Heat Kernel and Scaling of Gravitational Constants

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Abstract

We consider the non-local energy-momentum tensor of quantum scalar and spinor fields in 2w-dimensional curved spaces. Working to lowest order in the curvature we show that, while the non-local terms proportional to $\mathcal{R}$, $\mathcal{R}$, $\ldots$, $w^{-2}\mathcal{R}$ are fully determined by the early-time behaviour of the heat kernel, the terms proportional to $\mathcal{R}$ depend on the asymptotic late-time behaviour. This fact explains a discrepancy between the running of the Newton constant dictated by the RG equations and the quantum corrections to the Newtonian potential.
I. INTRODUCTION

In a recent paper [1] we have computed the corrections to the Newtonian potential due to a quantum massive scalar field coupled to the metric in a $R^2$-theory of gravitation (for a similar calculation see also [2]). This computation was carried out by means of a non-local approximation to the Effective Action (EA) [3,4], from which the effective gravitational equations of motion were deduced. Expanding in powers of $-\frac{m^2}{\mu^2}$ they read

\[
\left[ \alpha_0 - \frac{1}{64\pi^2} \left( \xi - \frac{1}{6} \right)^2 - \frac{1}{90} \right] \ln\left(-\frac{m^2}{\mu^2}\right) H^{(1)}_{\mu\nu} + \left[ \beta_0 - \frac{1}{1920\pi^2} \ln\left(-\frac{m^2}{\mu^2}\right) \right] H^{(2)}_{\mu\nu} + \\
\left[ -\frac{1}{8\pi G} + \frac{m^2}{16\pi^2} \left( \xi - \frac{1}{6} \right) \ln\left(-\frac{m^2}{\mu^2}\right) \left( R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) \right] - \frac{m^2}{384\pi^2} \ln\left(-\frac{m^2}{\mu^2}\right) \left[ (1 - 12\xi^2) H^{(1)}_{\mu\nu} - 2H^{(2)}_{\mu\nu} \right] = O(\mathcal{R}^2)
\]  

(1)

where

\[
H^{(1)}_{\mu\nu} = 4\nabla_\mu \nabla_\nu R - 4g_{\mu\nu} R + O(\mathcal{R}^2)
\]

\[
H^{(2)}_{\mu\nu} = 2\nabla_\mu \nabla_\nu R - g_{\mu\nu} R - 2R_{\mu\nu} + O(\mathcal{R}^2)
\]  

(2)

One alternative and natural way to evaluate the quantum corrections to the classical Einstein equations is to replace the classical gravitational parameters by their running counterparts, which are given by the renormalization group equations (RGEs). These are basically obtained from the Schwinger-DeWitt (SDW) coefficients and read [5]

\[
\mu \frac{d\alpha_0}{d\mu} = -\frac{1}{32\pi^2} \left[ \left( \frac{1}{6} - \xi \right)^2 - \frac{1}{90} \right]
\]

(3)

\[
\mu \frac{d\beta_0}{d\mu} = -\frac{1}{960\pi^2}
\]

(4)

\[
\mu \frac{dG}{d\mu} = \frac{G_0^2 m^2}{\pi} \left( \xi - \frac{1}{6} \right)
\]

(5)

When implemented in configuration space, this ‘Wilsonian’ procedure implies the replacement $\alpha \to \alpha + const \times \ln(-)$, where $\alpha$ denotes a general gravitational parameter and the value of the constant is obtained from the RGEs. Comparing the result of this procedure
with Eqn(1) one readily notes that, while the corrections proportional to \(\ln(-\frac{\mu}{r})\), interpreted as modifying \(\alpha_0\) and \(\beta_0\), have the same numerical coefficients as those deduced from the Wilsonian approach, this is not the case for the Newton constant. Indeed, because of the identity

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{4} (H^{(1)}_{\mu\nu} - 2H^{(2)}_{\mu\nu}) + O(R^2)
\]  

the non-analytic corrections proportional to \(-\frac{m^2}{r}\) can be interpreted as modifying \(G\) only for \(\xi = 0\) (see also [6]).

