INSTITUTE FOR NUCLEAR STUDY
UNIVERSITY OF TOKYO
Tanashi, Tokyo 188
Japan

INS-Rep.-1055
Sep. 1994

Double-Strangeness Five-Body System

Khin Swe MYINT and Yoshinori AKAISHI[†]

Department of Physics, Mandalay University, Mandalay, Union of Myanmar

[†]Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo 188, Japan

SC 2443

Double-Strangeness Five-Body System

Khin Swe MYINT and Yoshinori AKAISHI[†]

Department of Physics, Mandalay University, Mandalay, Union of Myanmar

[†]Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo 188, Japan

We perform theoretical analysis on the structure and decay of a double-strangeness five-body system which consists of ${}_{\Lambda\Lambda}^5\text{H}$ and ${}_{\Xi}^5\text{H}$ states. In this $S=-2$ five-body system the thresholds of the $t\Lambda\Lambda$ channel and the $\alpha\Xi^-$ channel come closer with only 8.51 MeV difference. We treat both bound and resonant states of the three-body channels $t\Lambda\Lambda$ and $tp\Xi^-$ by applying a complex rotation method. It is found that there is a bound ${}_{\Lambda\Lambda}^5\text{H}$ state with 6.3 MeV below the threshold of $t+\Lambda+\Lambda$. In the Ξ^- channel a resonant ${}_{\Xi}^5\text{H}$ state appears at 1.7 MeV below the threshold of $\alpha + \Xi^-$. Though the existence of this state is ensured by the Coulomb interaction, it is a 'halo' nuclear state rather than an atomic state as judged from its size. The conversion width of this state is 0.2 MeV which is extremely narrow. It is also found that Ξ mixing into the ${}_{\Lambda\Lambda}^5\text{H}$ ground state is small with 1.0 %. For the ${}_{\Lambda\Lambda}^5\text{H}$ state, the weak decay to the $\alpha+\Sigma^-$ final state produces a high mono-energetic Σ^- with branching ratio of 5.5 %. Thus the Σ^- with discrete energy would become a clear signature of the forming of the $\Lambda\Lambda$ hypernucleus.

§1. Introduction

In order to complete the knowledge of baryon-baryon interactions it is essential to understand $\Lambda\Lambda$ and $N\Xi$ interactions which can be obtained from double- Λ hypernuclei and Ξ hypernuclei. Thus the significance of strangeness $S=-2$ hypernuclei is prominent.

There are, however, only a few existing data on $\Lambda\Lambda$ hyperfragments which have been produced in emulsion targets exposed to K^- incident beams. The first two events were reported more than 20 years ago and identified as ${}_{\Lambda\Lambda}^{10}\text{Be}$ 1) and ${}_{\Lambda\Lambda}^6\text{He}$ 2). In the recent experiment performed at KEK, the emulsion-counter hybrid technique has been used 3). The scattered K^+ meson from the emulsion target in the reaction

$K^- + p \rightarrow K^+ + \Xi^-$ is tagged by a K^+ spectrometer which ensures that the strangeness of -2 is transferred to a nucleus in the emulsion. The track of the escaped Ξ^- is followed up to its end point where a double- Λ hyperfragment is formed. This hyperfragment is assigned to be either ${}_{\Lambda\Lambda}^{10}\text{Be}$ or ${}_{\Lambda\Lambda}^{13}\text{B}$ 4,5). Improvement of experimental technique in search of double-strangeness systems encourages us to do theoretical investigation of the systems.

The purpose of the present work is to investigate the structure and decay of the double-strangeness five-body hypernuclear system which is composed of ${}_{\Xi}^5\text{H}$ and ${}_{\Lambda\Lambda}^5\text{H}$. The interesting feature of this $S=-2$ five-body system is that, due to the large binding energy of the α particle, the Ξ^- channel comes closer to the $\Lambda\Lambda$ channel whereas they are usually far apart from each other by 28.3 MeV. We show the various threshold energies in $\Lambda\Lambda$ and Ξ channels of the five-body system in Fig. 1. It may be expected that there is a significant amount of Ξ contribution in the double- Λ hypernuclear state which would be able to enhance the production rate of ${}_{\Lambda\Lambda}^5\text{H}$ through the reaction ${}^6\text{Li}(K^-, K^+n)_{\Lambda\Lambda}^5\text{H}$. The mixture of Ξ^- and $\Lambda\Lambda$ components is an interesting quantity since the H dibaryon in the pure flavor SU(3) singlet state is expressed in terms of two-baryon configurations as $|H\rangle = \sqrt{\frac{2}{3}}|\Lambda\Lambda\rangle + \sqrt{\frac{1}{3}}|N\Xi\rangle - \sqrt{\frac{1}{3}}|\Sigma\Sigma\rangle$. We investigate the amount of Ξ^- component in the ${}_{\Lambda\Lambda}^5\text{H}$ state. The large binding energy of the α particle would also provide an interesting feature in the weak decay of the ${}_{\Lambda\Lambda}^5\text{H}$ ground state.

Fig.1

This paper provides the details of a talk which has been presented by one of the authors (K. S. M.)⁶⁾ at the 20th INS Symposium held in 1991.

