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AND HIGGS BOSON IN FOUR-GENERATION
FERMION CONDENSATE SCHEME**

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MIRAMARE-TRIESTE

International Atomic Energy Agency
and
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**MASSES OF HEAVY FERMIONS AND HIGGS BOSON
IN FOUR-GENERATION FERMION CONDENSATE SCHEME**

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ABSTRACT

The renormalization group analyses based on low-energy effective Lagrangian indicate that a model of electroweak symmetry breaking of Nambu-Jona-Lasinio (NJL) type with four generations of quarks and leptons could accommodate itself to the top-quark mass $\sim 174 \text{ GeV}$ for the acceptable momentum cut-off $\Lambda \sim 10^6 - 5 \times 10^3 \text{ GeV}$. The fourth generation of quarks will always be heavier than the top-quark but the corresponding leptons, though being heavier than the Z boson, can be either lighter or heavier than the top-quark depending on Λ being either greater or less than $\sim 2.5 \times 10^4 \text{ GeV}$. The mass of the Higgs boson is predicted to be in the region $287-481 \text{ GeV}$ which could provide an important experimental test of the model.

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The top-quark condensate scheme [1,2] of electroweak symmetry breaking and its many fermion-generation extension [3-7] provide a dynamical explanation of the origin of the Higgs boson and the large top-quark mass. In the model with heavier fermions than the top-quark, the momentum cut-off Λ could be lower and the fine-tuning problem of the coupling constants will no longer be a serious one [5]. A model of this kind with a heavier degenerate fourth generation of U -fermions has been expounded in the bubble approximation [5,7]. It has been obtained that the acceptable momentum cut-off Λ could be in the region $10^6 - 5 \times 10^3 \text{ GeV}$ and the U -fermion mass m_U in the range $163 - 353 \text{ GeV}$ if the top-quark mass m_t is taken to be 160 GeV [5]. The mass m_{h^0} of the composite Higgs boson h^0 turns out to be in the range $324 - 689 \text{ GeV}$ and obeys the constraints $m_U + m_t \leq m_{h^0} \leq 2m_U$ [7]. Although these results in the bubble approximation represent the main feature of the model, they can not be considered as precise predictions of the masses of relevant particles since the effects of the color and electroweak gauge interactions and the dynamics of the composite Higgs have been omitted from the discussions.

In order to take these effects into account, we will follow the approach of the low-energy effective Lagrangian proposed in Ref.[2]. When a heavy fourth generation of quarks and leptons are included, the Lagrangian of Yukawa-form, which is equivalent to the effective four-fermion Lagrangian responsible for spontaneous breaking of the electroweak $SU_L(2) \times U_Y(1)$ gauge group at the momentum cut-off Λ , can be expressed by

$$\mathcal{L}_Y = - \left\{ H^\dagger \sum_{\alpha=3,4} \left[g_{U_\alpha}^0 (-i\sigma_2) (\bar{Q}_{\alpha L}^T U_{\alpha R}) + g_{D_\alpha}^0 (\bar{D}_{\alpha L} Q_{\alpha R}) \right] + h.c. \right\} - m_0^2 H^\dagger H \quad (1)$$

where

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \quad (2)$$

is the static Higgs doublet to be regarded as a manifestation of the composite Higgs field. The fermion doublets

$$Q_{\alpha L} = \begin{pmatrix} U_\alpha \\ D_\alpha \end{pmatrix}_L, Q_{\alpha R} = U_{\alpha R}, D_{\alpha R} \quad (\alpha = 3, 4) \quad (3)$$

are assigned in the $SU_c(3)$ representation R with dimension $d_{Q_\alpha}(R)$. They are all assumed to be in their mass eigenstates [6] and limited to only those generations containing heavy

fermions, i.e. $\alpha = 3$ corresponds to the third generation of quarks (t, b) and $\alpha = 4$ to the fourth generation of quarks and leptons (U, D) and (N, E). The denotations g_Q^0 ($Q = U_\alpha, D_\alpha$) represent the Higgs-Yukawa coupling constants and m_0 is the bare mass of the Higgs fields.

