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INS-Rep.-1064
Sep. 1994

845443

HADRON-NUCLEON SCATTERING LENGTHS IN QCD SUM RULES

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Invited talk at the "International Symposium on Frontiers in Nuclear Structure Physics" at RIKEN, Japan (March 2-5, 1994)

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ABSTRACT

Recent progress of the QCD sum rule in its application to the calculation of hadron-nucleon scattering lengths is reported. After a brief introduction to the QCD sum rule, how to obtain scattering lengths in the QCD sum rule is explained. Then the results of its application are presented first for the pion-nucleon and kaon-nucleon channels and secondly for the nucleon-nucleon and hyperon-nucleon channels.

1. What is QCD Sum Rule ?

Let me start by explaining what the QCD sum rule^{1,2} is about. We show two examples of the QCD sum rules, which are related with our following discussion:

$$M_N = -\frac{8\pi^2}{M_B^2} \langle \bar{u}u \rangle_0, \quad (1)$$

$$2m_\pi^2 f_\pi^2 = -(m_u + m_d) \left(\langle \bar{u}u \rangle_0 + \langle \bar{d}d \rangle_0 \right), \quad (2)$$

where M_B is the Borel mass. The former is derived by Ioffe³ and shows that the condensation of quarks is responsible for the nucleon mass. The latter is the well-known Gell-Mann-Oakes-Renner relation,⁴ which was rederived in the context of the QCD sum rule by Shifman, Vainshtein and Zakharov.¹ Both of these are relations between observables of hadrons, such as masses and decay constants, and vacuum expectation values of quark-gluon operators.

The QCD sum rule is a nonperturbative method with which one can derive relations between physical quantities. However, it does not directly solve the QCD. Therefore, it cannot answer questions concerning such as confinement and/or spontaneous chiral symmetry breaking.

The structure of the QCD sum rules, eqs. (1) and (2), can be illustrated as follows:

Observables of 1-Hadron States $\Leftarrow$ Information on the 0-Hadron State.
(mass, decay constant) (vacuum)

What was proposed in ref. 5,6 is to extend the QCD sum rule by increasing the number of hadrons by one on both sides of the above relation:

Observables of 2-Hadron States $\Leftarrow$ Information on the 1-Hadron State.
(S-matrix)

Namely, the scattering quantities of hadrons are related with the expectation values of quark-gluon operators with respect to the one-hadron state. To be specific, we show here two examples of new QCD sum rules:^{5,6}

$$a_{NN}^1 = \frac{4\pi}{M_B^2} M_N \left(\langle \bar{d}d \rangle_p + 3 \langle u^{\dagger}u \rangle_p + \langle d^{\dagger}d \rangle_p \right), \quad (3)$$

$$\frac{1}{3} \left(a_{\pi N}^{I=1/2} - a_{\pi N}^{I=3/2} \right) = \frac{1}{4\pi} \frac{m_\pi M_N}{m_\pi + M_N} \left(\langle u^{\dagger}u \rangle_p - \langle d^{\dagger}d \rangle_p \right). \quad (4)$$

The former shows that the nucleon-nucleon scattering length is determined by the quark densities in the nucleon. The latter is a similar relation for the pion-nucleon scattering length, which reproduces the well-known Tomozawa-Weinberg relation.^{7,8}

In the next section we will see how to obtain scattering lengths in the QCD sum rule.

2. How to obtain scattering lengths in QCD Sum Rules

The central quantity we deal with is the correlation function,

$$\Pi^H(q) = -i \int d^4x e^{iqx} \langle \psi | T(\eta_H(x) \eta_H^{\dagger}(0)) | \psi \rangle. \quad (5)$$

η_H is the interpolating field for the hadron, H , i.e. a quark-gluon composite operator which creates H . The spectral function is then defined in terms of the correlation function as

$$\rho^H(\omega, \mathbf{q}) = \frac{i}{2\pi} \left\{ \Pi^H(\omega + i\eta, \mathbf{q}) - \Pi^H(\omega - i\eta, \mathbf{q}) \right\}. \quad (6)$$

The spectral function contains the information on the physical spectrum of the system:

$$\rho^H(q) = (2\pi)^3 \sum_n \left\{ \delta^4(p_\psi - p_n - q) |\langle n | \eta_H^{\dagger}(0) | \psi \rangle|^2 + \delta^4(p_\psi + p_n - q) |\langle \psi | \eta_H(0) | n \rangle|^2 \right\}. \quad (7)$$

In the physical region, $q^2 \sim m_H^2$, Π^H is nonperturbative and difficult to calculate. However, in the deep Euclid region, $q^2 \rightarrow -\infty$, Π^H is perturbative so that the operator product expansion (OPE),

$$-i \int d^4x e^{iqx} T(\eta_H(x) \eta_H^{\dagger}(0)) = C_I(q) I + C_{\bar{q}q}(q) \bar{q}q + C_{G^2}(q) G_{\mu\nu} G^{\mu\nu} + \cdots \quad (8)$$

makes sense. Namely, in the deep Euclid region the correlation function can be expressed in terms of the expectation values of quark-gluon operators.