§2. Coupled-channel calculations

2.1. Interaction between hyperon and nucleon

We are going to solve a coupled Schrödinger equation of $t\Lambda\Lambda$ and $tp\Xi^-$ channels which are three-body systems. The coupled equation is

$$(T_{11} + V_{\Lambda\Lambda} + V_{\Lambda\Lambda} + V_{\Lambda\Lambda})\Psi_1 + V_{\text{cpl}}\Psi_2 = E\Psi_1, \quad (2.1a)$$

$$(T_{22} + V_{\Xi^-} + V_{\Xi^-} + V_{\Xi^-} + V_{\Xi^-} + V_{\Xi^-} + V_{\Xi^-} + V_{\Xi^-} + V_{\Xi^-} + V_{\Xi^-})\Psi_2 + V_{\text{cpl}}\Psi_1 = E\Psi_2, \quad (2.1b)$$

where T_{11} and T_{22} are kinetic energy operators of the three-body systems of channel 1 ($t\Lambda\Lambda$) and channel 2 ($tp\Xi^-$) respectively, V_{cpl} is the coupling potential between the two channels, V_{Ξ^-} is sum of the Coulomb potentials and Q_0 is the reaction Q-value of the elementary process $\Xi^- + p \rightarrow \Lambda + \Lambda$,

$$Q_0 = M_p c^2 + M_{\Xi^-} c^2 - 2M_{\Lambda} c^2 = 28.33 \text{ MeV}. \quad (2.2)$$

Due to the strong interaction between t and p , the threshold energy difference Q between the $\alpha\Xi^-$ and the $t\Lambda\Lambda$ channels is largely reduced to be

$$Q = Q_0 - B_{\alpha} + B_t = 8.51 \text{ MeV}, \quad (2.3)$$

where B_{α} and B_t are binding energies of α and t , respectively.

For the $t\Lambda$ potential we apply Kurihara's isle-type potential ⁷⁾ which is derived from Dalitz's hard core ΛN interaction ⁸⁾. It is in two-range Gaussian form and is given for the effective $\alpha\Lambda$ potential. We adjust the strengths of the $\alpha\Lambda$ potential to reproduce the experimental ground state energy of ${}^4\text{H}$ ($E = -2.04 \pm 0.04 \text{ MeV}$) for the singlet $t\Lambda$ state and the first excited state energy ($E = -0.99 \pm 0.04 \text{ MeV}$) for the triplet $t\Lambda$ state ⁹⁾ respectively. The potential between t and Λ in the $t\Lambda\Lambda$ channel is given, in unit of MeV and fm, as

$$V_{\Lambda\Lambda}(r) = \frac{1}{4}V'_{\Lambda\Lambda} + \frac{3}{4}V'_{\Lambda\Lambda} = 359.2 \exp\left[-\left(\frac{r}{1.25}\right)^2\right] - 324.9 \exp\left[-\left(\frac{r}{1.41}\right)^2\right], \quad (2.4)$$

and is shown in Fig.2.

Fig.2

The potential between t and p is the following two-range Gaussian folding potential which is derived from the Volkov No.2 force ¹⁰⁾.

$$V_{\text{p}}(r) = 50.56 \exp\left[-\left(\frac{r}{1.707}\right)^2\right] - 102.95 \exp\left[-\left(\frac{r}{2.237}\right)^2\right]. \quad (2.5)$$

This potential is adjusted to give the energy of the tp system $E_{\text{tp}} = -19.81 \text{ MeV}$ which corresponds to the α binding energy.

The remaining potentials $\Lambda\Lambda$, $p\Xi^-$, $t\Xi^-$ and the coupling term are obtained from Shinmura's potential ¹¹⁾ which is based on the Nijmegen model-D potential ¹²⁾. In $S=-2$ two-body interactions, there are couplings among $\Lambda\Lambda$, $N\Xi$, $\Lambda\Sigma$ and $\Sigma\Sigma$ channels as shown in Table I. But it is difficult to treat all of these channels explicitly,

Table I

and in Shinmura's potential only $\Lambda\Lambda$ and $N\Xi^-$ are explicitly included. In this five-body system the $n\Xi^0$ channel is also truncated out since it is well separated from the $p\Xi^-$ channel by 14.7 MeV which can be seen from Fig. 1. In the two-body system they are only about 5 MeV apart and are treated in the same footing. The effect of the truncated channels are implicitly included by reproducing at low energies the S-matrix of the Nijmegen model-D potential. Table II shows the $S = -2$ two-body Shinmura's potentials which are in two-range Gaussian form.

Table II

The hard core radius of the Nijmegen model-D potential is a free parameter and we determine it by fitting the experimental binding energy of ${}^6_{\Lambda\Lambda}\text{He}$ ²⁾. We solve the $\alpha\Lambda\Lambda$ three-body system in a Gaussian-basis treatment ¹³⁾ to get the binding energy of ${}^6_{\Lambda\Lambda}\text{He}$. By employing the different hard core radius $r_c = 0.46, 0.50$ and 0.56 fm , the calculated binding energy is $-15.4, -10.5$ and -7.7 MeV , respectively, while the experimental value is -10.9 MeV . It is found that the hard core radius 0.50 fm gives a good fit to the experimental data.