From the two-point and four-point functions of the Higgs fields calculated based on $\mathcal{L}_Y$ in the bubble approximation, it is found that the static Higgs fields H will acquire an induced kinetic and quartic self-interaction term. When all of gauge interactions are opened, the induced effective Lagrangian at some low-energy scale μ below the momentum cut-off Λ can be expressed by

$$\mathcal{L} = \mathcal{L}_{\text{kinetic}} - (H^\dagger C + C^\dagger H) + \Delta L_{\text{gauge}} + Z_H |D_\mu H|^2 - m_H^2 H^\dagger H - \frac{\lambda_0}{2} (H^\dagger H)^2, \quad (4)$$

$$C = \sum_{\alpha=3,4} \left[g_{U_\alpha}^0 (-i\sigma_2) (\bar{Q}_{\alpha L}^T U_{\alpha R}) + g_{D_\alpha}^0 (\bar{D}_{\alpha L} Q_{\alpha R}) \right]$$

where D_μ is the gauge covariant derivative and all loops are now defined with respect to the low-energy scale μ . $\mathcal{L}_{\text{kinetic}}$ represents the $SU_c(3) \times SU_L(2) \times U_Y(1)$ gauge-invariant kinetic terms of the gauge and Q -fermion fields. ΔL_{gauge} represents the fermion loop contributions after deducting the low-energy loop terms to the gauge coupling constants in the bubble approximation of the effective four-fermion interactions [6]. The constants Z_H, m_H^2 and λ_0 turn out to be as follows

$$Z_H = \sum_Q d_Q(R) (g_Q^0)^2 (4\pi)^{-2} \ln \frac{\Lambda^2}{\mu^2},$$

$$m_H^2 = m_0^2 - \sum_Q 2d_Q(R) (g_Q^0)^2 (4\pi)^{-2} (\Lambda^2 - \mu^2),$$

$$\lambda_0 = \sum_Q 2d_Q(R) (g_Q^0)^4 (4\pi)^{-2} \ln \frac{\Lambda^2}{\mu^2} \quad (5)$$

where the Q in the sums run over all the heavy fermions in the third and the fourth generation except the light bottom quark (this is equivalent to assuming $g_b^0 = 0$). Eq.(5) is just the generalization of the formula (3.3) in Ref.[2]. We notice that

$$Z_H \rightarrow 0, \quad m_H^2 \rightarrow m_0^2, \quad \lambda_0 \rightarrow 0, \quad g_Q^0 \rightarrow \text{constant}, \quad \text{when } \mu \rightarrow \Lambda \quad (6)$$

and the Higgs fields H will return to the static one in this case. Hence Eq.(6) simply represents the compositeness boundary conditions in the bubble approximation.

By rescaling the Higgs fields by $H \rightarrow H/\sqrt{Z_H}$ and taking the unitary gauge $H = (0, h^0)/\sqrt{2}$ we may find out from the Lagrangian (4) and the expressions (5) the low-energy physical tree spectrum as follows:

$$m_Q = g_Q^0 < h^0 > / \sqrt{2Z_H} \quad (7)$$

$$m_{h^0}^2 = -2m_H^2/Z_H = \lambda_0 < h^0 >^2 / Z_H^2 \quad (8)$$

with

$$< h^0 >^2 = 2 \sum_Q d_Q(R) m_Q^2 (4\pi)^{-2} \ln \frac{\Lambda^2}{\mu^2} \quad (9)$$

Eq.(8) can be further reduced to that

$$m_{h^0}^2 = 4 \sum_Q d_Q(R) m_Q^4 / \sum_Q d_Q(R) m_Q^2 \quad (10)$$

Therefore, the tree mass formula (10) of the composite Higgs boson is precisely coincident with the approximate mass formula (7) in Ref.[7].

The complete dynamics contained in the low-energy effective Lagrangian (4) is embodied in the renormalization group evolutions of the gauge coupling constants and the Higgs-Yukawa and the Higgs-quartic coupling constants. After rescaling $H \rightarrow H/\sqrt{Z_H}$, Eq.(4) will have the same form as the Lagrangian of the standard model of strong and electroweak interactions. Therefore, we can use the renormalization group evolutions of these coupling constants in the standard model but necessary compositeness boundary conditions consistent with Eq.(6) must be imposed. Under the conventional normalization,

$$H \rightarrow H/\sqrt{Z_H}, \quad Q_L \rightarrow Q_L/\sqrt{Z_{Q_L}}, \quad Q_R \rightarrow Q_R/\sqrt{Z_{Q_R}} \quad (11)$$