This discrepancy can also be seen at the level of the Newtonian potential, which has \(\frac{\log r}{r}\) and \(r^{-3}\) quantum corrections [1]. On the one hand, one could interpret this result as inducing an effective Newton constant that varies with the distance, \(V(r) = -G(r)M/r\). On the other hand, one can make once again a Wilsonian argument, replacing in the classical potential \(V_d(r)\) the Newtonian constant by its running counterpart and identifying \(\mu \leftrightarrow r^{-1}\). In this fashion, one can also obtain a modified Newtonian potential \(V(r) = -G(\mu = r^{-1})M/r\), but it does not show a \(r^{-3}\) dependency. As regards the logarithmic term, both potentials show a similar behaviour but their respective coefficients coincide only for minimal and conformal coupling. \(^1\)

The aim of this work is to elucidate the origin of the discrepancy between the scaling behaviour of the Newton constant deduced from the effective equations of motion and that obtained through the RGEs. To this end we will show that there is a qualitative difference between the non-local corrections proportional to \(\ln(-)\) and those proportional to \(-\frac{m^2}{r}\ln(-)\). While the former are linked to the early-time behaviour of the heat kernel [7] (and consequently are determined by the \(\hat{a}_2\) SDW coefficient), the latter depend on the late-time behaviour and produce the above-mentioned discrepancy. We will prove this claim in Section II, where we will also extend the four-dimensional results to arbitrary dimensions. In Section III we will analyze the same problem for spinor fields.

\(^1\)The coincidence at \(\xi = 1/6\) takes place only after tracing the equations of motion.
II. SCALING FOR SCALAR FIELDS

Let us consider the evaluation of the one-loop contribution of a massive quantum scalar field to the gravitational EA

$$\Gamma = \frac{1}{2} \ln \det \left( -m^2 + \xi R \right)$$

(7)

The task of evaluating this functional determinant on an arbitrary background is quite complicated and approximation methods are compelling. Using the early-time expansion of the heat kernel, the EA reads [3,4,7]

$$\Gamma = -\frac{1}{2} \lim_{L^2 \to \infty} \frac{1}{(4\pi)^w} \int_{1/L^2}^{\infty} \frac{ds}{s^{w+1}} \exp(-sm^2) \sum_{n=0}^{\infty} s^n \int d^3 x \sqrt{\tilde{g}} \hat{a}_n(x)$$

(8)

where the ultraviolet divergence is regularized by the introduction of a positive lower limit in the proper-time integral. Here all the functions $\hat{a}_n(x)$ are the coincident limit of the SDW coefficients.

As suggested by Vilkovisky [7], when the background fields are weak but rapidly varying, one can obtain a non-local expansion of the EA that is well-behaved in the massless limit, namely

$$\Gamma = -\frac{1}{2} \lim_{L^2 \to \infty} \frac{1}{(4\pi)^w} \int d^3 x \sqrt{\tilde{g}} \left( h_0 + h_4 \left( \frac{1}{6} - \xi \right) R + R \left[ \int_{1/L^2}^{\infty} ds \frac{e^{-sm^2}}{s^{w-1}} f_1(-s) \right] \right) R +$$

$$R_{\mu\nu} \left[ \int_{1/L^2}^{\infty} ds \frac{e^{-sm^2}}{s^{w-1}} f_2(-s) \right] R_{\mu\nu} + O(R^3)$$

(9)

where $h_n = \int_{1/L^2}^{\infty} ds s^{n-w-1} e^{-sm^2}$ and the form factors $f_i$ are functions of the operator $-s \equiv \eta$ to be defined afterwards.

Up to here no assumptions about the mass $m$ have been made. In the large mass limit, $m^2 \mathcal{R} \gg \nabla \nabla \mathcal{R}$, the SDW expansion is recovered, while in the opposite one, the form factors can be expanded in powers of $z \equiv -m^2$. We shall be working in the latter limit. We have to evaluate the integral

$$I_w \overset{\text{def}}{=} \lim_{L^2 \to \infty} \int_{1/L^2}^{\infty} ds \frac{e^{-m^2 s}}{s^{w-1}} \sigma(-s)$$

(10)
where $\sigma$ denotes generically the $f_i$’s. In order to study the behaviour of $I_w$ in terms of the small quantity $z$, we split up the integral into two terms

$$I_w = \lim_{L^2 \to \infty} (A_w + B_w)$$

$$A_w = (-)^{w-2} \int_{-L^2}^{L^2} \frac{d\eta}{\eta^{w-1}} e^{-\eta z} \sigma(\eta)$$

$$B_w = (-)^{w-2} \int_{L^2}^\infty \frac{d\eta}{\eta^{w-1}} e^{-\eta z} \sigma(\eta)$$

(11)

where $C$ is chosen such that $z^{-1} \gg C \gg 1$. Let us analyze the two integrals separately.