The isospin-spin averaged potential of $V_{N\Xi}$ is derived as follows. Assuming a symmetric function for the radial part, we take the isospin-spin function of the channel

tp Ξ^- to be

$$\chi = \frac{1}{\sqrt{2}} \left| \frac{\alpha_{123}\beta_4 - \beta_{123}\alpha_4}{\sqrt{2}} \alpha_5 \right\rangle_{\text{spin}} \left| \frac{h_{123}n_4 - t_{123}p_4}{\sqrt{2}} \Xi_5^- \right\rangle_{\text{isospin}} - \frac{1}{\sqrt{2}} \left| \frac{\alpha_{123}\beta_5 - \beta_{123}\alpha_5}{\sqrt{2}} \alpha_4 \right\rangle_{\text{spin}} \left| \frac{h_{123}n_5 - t_{123}p_5}{\sqrt{2}} \Xi_4^- \right\rangle_{\text{isospin}}, \quad (2.6)$$

where t and p are formed into α . It should be noted that the α particle is not a simple tp bound state but a tp-hn coupled state with isospin 0 and spin 0. By rewriting χ as

$$\begin{aligned} \chi = & \frac{1}{\sqrt{8}} |\alpha_{123}t_{123}\rangle \left| \frac{\alpha_4\beta_5 - \beta_4\alpha_5}{\sqrt{2}} \frac{p_4\Xi_5^- + \Xi_4^-p_5}{\sqrt{2}} \right\rangle \\ & + \frac{1}{\sqrt{8}} |\alpha_{123}t_{123}\rangle \left| \frac{\alpha_4\beta_5 + \beta_4\alpha_5}{\sqrt{2}} \frac{p_4\Xi_5^- - \Xi_4^-p_5}{\sqrt{2}} \right\rangle + \frac{1}{2} |\beta_{123}t_{123}\rangle \left| \alpha_4\alpha_5 \frac{p_4\Xi_5^- - \Xi_4^-p_5}{\sqrt{2}} \right\rangle \\ & - \frac{1}{\sqrt{8}} |\alpha_{123}h_{123}\rangle \left| \frac{\alpha_4\beta_5 - \beta_4\alpha_5}{\sqrt{2}} \frac{n_4\Xi_5^- + \Xi_4^-n_5}{\sqrt{2}} \right\rangle \\ & + \frac{1}{\sqrt{8}} |\alpha_{123}h_{123}\rangle \left| \frac{\alpha_4\beta_5 + \beta_4\alpha_5}{\sqrt{2}} \frac{n_4\Xi_5^- - \Xi_4^-n_5}{\sqrt{2}} \right\rangle - \frac{1}{2} |\beta_{123}h_{123}\rangle \left| \alpha_4\alpha_5 \frac{n_4\Xi_5^- - \Xi_4^-n_5}{\sqrt{2}} \right\rangle, \end{aligned} \quad (2.7)$$

we obtain the isospin-spin averaged N Ξ interaction

$$V_{N\Xi^-} = \frac{1}{8} V_{p\Xi^-}^{1S_0} + \frac{3}{8} V_{p\Xi^-}^{3S_1} + \frac{1}{8} V_{n\Xi^-}^{1S_0} + \frac{3}{8} V_{n\Xi^-}^{3S_1}. \quad (2.8)$$

Shinmura's two-body potential gives a simple parametrization

$$V_{N\Xi^-}(r) = \sum_{k=1}^3 v_k \exp \left[- \left(\frac{r}{\mu_k} \right)^2 \right], \quad (2.9)$$

with $v_1 = 5000.0$ MeV, $v_2 = -106.06$ MeV, $v_3 = -76.64$ MeV, $\mu_1 = 0.3550$ fm, $\mu_2 = 0.8550$ fm and $\mu_3 = 0.9279$ fm.

The coupling potential between the two channels arises from the conversion of 1S_0 p Ξ^- into $\Lambda\Lambda$ particles. We obtain the coupling potential by using the number representation which is easy to handle totally-antisymmetrized states. The coupling potential is expressed with creation and annihilation operators $a^\dagger$ and a for particles as

$$a_{\Lambda\uparrow}^\dagger a_{\Lambda\downarrow}^\dagger V_{\Lambda\Lambda, p\Xi^-} \frac{1}{\sqrt{2}} (a_{\Xi^- \uparrow} a_{p\downarrow} - a_{\Xi^- \downarrow} a_{p\uparrow}), \quad (2.10)$$

where $\uparrow$ and $\downarrow$ denote spin-up and spin-down, respectively. The initial $\alpha\Xi^-$ and the final t $\Lambda\Lambda$ states are given by

$$a_{\Xi^- \uparrow}^\dagger a_{p\downarrow}^\dagger a_{p\uparrow}^\dagger a_{n\downarrow}^\dagger a_{n\uparrow}^\dagger |0\rangle, \quad (2.11)$$

$$a_{\Lambda\uparrow}^\dagger a_{\Lambda\downarrow}^\dagger a_{p\uparrow}^\dagger a_{n\downarrow}^\dagger a_{n\uparrow}^\dagger |0\rangle, \quad (2.12)$$

and the matrix element of the coupling potential between these states is obtained

$$V_{\text{cpl}} = \frac{1}{\sqrt{2}} V_{\Lambda\Lambda, p\Xi^-}, \quad (2.13)$$

of which strength and range are given in Table II.