where Z_H, Z_{Q_L} and Z_{Q_R} are respectively the divergent wave function normalization constants of the Higgs fields, the left-handed and the right-handed Q fermion fields, the kinetic terms in Eq.(4) will have the form of the free field theory. The physical couplings $\bar{g}_Q$ and $\bar{\lambda}$ are defined by

$$\bar{g}_Q = \frac{Z_{HQ}}{\sqrt{Z_H Z_{Q_L} Z_{Q_R}}} g_Q^0,$$

$$\bar{\lambda} = \frac{Z_{4H}}{Z_H^2} \lambda_0 \quad (12)$$

where Z_{HQ} and Z_{4H} are the proper vertex renormalization constants for the Higgs-Yukawa and the Higgs-quartic interactions separately.

In order to describe the compositeness boundary conditions consistent with Eq.(6), following the method used in Ref.[2], we may further take the "unconventional" normalization convention

$$H \rightarrow H/\bar{g}_t^2(\mu) \quad (13)$$

where t represents the top-quark. Thus we will obtain the wave function normalization constant $\tilde{Z}_H$ in the Higgs kinetic term and the Higgs quartic self-coupling constant $\tilde{\lambda}$ in this convention as follows:

$$\tilde{Z}_H(\mu) = \frac{1}{\bar{g}_t^2(\mu)} \quad (14)$$

$$\tilde{\lambda}(\mu) = \frac{\bar{\lambda}(\mu)}{\bar{g}_t^4(\mu)} \quad (15)$$

In addition, we also define that

$$\tilde{Z}_Q(\mu) = \frac{1}{\bar{g}_Q^2(\mu)}, \text{ for } Q = U, D, N, E \quad (16)$$

It is easy to verify that owing to Eq.(6) we will have

$$\tilde{Z}_H(\mu) \rightarrow 0, \tilde{\lambda}(\mu) \rightarrow 0 \text{ and } \tilde{Z}_Q(\mu) \rightarrow 0 \text{ (} Q = U, D, N, E \text{), when } \mu \rightarrow \Lambda \quad (17)$$

However, the ratio between the Yukawa coupling constants $\bar{g}_Q^2$ and $\bar{g}_t^2$ will be ultraviolet finite, i.e.

$$\frac{\tilde{Z}_H(\mu)}{\tilde{Z}_Q(\mu)} \equiv \frac{\bar{g}_Q^2(\mu)}{\bar{g}_t^2(\mu)} = \eta_{Qt} \left[\frac{Z_{HQ}^{G+F}(\mu)}{Z_{Ht}^{G+F}(\mu)} \right]^2 \frac{Z_{tL}^{G+F}(\mu) Z_{tR}^{G+F}(\mu)}{Z_{Q_L}^{G+F}(\mu) Z_{Q_R}^{G+F}(\mu)} \rightarrow \text{constant, when } \mu \rightarrow \Lambda \quad (18)$$

where the parameter η_{Qt} is defined by

$$\eta_{Qt} = (g_Q^0/g_t^0)^2 \quad (19)$$

In Eq.(18) we have kept only the contributions Z_{HQ}^{G+F} , $Z_{Q_L}^{G+F}$ and $Z_{Q_R}^{G+F}$ coming from the fermion and gauge-boson loops in the corresponding wave function normalization constants Z_{HQ} , Z_{Q_L} and Z_{Q_R} separately, this is because when $\mu \rightarrow \Lambda$, the kinetic term of

the Higgs fields will disappear and the Higgs boson will cease to propagate. To one-loop order we have the following expressions for these constants

$$Z_{HQ}^{G+F}(\mu) = 1 + \left[2e_Q Y_{Q_L} \bar{g}_t^2(\mu) + 4C_2(R_Q) \bar{g}_3^2(\mu) \right] (4\pi)^{-2} \ln \frac{\mu^2}{M_Z^2},$$

$$Z_{Q_L}^{G+F}(\mu) = 1 + \left[\frac{1}{4} Y_{Q_L}^2 \bar{g}_1^2(\mu) + \frac{3}{4} \bar{g}_2^2(\mu) + C_2(R_Q) \bar{g}_3^2(\mu) \right] (4\pi)^{-2} \ln \frac{\mu^2}{M_Z^2},$$

$$Z_{Q_R}^{G+F}(\mu) = 1 + \left[e_Q^2 \bar{g}_1^2(\mu) + C_2(R_Q) \bar{g}_3^2(\mu) \right] (4\pi)^{-2} \ln \frac{\mu^2}{M_Z^2} \quad (20)$$

where e_Q is the value of the electric charge of the Q -fermions defined by

$$e_Q = \frac{1}{2}(\delta_Q + Y_{Q_L}) \quad (21)$$

with the sign function

$$\delta_Q = \begin{cases} 1 & \text{for } Q = U_\alpha \\ -1 & \text{for } Q = D_\alpha, \end{cases} \quad (22)$$