For the $A_w$ integral, one can use the Taylor expansion of the form factor, namely $\sigma(\eta) = \sum_{n=2}^\infty \sigma_n \eta^{n-2}$. The constants $\sigma_n$ can be read from the corresponding SDW coefficient $\hat{a}_n$, as follows from Eqns(8,9). The $n \geq w + 1$ terms have a finite $L^2 \to \infty$ limit that gives a $\eta$-dependent contribution that is analytic in the variable $z$, while the $2 \leq n \leq w$ terms are UV divergent. Expanding the exponential in $A_w$ in powers of the small quantity $\eta z$ we obtain its final expression

$$A_w = -(-)^{w-2} \log(-\frac{L^2}{\eta^2}) \sum_{n=2}^w \frac{\sigma_n}{(w-n)!} \eta^{w-n} +$$

$$(-)^{w-2} \sum_{n=2}^w \sum_{k=0}^{w-n-1} \frac{\sigma_n}{(w-n-k)!} \eta^{w-n-k}$$

(12)

where the dots denote finite terms, analytic in the small quantity $-\frac{m^2}{\mu}$. Note that both the divergent and non-analytic parts of $A_w$ are determined by the first $w$ SDW coefficients. In order to renormalize the theory, the infinities have to be cancelled by means of suitable counterterms in the classical lagrangian of the form $R R, R^2 R, R^3 R, \ldots, R^{w-2} R$, these being the only quadratic counterterms that can appear. The UV divergences proportional to $\log(L^2)$ that appear in both $A_w$ and the $h_n$ integrals are absorbed in the bare constants, being renormalized by terms of the form $\log(\frac{L^2}{\mu^2})$, where $\mu$ is an arbitrary arbitrary scale parameter with units of mass. The fact that the EA must not depend on this arbitrary parameter implies that the gravitational constants scale with $\mu$, the scaling being given by the RGEs (see Eqns(3,4,5) for the $w = 2$ case).

As to the $B_w$ integral, its leading behaviour in powers of $-\frac{m^2}{\mu}$ is governed by the asymptotic expansion of the form factor. Assuming that $\sigma(\eta) = \frac{k}{\eta}$ as $\eta \to \infty$, where $k$ is a
numerical factor, the integral $B_w$ reads

$$B_w = k \frac{(-1)^w}{(w-1)!} (-1)^{w-2}(\frac{m^2}{\mu^2})^{w-1} \ln(\frac{m^2}{\mu^2}) + \ldots$$

(13)

the dots being analytic terms.

Given the EA one can derive the effective gravitational field equations. After a straightforward calculation we find

$$\left( -\frac{1}{8\pi G} + \frac{(-1)^w(m^2)^{w-1}}{(4\pi)^w(w-1)!} (\xi - \frac{1}{6} \ln(\frac{m^2}{\mu^2})) (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) + \sum_{j=0}^{w-2} [\alpha_j H^{(1)}_{\mu\nu} + \beta_j H^{(2)}_{\mu\nu}] =
\right.$$

$$< T_{\mu\nu} > \equiv -\frac{1}{2(4\pi)^w} \left[ F_1() H^{(1)}_{\mu\nu} + F_2() H^{(2)}_{\mu\nu} \right] + O(R^2)$$

(14)

where $F_1()$ and $F_2()$ are the form factor integrals appearing in Eqn(9), respectively associated with the $R^2$ and $R_{\mu\nu}R_{\mu\nu}$ terms. In this equation the cosmological constant term has been omitted and $\alpha_j$ and $\beta_j$ denote the gravitational constants associated with the higher order terms in the classical lagrangian.