The potential between t and Ξ^- is calculated by folding an effective N Ξ potential with the triton density.

$$V_{\Xi^-}(R) = \int dr \rho(r) V_{N\Xi^-}^{\text{eff}}(R-r), \quad (2.14)$$

where R is the radial vector of Ξ from the centre-of-mass of the nucleus. The triton density is obtained from a harmonic oscillator model

$$\rho(r) = 3 \left(\frac{3\gamma}{2\pi} \right)^{\frac{3}{2}} \exp \left(- \frac{3}{2} \gamma r^2 \right), \quad \gamma = 0.386 \text{ fm}^{-2}, \quad (2.15)$$

where γ is the harmonic oscillator strength which reproduces the matter r.m.s. radius 1.61 fm extracted from the charge r.m.s. radius by electron-scattering experiment ¹⁴). The effective potential $V_{N\Xi^-}^{\text{eff}}$ is obtained from $V_{N\Xi^-}$ of Eq. (2.8) by taking into account the short-range correlation between N and Ξ . The correlation effect is included by introducing a factor f_{sc} in the repulsive part of Shinmura's potential. This factor is determined to be 0.39 by simulating V_{Ξ^-} potential derived from pnn Ξ^- four-body microscopic calculation by Harada ¹⁵). Then, the folding potential between t and Ξ^- is

$$V_{\Xi^-}(R) = \sum_{k=1}^3 v_k \exp \left[- \left(\frac{R}{\mu_k} \right)^2 \right], \quad (2.16)$$

with $v_1 = 103.67$ MeV, $v_2 = -51.57$ MeV, $v_3 = -44.10$ MeV, $\mu_1 = 1.362$ fm, $\mu_2 = 1.568$ fm and $\mu_3 = 1.610$ fm. Figure 3 shows the $t\Xi^-$ potential which is less attractive than the $t\Lambda$ potential. The $t\Xi^-$ potential has no bound state.

Fig.3

2.2. Three-body calculation

We treat the system of three bodies with masses m_1, m_2 and m_3 in terms of Gaussian basis functions which are spanned over three rearrangement channels ¹²⁾ shown in Fig.4. The kinetic energy operator of Eq.(2.1) is expressed in the Jacobi coordinate system (r_c, R_c) as

$$T = -\frac{\hbar^2}{2\mu_c} \nabla_{r_c}^2 - \frac{\hbar^2}{2M_c} \nabla_{R_c}^2, \quad (2.17)$$

where $c = 1, 2$ or 3 and μ_c and M_c are reduced masses defined as below;

$$M_1 = \frac{m_1(m_2 + m_3)}{m_1 + m_2 + m_3}, \quad M_2 = \frac{m_2(m_3 + m_1)}{m_1 + m_2 + m_3}, \quad M_3 = \frac{m_3(m_1 + m_2)}{m_1 + m_2 + m_3},$$

$$\mu_1 = \frac{m_2 m_3}{m_2 + m_3}, \quad \mu_2 = \frac{m_3 m_1}{m_3 + m_1}, \quad \mu_3 = \frac{m_1 m_2}{m_1 + m_2}. \quad (2.18)$$

Fig.4

The total wave function is written as:

$$\Psi = \sum_{c=1}^3 \sum_{i,j} A_{ij}^c \exp \left[- \left(\frac{r_c}{b_i^c} \right)^2 \right] \exp \left[- \left(\frac{R_c}{b_j^c} \right)^2 \right], \quad (2.19)$$

where

$$b_{i+1} = (b_N/b_1)^{\frac{1}{N-1}} b_i$$

with b_1 and b_N are Gaussian basis parameters and N is the number of the bases.

We solve the coupled-channel Schrödinger equation (2.1) by the complex rotation method which is based on the ABC theorem ¹⁶⁾. The method is convenient to treat both bound and resonant states. A $\hat{U}(\theta)$ transformation of the theorem is defined by

$$f_\theta(r) \equiv \hat{U}(\theta) f(r) = e^{i\frac{3}{2}\theta} f(re^{i\theta}), \quad (2.20)$$

where θ is a rotation parameter. Under this transformation, divergent resonance wave function would become convergent, and the resonance position and width can be obtained by using bound-state type wave functions. In the method of coordinate rotation, the Schrödinger equation transforms into the following form:

$$\hat{H}_\theta(r) \Psi_\theta(r) = E \Psi_\theta(r), \quad (2.21)$$

where

$$\hat{H}_\theta(r) = \hat{U}(\theta) \hat{H}(r) \hat{U}^{-1}(\theta). \quad (2.22)$$

The spectrum of the rotated Hamiltonian $\hat{H}_\theta(r)$ exhibits the following properties: (1) The bound eigenstates of H are the eigenstates of $\hat{H}_\theta$. (2) The continuum spectrum of H is rotated by an angle 2θ . (3) A complex resonance energy E_{res} belongs to the proper spectrum of $\hat{H}_\theta$ provided 2θ is greater than $|\arg E_{\text{res}}|$. The complex energy

$$E_{\text{res}} = E_r - i \frac{\Gamma}{2} \quad (2.23)$$

gives the usual resonance position E_r and width Γ .