Y_{Q_L} is the $U_Y(1)$ charge of the left-handed Q -fermions and $C_2(R_Q)$ is the eigenvalue of the $SU_c(3)$ quadratic Casimir operator in the representation R of the Q -fermions. The running coupling constants $\bar{g}_1^2(\mu)$, $\bar{g}_2^2(\mu)$ and $\bar{g}_3^2(\mu)$ of the $U_Y(1)$, $SU_L(2)$ and $SU_c(3)$ gauge group will be determined by the renormalization group evolution formulas in the standard model [8]

$$\bar{g}_i^2(\mu) = \bar{g}_i^2(M_Z) / \left[1 + \bar{g}_i^2(M_Z) \frac{\beta_0^{(i)}}{8\pi^2} \ln \frac{\mu}{M_Z} \right], \quad i = 1, 2, 3 \quad (23)$$

with

$$\beta_0^{(1)} = -\frac{163}{18}, \quad \beta_0^{(2)} = \frac{11}{6} \text{ and } \beta_0^{(3)} = \frac{17}{3} \quad (24)$$

for the model with four generations of quarks and leptons. Here we have taken the Z -boson mass M_Z as the low-energy scale parameter and this amounts to specify a subtraction scheme of the renormalization. Changing the scale parameter will only lead to an additional finite renormalization and not bring about essential influence on the results. In this paper we will also ignore the effect of the large mass difference $m_t - m_b$ on the conventional running of the gauge couplings $\bar{g}_1^2(\mu)$ and $\bar{g}_2^2(\mu)$ which was indicated in Ref.[7].

In the conventional normalization (11), Eq.(4) has the same form as the Lagrangian of the standard model, hence by means of standard calculations we can derive out the renormalization group evolution equations of the Higgs-Yukawa couplings $\bar{g}_Q(\mu)$ and the Higgs quartic coupling $\bar{\lambda}(\mu)$. The results are that

$$(4\pi)^2 \mu \frac{d\bar{g}_Q}{d\mu} = \left\{ \left[\frac{3}{2} + d_Q(R) \right] \bar{g}_Q^2 + \sum_{Q' \neq Q} d_{Q'}(R) \bar{g}_{Q'}^2 - \frac{3}{2} (\delta_{QU} \bar{g}_D^2 + \delta_{QD} \bar{g}_U^2) - 3(e_Q^2 + \frac{1}{4} Y_{QL}^2) \bar{g}_1^2 - \frac{9}{4} \bar{g}_2^2 - 6C_2(R_Q) \bar{g}_3^2 \right\} \bar{g}_Q, \quad Q = t, U, D, N, E \quad (25)$$

and

$$(4\pi)^2 \mu \frac{d\bar{\lambda}}{d\mu} = 12 \left\{ \bar{\lambda}^2 + \left[\frac{1}{3} \sum_Q d_Q(R) \bar{g}_Q^2 - A \right] \bar{\lambda} + B - \frac{1}{3} \sum_Q d_Q(R) \bar{g}_Q^4 \right\}, \quad A = \frac{1}{4} \bar{g}_1^2 + \frac{3}{4} \bar{g}_2^2, \quad B = \frac{1}{16} (\bar{g}_1^4 + 2\bar{g}_1^2 \bar{g}_2^2 + 3\bar{g}_2^4) \quad (26)$$

where we have used the definitions

$$\left. \begin{matrix} \delta_{QU} \\ \delta_{QD} \end{matrix} \right\} = \frac{1}{2} (1 \pm \delta_Q) \quad (27)$$