In four-dimensional spacetime the basic integral $I_w$ can be calculated using Eqns(12,13). Up to analytic terms in $-\frac{m^2}{\mu^2}$ it is given by

$$I_{w-2} = -\sigma_2 Log(-\frac{m^2}{\mu^2}) - k \frac{m^2}{\mu^2} Log(-\frac{m^2}{\mu^2}) + O(-\frac{m^2}{\mu^2})$$

(15)

The corresponding stress tensor reads

$$< T_{\mu\nu} > = \frac{1}{32\pi^2} \left( \log(-\frac{m^2}{\mu^2}) [\sigma_2^{(1)} H^{(1)}_{\mu\nu} + \sigma_2^{(2)} H^{(2)}_{\mu\nu}] + m^2 \log(-\frac{m^2}{\mu^2}) [k^{(1)} H^{(1)}_{\mu\nu} + k^{(2)} H^{(2)}_{\mu\nu}] \right)$$

(16)

the $\sigma_2^{(i)}$ and $k^{(i)}$ being the numerical constants in Eqn(15), respectively associated with the $R^2$ and $R_{\mu\nu}R_{\mu\nu}$ terms in the EA.

The $m^2$-independent terms in $< T_{\mu\nu} >$ can be interpreted as being quantum corrections to the gravitational constants $\alpha_0$ and $\beta_0$. As was already mentioned, the numerical coefficients $\sigma_2^{(i)}$ associated with these corrections are basically given by the $\hat{a}_2$ SDW coefficient (early-time behaviour of the heat kernel). One can express $\hat{a}_2$ in terms of just the Ricci tensor and
scalar \(^2\) and then verify that the \(\sigma^{(i)}\)'s coincide with the numerical coefficients obtained in the RGEs. When the equations of motion are traced and solved, these terms produce \(r^{-3}\) quantum corrections to the Newtonian potential [1].

In an analogous way, one would expect that the \(m^2\)-dependent terms in \(< T_{\mu\nu} >\), namely

\[
\frac{m^2 k^{(1)}}{32\pi^2} \log(-\frac{m^2}{m^2}) - \frac{m^2}{32\pi^2} \left( H_{\mu\nu}^{(1)} + \frac{k^{(2)}}{k^{(1)}} H_{\mu\nu}^{(2)} \right)
\]

(17)
could be expressed in a combination proportional to \(m^2 \log(-\frac{m^2}{m^2}) (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu})\), so that they can be interpreted as a quantum correction to the Newton constant. From Eqn(6) we see that the aforementioned combination comes up only for \(k^{(2)}/k^{(1)} = -2\), a condition that is not always met. Also note that the correction depends on the numerical coefficients \(k^{(i)}\), which are given by the asymptotic late-time behaviour of the heat kernel. The terms in Eqn(17) produce a \(\frac{\log}{r^4}\) correction to the Newtonian potential [1].

The coefficients \(\sigma_n\)'s and \(k\)'s can be evaluated from the form factors \(f_i\)'s. These are defined through the basic form factor \(f(\eta) = \int_0^1 dt e^{-t(1-\eta)}\) as follows [4,8]

\[
f_1(\eta) = \frac{f(\eta)}{8} \left[ \frac{1}{36} + \frac{1}{3\eta} - \frac{1}{\eta^2} \right] - \frac{1}{16\eta} + \frac{1}{8\eta^2} + (\xi - \frac{1}{6}) \left[ \frac{f(\eta)}{12} + \frac{f(\eta) - 1}{2\eta} \right] + \frac{1}{2}(\xi - \frac{1}{6})^2 f(\eta)
\]

\[
f_2(\eta) = \frac{f(\eta) - 1 + \eta/6}{\eta^2}
\]

(18)

From here the relevant coefficients for the four-dimensional theory can be calculated: \(\sigma^{(i)} = f_i(0)\) and \(k^{(i)} = \lim_{\eta \to \infty} \eta f_i(\eta)\). Therefore we have

\[
\sigma^{(1)}_2 = \frac{1}{2} \left[ \left( \frac{1}{6} - \xi \right)^2 - \frac{1}{90} \right], \quad \sigma^{(2)}_2 = \frac{1}{60}, \quad k^{(1)} = \xi^2 - \frac{1}{12}, \quad k^{(2)} = \frac{1}{6}
\]

(19)

It is straightforward to see that only for minimal coupling (\(\xi = 0\)) can the \(m^2\)-dependent part of \(< T_{\mu\nu} >\) be interpreted as correcting the Newton constant.