Since we use Gaussian-type basis functions, the Schrödinger equation under the complex rotation is rewritten as

$$\hat{H}(r') \sum_i C_i \exp \left[- \left(\frac{r'}{b_i'} \right)^2 \right] = E \sum_i C_i \exp \left[- \left(\frac{r'}{b_i'} \right)^2 \right], \quad (2.24)$$

with $r' = re^{i\theta}$ and $b_i' = b_i e^{i\theta}$. This leads to a linear equation for complex coefficients C_i 's, of which Hamiltonian matrix elements are given with the complex basis

parameters b_i 's. The parameters used in the present calculation are $(b_1, b_N) = (0.1, 19.0)$ fm, $N=10$ for $\Lambda\Lambda$ and for $p\Xi^-$, $(b_1, b_N) = (0.5, 10.0)$ fm, $N=8$ for $t\Lambda$, $(b_1, b_N) = (0.5, 11.0)$ fm, $N=8$ for $t\Xi^-$, $(b_1, b_N) = (0.5, 13.0)$ fm, $N=8$ for tp . The Hamiltonian matrix to be solved is of 456×456 size. We obtain complex eigenvalues of the coupled Schrödinger equation with two rotation angles $\theta = 0.03$ and 0.05 . Only the result of $\theta = 0.03$ is shown in Fig. 5. There appear a bound state

Fig.5

and a resonance state. It is found that the bound and resonant states do not change with the change of the rotation angle, while continuum states rotate through the angle 2θ .

§3. Characteristics of ${}_{\Lambda\Lambda}^5\text{H}$ and ${}_{\Xi\Xi}^5\text{H}$

3.1. Structure of the ${}_{\Lambda\Lambda}^5\text{H}$ state

It is found that there appears a bound state ${}_{\Lambda\Lambda}^5\text{H}$ with

$$E = -6.3 \text{ MeV} \quad (3.1)$$

below the $t+\Lambda+\Lambda$ threshold as shown in Fig. 6. The uncoupled level of ${}_{\Lambda\Lambda}^5\text{H}$ is $E = -5.8 \text{ MeV}$. By comparing these two energies we know that the energy level of the ${}_{\Lambda\Lambda}^5\text{H}$ state is slightly pushed down from the uncoupled level due to the coupling of the two channels.

Fig.6

We also calculate the mixture of Ξ^- component in this state, and it is found to be

$$P(\Xi^-) = 1.0 \%. \quad (3.2)$$

This Ξ^- component is very small: It has been expected that, because of the small energy difference between the $t+\Lambda+\Lambda$ and $\alpha+\Xi^-$ thresholds, the Ξ^- contribution in this state may be considerably large. The small Ξ^- mixture indicates that the coupling potential between the Ξ^- and $\Lambda\Lambda$ channels is weak. The coupling arises from the conversion of $p\Xi^-$ to $\Lambda\Lambda$, and only the singlet state of $p\Xi^-$ contributes to the conversion process. We show in Eq.(2.13) that $V_{\text{cpl}} = \sqrt{1/2} V_{\Lambda\Lambda, p\Xi^-}$, where the factor $\sqrt{1/2}$ corresponds to the isospin-spin weight of the $p\Xi^-$ singlet state in the α ground state. The formation of α from t and p lowers the threshold of the Ξ^- channel but at the same time weakens the coupling potential. Thus the coupling potential is weak and consequently the Ξ^- component in the $\Lambda\Lambda$ state becomes very small.

The effect of the Ξ^- threshold and the isospin-spin weight of $p\Xi^-$ upon the Ξ^- contribution is interesting to be investigated. The mixture of Ξ^- component is calculated by arbitrarily changing the value of the threshold-energy difference. The relation between the Ξ^- contribution and the threshold difference is plotted in Fig. 7.

Fig.7

The Ξ^- component increases, according as the threshold difference Q decreases, especially at low Q values. Dropping off the isospin-spin reduction factor increases the Ξ^- component about two times the amount in which the factor is included.

From the above calculations it is concluded that the ${}^5_{\Lambda\Lambda}\text{H}$ ground state is an almost pure $\Lambda\Lambda$ state and the prospect of producing the double- Λ state through the Ξ^- channel is not very high.

3.2. Structure of the ${}^5_{\Xi}\text{H}$ state

We are going to discuss the existence of the Ξ -hypernuclear state. As shown in Fig. 6, the energy of the ${}^5_{\Xi}\text{H}$ state is

$$E = 6.8 \text{ MeV} \quad (3.3)$$

above the $t+\Lambda+\Lambda$ threshold. This is 1.7 MeV below the $\alpha+\Xi^-$ threshold. The energy position of this state is not much shifted from the uncoupled state, i.e., $E = 6.9 \text{ MeV}$.

The binding mechanism of this system is interesting. In the $\alpha\Xi^-$ system there is an atomic $1s$ state with r.m.s. distance of 24.0 fm. The Coulomb potential energy of this atomic state is only -0.2 MeV , and its binding energy is 0.1 MeV . For the strong interaction between α and Ξ^- there is a shallow bound state with binding energy 0.5 MeV . This nuclear state has r.m.s. distance of 5.3 fm. Since the Ξ^- hyperon comes closer to the α particle than it does in the atomic $1s$ state, the Coulomb potential energy increases from -0.2 to -1.0 MeV . If we combine both the strong and Coulomb interactions, we have a bound state in the $\alpha\Xi^-$ system with 1.6 MeV binding energy. The bound state of the combined Coulomb and strong interactions has 4.0 fm in the r.m.s. distance which is small. Thus the state is not an atomic state. Rather it is a 'halo' nuclear state as seen from the Ξ^- distribution of this state in Fig. 8. The density of the atomic $1s$ state at 2.0 fm is only about 0.007 times as large as that of the 'halo' nuclear state. The Coulomb interaction ensures the existence of this state while the strong interaction maintains the nature of a nuclear state. The combination of the Coulomb and strong interactions plays an important role in the binding mechanism of the $\alpha\Xi^-$ system. This reminds us similar situations in Σ^- -hypernuclei ^{17,18}.