and omitted the coupling constant $\bar{g}_b$ for the light bottom-quark. By means of Eqs.(14)-(16), Eqs.(25) and (26) can be transformed into coupled equations about $\tilde{Z}_H$ and $\tilde{Z}_Q(Q = U, D, N, E)$ and an equation about $\tilde{\lambda}$. They must be solved together with the compositeness boundary conditions given by Eqs.(17) and (18). In addition, in the equation of $\tilde{\lambda}$ there are the terms like $\tilde{\lambda}/\tilde{Z}_H$ and $(\tilde{\lambda}/\tilde{Z}_H)^2$. Since $\tilde{\lambda}(\Lambda)/\tilde{Z}_H(\Lambda)$ is of the indefinite form of 0/0 by Eq.(17), we must also determine the boundary condition satisfied by $\tilde{\lambda}/\tilde{Z}_H$ if we intend to solve the equation numerically. In fact, the function

$$x = \tilde{\lambda}/\tilde{Z}_H \quad (28)$$

obeys the equation

$$(4\pi)^2 \frac{dx}{dt} = 12\bar{g}_t^2 \left[x^2 + \left(\frac{1}{4} + E \right) x - (1 + D) \right] \quad (29)$$

where the variant t is defined by

$$t = \ln \frac{\mu}{M_Z} \quad (30)$$

and the denotations

$$E = \tilde{Z}_H \left[\frac{1}{6} \sum_{Q \neq t} \frac{d_Q(R)}{\tilde{Z}_Q} - \frac{1}{72} \bar{g}_1^2 - \frac{3}{8} \bar{g}_2^2 + \frac{4}{3} \bar{g}_3^2 \right], \quad D = \tilde{Z}_H^2 \left[\frac{1}{3} \sum_{Q \neq t} \frac{d_Q(R)}{\tilde{Z}_Q^2} - B \right] \quad (31)$$

We may assume that $E > 0$ and $D > 0$ because of weakness of the gauge couplings $\bar{g}_i^2 (i = 1, 2)$. Then from the zero points of the right-handed side of Eq.(29)

$$x_{\pm} = \frac{1}{8} \left\{ -(1 + 4E) \pm [(1 + 4E)^2 + 64(1 + D)]^{\frac{1}{2}} \right\}, \quad (32)$$

it is easy to find that $x_-(t \rightarrow \infty)$ and $x_+(t \rightarrow 0)$ will respectively be the ultraviolet (UV) and infrared (IR) stable fixed point [9]. Considering that in the model with heavier fermions than the top-quark, the momentum cut-off Λ may be lower, it is more reliable to use the IR stable fixed point $x_+(t \rightarrow 0)$ to specify the boundary condition of $\tilde{\lambda}$, i.e.

$$\tilde{\lambda}(0) = \tilde{Z}_H(0) x_+(0) \quad (33)$$

where $x_+(0)$ may be calculated by Eqs.(31) and (32) when $\tilde{Z}_H$ and $\tilde{Z}_Q(Q = U, D, N, E)$ have been solved in advance. In this case, as long as $x_- < x \leq x_+$, then x will be driven toward the constant $x_-(\Lambda)$ when $\mu \rightarrow \Lambda$ and $\tilde{\lambda}(\Lambda) = \tilde{Z}_H(\Lambda) x_-(\Lambda)$ will tend to zero, i.e. the UV boundary condition (17) may be automatically satisfied. The boundary values of the gauge coupling constants will be taken to be that

$$\bar{g}_1^2(M_Z) = 0.127, \quad \bar{g}_2^2(M_Z) = 0.446, \quad \bar{g}_3^2(M_Z) = 1.44 \quad (34)$$

and $M_Z = 91.17 \text{ GeV}$, following Ref.[2].

Without loss of essentiality, we solve the equations of $\tilde{Z}_Q(Q = H, U, D, N, E)$ with the assumption that

$$g_Q^0 = \text{constant, for } Q = U, D, N, E \quad (35)$$

i.e. the masses of the fourth generation of fermions are degenerate in the bubble approximation. In this case the boundary conditions of $\tilde{Z}_Q$ will contain only a single free parameter η_{Q_i} expressed by Eq.(19). Once the value of η_{Q_i} is specified, all the boundary

conditions of $\tilde{Z}_H/\tilde{Z}_Q$ will be completely determined by Eq.(18), where the wave function normalization constants can be calculated by Eq.(20). Furthermore, after all the $\tilde{Z}_Q$ are solved explicitly, the boundary condition of $\tilde{\lambda}$ will also be determined by Eq.(33). Therefore, η_{Q_t} in fact becomes the only free parameter for solving of the total equations. The masses m_Q of the heavy fermions and the mass m_{h^0} of the Higgs boson will respectively be given by the roots of the equations

$$\frac{v}{\sqrt{2}}\bar{g}_Q(m_Q) = m_Q, \quad Q = t, U, D, N, E \quad (36)$$

and

$$v^2\tilde{\lambda}(m_{h^0}^2) = m_{h^0}^2 \quad (37)$$

where $v = (\sqrt{2}G_F)^{-1/2} = 246 \text{ GeV}$ is the standard -model Higgs vacuum expectation value and G_F is the Fermi constant [10].