\(^2\)To this end one uses the non-local expansion \(R_{\alpha\beta\mu\nu} = \frac{1}{2} \{ \nabla_{\mu} \nabla_{\alpha} R_{\nu\beta} + \nabla_{\nu} \nabla_{\beta} R_{\mu\alpha} - \nabla_{\nu} \nabla_{\alpha} R_{\mu\beta} - \nabla_{\mu} \nabla_{\beta} R_{\nu\alpha} \} + O(\mathcal{R}^2)\) (see [4,8])
All this reasoning case be extended for arbitrary values of $w$. All terms in the energy-momentum tensor that depend on the $\sigma_n$'s can be interpreted as being quantum corrections to the gravitational constants associated with the corresponding $R^{n-2}R_n$ ($2 \leq n \leq w$) terms in the classical lagrangian. The numerical coefficients $\sigma_n$'s in these corrections depend on the $\hat{a}_n$ SDW coefficient. On the contrary, the terms with higher power of the mass ($k$-dependent ones) involve the asymptotic behaviour of the non-local form factors and can be viewed as correcting the Newton constant only for $\xi = 0$.

For example, in six-dimensional spacetime the integral $I_w$ can be calculated using Eqns(12,13) and is given by

$$I_{w=3} = \sigma_3 \log\left(-\frac{m^2}{\mu^2}\right) + \sigma_2 m^2 \log\left(-\frac{m^2}{\mu^2}\right) - k \frac{m^4}{2} \log\left(-\frac{m^2}{\mu^2}\right)$$

(20)

For this theory the coefficients $\sigma_2$ and $k$ are the same as those of the four-dimensional one, while the $\sigma_3$ coefficients are obtained from the term of the form factors that is linear in $\eta$ and read

$$\sigma_3^{(1)} = \frac{1}{336} + \frac{\xi}{30} - \frac{\xi^2}{12}, \quad \sigma_3^{(2)} = -\frac{1}{840}$$

(21)

In this case one obtains that the $m_2^0 (m^2)$ terms in $< T_{\mu\nu} >$ are interpreted as quantum corrections to the gravitational coefficients $\alpha_0$, $\beta_0$ ($\alpha_1$, $\beta_1$) and depend on the $\hat{a}_2$ ($\hat{a}_3$) SDW coefficient. As before, one can view the $m^4$ terms as a quantum correction to the Newton constant only for minimal coupling.

Having evaluated the energy-momentum tensor, we shall make a brief comment on the trace anomaly. As is well-known [5], the classical theory is conformally invariant for $m = 0$ and $\xi = \frac{1}{4} \frac{2w-2}{2w-1}$. Due to quantum effects, a trace anomaly in $< T_{\mu\mu} >$ appears, which is local and proportional to the $\hat{a}_w$ SDW coefficient. In our computation of the energy-momentum tensor we have concentrated on the non-local terms and we have absorbed the local ones into the renormalized classical gravitational constants. Using the expressions for the coefficients $\sigma_w^{(i)}$ evaluated at conformal coupling (see Eqns(19,21) for the $w = 2$ and $w = 3$ cases) one can readily prove that the trace of the non-local and mass-independent terms of the energy-momentum tensor vanishes. Although the local terms are irrelevant for the main point of
this work, which is thoroughly developed in previous paragraphs, their evaluation from the integral \( A_w \) is straightforward. At conformal coupling these terms give the correct trace anomaly, up to the order we are working \( (O(R^2)) \).

III. SCALING FOR SPINOR FIELDS

In this section we shall extend the reasoning to spinor fields in four dimensions. The one-loop contribution to EA of the free Dirac field on a gravitational background is

\[
\Gamma = -\frac{1}{2} Tr \ln \hat{K} \\
\hat{K} \Psi = (\gamma_{\mu} \nabla_{\mu} + m) (\gamma_{\nu} \nabla_{\nu} + m) \Psi = (- + m^2 + \frac{1}{4} R) \Psi
\]

(22)

Therefore we have to evaluate the trace of an operator similar to that associated with the scalar field for \( \xi = 1/4 \) and trace over the spinor indexes.