Fig.8

The ${}^5_{\Xi}\text{H}$ state is bound in the $\alpha\Xi^-$ channel while the $t\Lambda\Lambda$ and ${}^4_{\Lambda}\text{H}\Lambda$ channels are open to it, and therefore it would decay to these latter channels. The conversion width

of this state is composed of three-body and two-body breakup processes. We apply the complex scaling method to solve the coupled-channel Schrödinger equation. An advantage of using the complex rotation method is that it can successfully include the three-body breakup as well as the two-body breakup. The obtained conversion width is very narrow which is

$$\Gamma_{\Xi} = 0.2 \text{ MeV}. \quad (3.4)$$

The narrow width of the ${}^5_{\Xi}\text{H}$ state is due to the strong binding of t and p into the α particle which reduces the phase volume in the open channels. This is a particular phenomenon of the five-body system.

Finally we give a comment related to the binding mechanism seen in the $\alpha\Xi^-$ system. In heavy nuclei there is a possibility that Ξ^- -hypernuclear bound state would become lower than all Λ -channel thresholds due to the large Coulomb attraction. For instance, in $A=208$ system a bound ${}^{208}_{\Xi}\text{Hg}$ state becomes lower than the ${}^{207}_{\Lambda}\text{Pa}+\Lambda$ threshold. The ${}^{208}_{\Xi}\text{Hg}$ state decays eventually to the ground state ${}^{208}_{\Lambda\Lambda}\text{Hg}$. Then, the heavy $\Lambda\Lambda$ hypernucleus can be produced by (K^-, K^+) reaction through the formation of the Ξ -hypernuclear state. This will be discussed elsewhere.

§4. Weak decay of ${}_{\Lambda\Lambda}^5\text{H}$

Hypernuclei can be detected through their decay products. A free Λ hyperon decays via the π -mesonic decay mode, $\Lambda \rightarrow N + \pi$, while Λ in a nucleus may go through the non-mesonic decay mode $\Lambda + N \rightarrow N + N$ in addition to the mesonic decay mode. Motoba et al. discussed the identification of single and double- Λ hypernuclei with the pionic weak decay ¹⁹⁾.

In a double- Λ hypernucleus there is a possibility that the following characteristic non-mesonic weak decay modes may be observed ²⁰⁾:

The ${}_{\Lambda\Lambda}^5\text{H}$ hypernucleus may be identified through the second decay mode. Two Λ particles in the ${}_{\Lambda\Lambda}^5\text{H}$ system decay into p and Σ^- , and if the decay products p and n would form an α particle, it gives a larger Q value than that of Eq. (4.1),

Since the decay process is two-body breakup instead of three-body one, the emitted Σ^- is mono-energetic which carries high energy of 82.5 MeV. Thus the high-energy discrete Σ^- would be a clear signature of the formation of the ${}_{\Lambda\Lambda}^5\text{H}$ system.

We calculate the rate of the non-mesonic Σ^- decay for the five-body system with the Green's function method ²¹⁾. The total decay width is given by

$$\Gamma^{\text{tot}} = \frac{2\sqrt{2}}{\hbar^3} (2\pi)^5 M_Z^{3/2} |\langle t | \rangle|^2 \int_0^\infty dE_Z \sqrt{E_Z} \left(-\frac{1}{\pi}\right) \text{Im} \left\{ \int dr dr' f^*(r') G_{E'}(r', r) f(r) \right\}, \quad (4.3)$$

$$f(r) = e^{-i \left(1 - \frac{M_p}{M_p + M_\Lambda}\right) k_Z r} \Psi(r, \rho), \quad \rho = \frac{M_t}{M_t + M_\Lambda} r,$$

where $\Psi(r, \rho)$ is the initial double- Λ hypernuclear state. The r -representation of Green's function is

$$G_{E'}(r', r) = \left\langle r' \left| \frac{1}{E' - \hat{H}_p + i\epsilon} \right| r \right\rangle. \quad (4.4)$$

The derivation and the partial-wave decomposition of Eqs. (4.3) and (4.4) are easily obtained from formulae in Ref. 22). The distortion of emitted Σ^- wave function, which is determined from interaction between Σ^- and the residual nucleus, is also included. We calculate the decay width in an arbitrary unit since we have no knowledge about the elementary process $\Lambda\Lambda$ to $p\Sigma^-$ either theoretically or experimentally.

The energy spectrum of the emitted Σ^- is shown in Fig. 9. If there is no interaction between t and p and between t and Σ^- , the Σ^- spectrum from two free Λ particles has a sharp line at 41.98 MeV. However, two Λ hyperons inside ${}_{\Lambda\Lambda}^5\text{H}$ would provide a broad Σ^- energy spectrum due to the Fermi motion of the hyperons in the nucleus. If we take into account the tp interaction, the strength of the decay rate is shared among every possible final states of t and p . Thus there appear a broader continuous spectrum and a small peak which corresponds to the final ground state α . The dotted curve shows this nature in Fig. 9.

Fig.9

When we include the $t\Sigma^-$ interaction further, it is found that the decay rate to the ground state is much enhanced and the higher energy part of the continuum is slightly enhanced, while the lower energy part is slightly reduced. Since the $t\Sigma^-$ interaction has a central repulsion ²³⁾, it affects the high-momentum part of the Σ^- spectrum. The $t\Sigma^-$ interaction increases the bound state decay rate by about four times. The calculation shows that the branching ratio of the ground state to the continuum is 5.5%.