We list in Table 1 the results of masses of the heavy fermions and the Higgs boson when the top-quark mass is fitted to the present experimental value $m_t = 174 \text{ GeV}$ [11] by adopting appropriate parameter η_{Q_t} for different momentum cut-off Λ .

Table I. The masses of the fourth generation of fermions U, D, N, E and the Higgs boson h^0 for different values of the momentum cut-off Λ . The values of parameter η_{Q_t} have been chosen so as to fit the top-quark mass $m_t = 174 \text{ GeV}$.

$\Lambda(\text{GeV})$	10^7	10^6	10^5	5×10^4	2.5×10^4	10^4	5×10^3
η_{Q_t}	0.083	0.149	0.33	0.44	0.60	0.942	1.376
$m_U(\text{GeV})$	209	229	261	275	293	326	366
$m_D(\text{GeV})$	207	228	260	274	292	325	365
$m_N(\text{GeV})$	90	110	142	156	174	207	244
$m_E(\text{GeV})$	93	113	144	159	176	209	246
$m_{h^0}(\text{GeV})$	260	287	333	353	379	427	481

It can be seen from Table I that, for the model of minimal dynamical electroweak symmetry breaking of NJL-type with four generations of quarks and leptons, when all the gauge interactions and the dynamics of the composite Higgs boson are included it is entirely possible to accomodate itself to the present experimental top-quark mass $m_t \sim 174 \text{ GeV}$ for different acceptable momentum cut-off $\Lambda \sim 10^6 - 5 \times 10^3 \text{ GeV}$. Under the assumption that the fourth generation of fermions are mass-degenerate in the bubble approximation, once m_t is specified then the masses of all the other heavy particles in the model will be completely determined by the renormalization group equations. The leptons in the fourth generation without the color interactions are obviously the lightest particle among them. Their masses, m_N and m_E , could become smaller than the top-quark mass m_t . For instance, they could be even down to near or below the Z -boson mass m_Z when the momentum cut-off $\Lambda \geq 10^7 \text{ GeV}$. If one expects the masses of all the fourth generation of fermions including leptons to be close to or bigger than the top-quark mass, then the momentum cut-off Λ must be confined in the region $2.5 \times 10^4 - 5 \times 10^3 \text{ GeV}$ and this means quite a low compositeness scale. Table I also shows that for all the values of Λ we will have $m_U \approx m_D$ and $m_N \approx m_E$. This result merely reflects the fact that when the gauge interactions are neglected the ratios $\bar{g}_U^2(\mu)/\bar{g}_D^2(\mu)$ and $\bar{g}_N^2(\mu)/\bar{g}_E^2(\mu)$ have the unities as their infrared stable fixed points and the masses of particles actually depend on only the low-energy evolutions of corresponding running coupling constants.

The Higgs boson mass m_{h^0} will increase rapidly as the momentum cut-off Λ decreases. When $\Lambda = 10^6 - 5 \times 10^3 \text{ GeV}$, m_{h^0} will vary in the region $287 - 481 \text{ GeV}$. In the meantime m_U will vary in the region $229 - 336 \text{ GeV}$. These results coincide with the general pattern of that m_{h^0} varies as $\Lambda(m_U)$ in the bubble approximation [11]. In addition, the mass inequalities $2(m_Q)_{\min} < m_{h^0} < 2(m_Q)_{\max}$ ($Q = t, U, D, N, E$) valid in the bubble approximation are also satisfied in present case.

In conclusion, if experimentally one can find the Higgs boson with the mass in the region $287 - 481 \text{ GeV}$, then the theoretical model of dynamical electroweak symmetry breaking of NJL-type with the fourth generation of heavy quarks and leptons would get a strong support and deserve to be investigated further.

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