We shall evaluate the EA following the method described in the previous Section (see Eqn(9)). The second order term in curvatures can be written as \([3,4]\)

\[
\Gamma^{(2)} = \frac{1}{32 \pi^2} \int d^4 x \sqrt{g} [4 R F_1(\cdot) \hat{R} + 4 R_{\mu \nu} F_2(\cdot) R_{\mu \nu} + Tr(\mathcal{R}_{\mu \nu} F_3(\cdot) \mathcal{R}_{\mu \nu})]
\]

(23)

where \( \mathcal{R}_{\mu \nu} \Psi = \frac{i}{8} [\gamma_\alpha(x), \gamma_\beta(x)] \mathcal{R}_{\alpha \beta \mu \nu}(x) \Psi \) is the commutator of the covariant derivatives \([9]\). Here \( F_1(\cdot) \) and \( F_2(\cdot) \) are the scalar field-form factor integrals evaluated at \( \xi = 1/4 \). We have a new contribution due to \( F_3(\cdot) \), corresponding to the integral of an additional form factor \( f_3 = -\frac{f(\mu)-1}{2\eta} \). Using the expression for \( \mathcal{R}_{\mu \nu} \) and calculating the trace of the product of four gamma matrices, the last term in Eqn(23) can be written as \( Tr \mathcal{R}_{\mu \nu} F_3(\cdot) \mathcal{R}_{\mu \nu} = -\frac{1}{2} R_{\alpha \beta \mu \nu} F_3(\cdot) R_{\alpha \beta \mu \nu} \). Finally, using the non-local expansion of the Riemann tensor in terms of the Ricci tensor \([4,8]\), one can rewrite the last expression through a kind of generalized Gauss-Bonnet identity, namely

\[
\int d^4 x Tr \mathcal{R}_{\mu \nu} F_3(\cdot) \mathcal{R}_{\mu \nu} = \int d^4 x \left[ \frac{1}{2} RF_3(\cdot) \hat{R} - 2 R_{\mu \nu} F_3(\cdot) R_{\mu \nu} + O(R^3) \right]
\]

(24)

In view of this identity, the stress tensor is basically the one for the scalar field, modified as follows
\[ < T_{\mu\nu} > = -\frac{1}{32\pi^2} \left( \log \left( -\frac{\mu}{m^2} \right) \left[ (4\sigma_2^{(1)} + \frac{1}{2}\sigma_2^{(3)}) H^{(1)}_{\mu\nu} + (4\sigma_2^{(2)} - 2\sigma_2^{(3)}) H^{(2)}_{\mu\nu} \right] + \right. \\
\left. \frac{m^2}{\alpha^2} \log \left( -\frac{m^2}{\alpha^2} \right) \left[ (4k^{(1)} + \frac{1}{2}k^{(3)}) H^{(1)}_{\mu\nu} + (4k^{(2)} - 2k^{(3)}) H^{(2)}_{\mu\nu} \right] \right) \] (25)

The new coefficients, associated to the form factor integral \( F_{\beta} \), are given by \( \sigma_2^{(3)} = 1/12 \) (early-time behaviour) and \( k^{(3)} = 1/2 \) (late-time behaviour), and the other coefficients, written in Eqn(19), are evaluated at \( \xi = 1/4 \). Therefore the \( m^2 \)-dependent terms in \( < T_{\mu\nu} > \) can be seen as correcting the Newton constant since \( (4k^{(2)} - 2k^{(3)})/(4k^{(1)} + k^{(3)}) = -2 \). The spinor field behaves, in this respect, as the minimally-coupled scalar field.

Finally, after tracing and solving the equations of motion, the quantum correction to the Newtonian potential reads \( \delta V(r) = -\frac{G^2 M m^2}{3\pi} \frac{\log \frac{m}{r}}{r^2} \) which coincides with the Wilsonian potential, obtained from the RGE for the Newton constant \( G(\mu) \).

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