the OZI rule in the pseudoscalar channel. We remark that our results are obtained in the quenched approximation. In the presence of dynamical sea quarks topologically non-trivial gauge configurations are suppressed towards vanishing quark mass due to fermionic zero modes. It remains an intriguing problem to investigate how the \( \eta' \) mass survives the suppression in full QCD. The mechanism may be subtle involving a balance between the suppression and an increase of \( \eta' \) mass for larger values of topological charge as was shown in the present work.

A numerical elucidation of this effect however, will require very long runs since full QCD simulations carried out to date exhibit long-range correlations in the fluctuation of topological charge [13, 14].

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References

12. The flavour factor $\sqrt{N_f} = \sqrt{3}$ is included for $m_q$ in this figure to facilitate comparison with experiment.

Figure Captions

Fig. 1: Distribution of topological charge $Q$ in quenched lattice QCD at $\beta = 5.7$ on a $12^3 \times 20$ lattice. Bins are centered at integer values with unit width.

Fig. 2: Distribution of $(Q - \bar{Q})/\bar{Q}$ with $\bar{Q}$ the nearest integer value of $Q$. Configurations with $\bar{Q} = 0$ are excluded.

Fig. 3: Representative values of the ratio $R(t; \bar{Q})$ of two- and single-quark loop contributions to the $q'$ propagator for several values of $\langle \bar{Q} \rangle$ at $\beta = 5.7$ and $K = 0.1665$ on a $12^3 \times 20$ lattice.

Fig. 4: $q'$-octet mass splitting $m_q$ in lattice units as a function of $\langle \bar{Q} \rangle$ at $\beta = 5.7$ and $K = 0.1665$ on a $12^3 \times 20$ lattice in quenched QCD. The horizontal line represents the value for the entire ensemble.