The observation of mono-energetic Σ^- particles would be one clear and significant signature of the formation of ${}_{\Lambda\Lambda}^5\text{H}$. The discrete energy of this Σ^- would uniquely determine the double- Λ binding energy which in turn would provide new information concerning $\Lambda\Lambda$ interaction. The emitted Σ^- will decay into $n\pi^-$, whose neutron would be detected by a neutron counter ²⁴⁾.

§5. Summary and conclusions

The structure and the decay of the double-strangeness five-body system are investigated. The system consists of ${}_{\Lambda\Lambda}^5\text{H}$ and ${}_{\Xi}^5\text{H}$ states. It is found that the ${}_{\Lambda\Lambda}^5\text{H}$ state is an almost pure $\Lambda\Lambda$ state with a very small Ξ -component. Previous analyses of double- Λ hyperfragments which are taken as pure $\Lambda\Lambda$ states may be confirmed by the present study: Even in the five-body system where the $\Lambda\Lambda$ and Ξ thresholds are close to each other, the mixing between the two states is only 1 %.

There exists a ${}_{\Xi}^5\text{H}$ state which is bound by the cooperation of Coulomb and strong interactions. Both of the two interactions are significant in the binding mechanism of this state. While the Coulomb interaction ensures the existence of the bound state, the strong interaction keeps it as a 'halo' nuclear state. It is bound by 1.7 MeV from the $\alpha+\Xi^-$ threshold but is 6.8 MeV above the $t+\Lambda+\Lambda$ threshold. It can decay to the $\Lambda\Lambda$ channel through the $p\Xi^-$ to $\Lambda\Lambda$ conversion process. The conversion width of ${}_{\Xi}^5\text{H}$ is calculated by applying the complex rotation method. This method enables us to include not only two-body but also three-body breakup processes in the determination of the width. The conversion width is found to be only 0.2 MeV which is very narrow compared to the other Ξ -hypernuclear systems of which width is estimated to be about 1-2 MeV²⁵⁾. The narrowness is due to the forming of the α particle from t and p . It is a feature of the five-body system which is distinguished from the other few-body systems.

Another interesting feature of the $S=-2$ five-body system is found in the weak decay process of ${}_{\Lambda\Lambda}^5\text{H}$. This state can decay through the non-mesonic decay of $\Lambda+\Lambda\rightarrow\Sigma^-+p$. If the final state is $\alpha+\Sigma^-$, then high mono-energetic Σ^- may be a clear signature for detecting the formation of the ${}_{\Lambda\Lambda}^5\text{H}$ hyperfragment. The branching ratio of discrete to continuum decays is estimated to be 5.5 %.

In conclusion, the resonance state ${}_{\Xi}^5\text{H}$ is formed by the strong and Coulomb interactions and it has a very narrow conversion width of 0.2 MeV. The ${}_{\Lambda\Lambda}^5\text{H}$ state is an almost pure $\Lambda\Lambda$ state with only 1% mixture of Ξ^- component. High mono-energetic Σ^- , which is emitted from the ${}_{\Lambda\Lambda}^5\text{H}$ state through a characteristic mode of non-mesonic weak decay, may be a clear signature for the formation of the double- Λ hypernucleus ${}_{\Lambda\Lambda}^5\text{H}$.

Acknowledgments

The authors would like to dedicate this paper to late Professor Hiroharu Bandō who impressed them by his deep thought on hypernuclear physics through fruitful

communications. They benefitted so much from Bandō-Motoba-Žofka's review article 26). They are grateful to Dr. S. Shinmura and Dr. T. Harada for their kind collaborations and to Prof. Y. Yamamoto and Prof. T. Fukuda for valuable discussions. It is a pleasure to thank Prof. H. Tanaka and Prof. K. Katō for their encouragements. One of the author (K.S.M.)'s stay in Japan was financially supported by the Inoue Foundation for Science to which she owes her gratitude very much.

References

- 1) M. Danyasz et al., Phys. Rev. Lett. **11** (1963), 29.
- 2) D.J. Prowse, Phys. Rev. Lett. **17** (1966), 782.
- 3) K. Imai, Nucl. Phys. **A527** (1991), 181c.
- 4) S. Aoki et al., Prog. Theor. Phys. **85** (1991), 1287.
- 5) C.B.Dover, D.J. Millener, A. Gal and D.H. Davis, Phys. Rev. **C44** (1991), 1905.
- 6) Khin Swe Myint, Nucl. Phys. **A547** (1992), 227c.
- 7) Y. Kurihara, Y. Akaishi and H. Tanaka, Phys. Rev. **C31** (1985), 971.
- 8) R.H. Dalitz, R.C. Herndon and Y.C. Tang, Nucl. Phys. **B47** (1972), 109.
- 9) H. Piekarz, Proceedings of the International Conference on Hypernuclear and Kaon Physics (Heidelberg, 1982), 73.
- 10) A.B. Volkov, Nucl. Phys. **74** (1965), 33.
- 11) S. Shinmura, private communication.
- 12) M.M. Nagels, T.A. Rijken and J.J. de Swart, Phys. Rev. **D12** (1975), 744.; **D15** (1977), 2547; **D20** (1979), 1633.
P.M.M. Maessen, Th.A. Rijken and J.J. de Swart, Phys. Rev. **C40** (1989), 2226.
- 13) M. Kamimura, Phys. Rev. **A38** (1988), 621.
- 14) C. Ciofi degli Atti, Prog. Part. Nucl. Phys. **3** (1980), 163.
- 15) T. Harada, private communication.
- 16) J. Aguilar and J.M. Combes, Commun. Math. Phys. **22** (1971), 269.
E. Balslev and J.M. Combes, Commun. Math. Phys. **22** (1971), 280.
- 17) Khin Swe Myint and Y. Akaishi, Prog. Theor. Phys. **79** (1988), 454.
- 18) T. Yamazaki, R.S. Hayano, O. Morimatsu and K. Yazaki, Phys. Lett. **B207** (1988), 393.
- 19) T. Motoba, H. Bandō, T. Fukuda and J. Žofka, Nucl. Phys. **A534** (1991), 597.
- 20) M. May, Nuovo Cim. **102A** (1989), 401.
- 21) O. Morimatsu and K. Yazaki, Nucl. Phys. **A435** (1985), 727; **A483** (1988), 493.
- 22) I. Kumagai-Fuse and Y. Akaishi, to be published in Prog Theor. Phys.

- 23) T. Harada, S. Shinmura, Y. Akaishi and H. Tanaka,
Nucl. Phys. **A507** (1990), 715.
- 24) T. Fukuda, Preprint INS-Rep.-803.
- 25) Y. Yamamoto, Prog. Theor. Phys. **80** (1988), 757.
- 26) H. Bandō, T. Motoba and J. Žofka, Int. J. Mod. Phys. **A21** (1990), 4021.

Table I. The coupled channels in $S = -2$ two-body interactions.

	explicitly treated channels	truncated channels
1S_0	$\Lambda\Lambda \quad p\Xi^-$	$n\Xi^0 \quad \Lambda\Sigma^0 \quad \Sigma\Sigma(I=0)$
3S_1	$p\Xi^-$	$n\Xi^0 \quad \Lambda\Sigma^0 \quad \Sigma^+\Sigma^-(I=0)$
1S_0	$n\Xi^-$	$\Lambda\Sigma^-$
3S_1	$n\Xi^-$	$\Lambda\Sigma^- \quad \Sigma^0\Sigma^-(I=1)$

Table II. Shinmura's potential for $S = -2$ two-body systems in units of MeV and fm.

1S_0	$V_{\Lambda\Lambda} = V_{\text{core}} - 332.97 \exp[-(r/0.8550)^2]$
1S_0	$V_{\Lambda\Lambda, p\Xi^-} = 57.87 \exp[-(r/0.8550)^2]$
1S_0	$V_{p\Xi^-} = V_{\text{core}} - 310.93 \exp[-(r/0.8550)^2]$
3S_1	$V_{p\Xi^-} = V_{\text{core}} - 179.17 \exp[-(r/0.8550)^2]$
1S_0	$V_{n\Xi^-} = V_{\text{core}} - 188.57 \exp[-(r/0.9279)^2]$
3S_1	$V_{n\Xi^-} = V_{\text{core}} - 141.52 \exp[-(r/0.9279)^2]$
$V_{\text{core}} = 5000 \exp[-(r/0.355)^2]$	

Figure captions

Fig.1. Thresholds of the double-strangeness five-body system.

Fig.2. The $t\Lambda$ potential. The solid line is for the 1S_0 state and the dotted line is for the 3S_1 state.

Fig.3. The $t\Xi^-$ potential.

Fig.4. The three rearrangement channels with their Jacobi coordinates in a three-body system.

Fig.5. Energy position E_r and conversion width Γ of the bound ${}_{\Lambda\Lambda}^5\text{H}$ state and of the resonance ${}_{\Xi}^5\text{H}$ state with a rotation angle $\theta = 0.03$.

Fig.6. Energy levels of the coupled ${}_{\Lambda\Lambda}^5\text{H}$ and ${}_{\Xi}^5\text{H}$ states.

Fig.7. The mixture of Ξ^- component in the ${}_{\Lambda\Lambda}^5\text{H}$ state versus threshold energy of $\alpha + \Xi^-$. The dashed line represents the result when the isospin-spin factor is taken to be 1.

Fig.8. The density distribution of Ξ^- in the $\alpha\Xi^-$ system. The dot-dashed line is the $\alpha\Xi^-$ potential. The dotted line is the nucleon density distribution in the α particle.

Fig.9. The Σ^- energy spectrum from the decay of ${}_{\Lambda\Lambda}^5\text{H}$. The solid line is the result including the $t\Sigma^-$ interaction and the dotted line is the calculation without the $t\Sigma^-$ interaction.

$$28.33 \text{ --- } \overline{t + p + \Xi^-}$$

$$23.21 \text{ --- } \overline{t + n + \Xi^0}$$

$$\text{MeV} \quad 8.51 \text{ --- } \overline{\alpha + \Xi^-}$$

$$\begin{array}{c} 0 \text{ --- } \overline{t + \Lambda + \Lambda} \\ -1.27 \text{ --- } \overline{{}_{\Lambda}^4\text{H}^* + \Lambda} \\ \quad \quad \quad \overline{{}_{\Lambda}^4\text{H} + \Lambda} \end{array}$$

Fig.1

Fig.2

Fig.3

Fig.4

Fig.5

Fig.6

Fig.7

Fig.8

Fig.9