Now, a question arises: “Is it possible to relate the physical region and the deep Euclid region ?” The answer is “Yes”. These two regions can be related by the Lehman representation of the correlation function,

$$\Pi^H(\omega, q) = \int_{-\infty}^{\infty} d\omega' \frac{\rho^H(\omega', q)}{\omega - \omega'}. \quad (9)$$

ω' in the integral runs over the physical region while ω can be taken to be any value in the complex plane, therefore in the deep Euclid region. Thus, by evaluating the left hand side by the OPE and expressing the right hand side in terms of physical quantities of hadrons with ω in the deep Euclid region, one obtains relations between observables of hadrons and the expectation values of quark-gluon operators. These relations are the QCD sum rules. One can further transform both sides of eq. (9) in order to improve the convergence of the OPE and/or the integral and some variations of the QCD sum rules are obtained. The Borel sum rule, which we use later, is one of such variations.

As we have seen, there are two essential ideas employed in order to derive the QCD sum rule, i.e. the OPE and the Lehmann representation. These two ideas are valid independent of the choice of $|\psi\rangle$ with which the correlation function is defined. In the original sum rules^{1,2} $|\psi\rangle$ is taken to be the vacuum. In ref. 5, 6 it was proposed to take $|\psi\rangle$ to be the one-nucleon state, $|N(p=0)\rangle$. Then, the reduction formula tells us that the correlation function has second-order pole terms at $q^2 = m_H^2$ with their coefficients proportional to the T -matrices for the forward elastic HN and $\bar{H}N$ scatterings:

$$T_{(\bar{H}N)}^{HN}(qp \rightarrow qp) \propto \lim_{q^2 \rightarrow m_H^2} (q^2 - m_H^2)^2 \Pi_N^H(q). \quad (10)$$

Thus, the spectral function has the following structure,

$$\begin{aligned} \rho_N^H(\omega, q=0) &\propto \delta'(\omega - m_H) T_{HN} - \delta(\omega - m_H) \delta T_{HN} \\ &+ \delta'(\omega + m_H) T_{\bar{H}N} + \delta(\omega + m_H) \delta T_{\bar{H}N} \\ &+ (\text{continuum part}), \end{aligned} \quad (11)$$

where δT_{HN} and $\delta T_{\bar{H}N}$ are given by the T -matrix and its derivative. If there is a bound state in the hadron-nucleon channel the spectral function has another first-order pole term, which will be incorporated when it becomes relevant. By substituting the above expression to the r.h.s. of eq. (9), we obtain the relation between the T -matrix and the matrix elements of quark-gluon operators with respect to the nucleon.

Before closing this section we summarize here the parameters used in the following calculations: the quark masses are

$$m_u = m_d = 7 \text{ MeV}, \quad m_s = 170 \text{ MeV},$$

the condensates of quark-gluon operators are²

$$\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0 = -(225 \text{ MeV})^3, \quad \langle \bar{s}s \rangle_0 = -(217 \text{ MeV})^3,$$

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_0 = (340 \text{ MeV})^4,$$

and the expectation values of quark-gluon operators with the nucleon are^{9,10,11}

$$\begin{aligned} \langle u^\dagger u \rangle_p &= \langle d^\dagger d \rangle_n = 2, \quad \langle u^\dagger u \rangle_n = \langle d^\dagger d \rangle_p = 1, \quad \langle s^\dagger s \rangle_n = \langle s^\dagger s \rangle_p = 0, \\ \langle \bar{u}u \rangle_p &= \langle \bar{d}d \rangle_n = 3.46, \quad \langle \bar{u}u \rangle_n = \langle \bar{d}d \rangle_p = 2.96, \quad \langle \bar{s}s \rangle_p = \langle \bar{s}s \rangle_n = 0.77, \\ i\langle S[\bar{u}\gamma_\mu D_\nu u] \rangle_p &= i\langle S[d\gamma_\mu D_\nu d] \rangle_n = 222 \text{ MeV}, \\ i\langle S[d\gamma_\mu D_\nu d] \rangle_p &= i\langle S[\bar{u}\gamma_\mu D_\nu u] \rangle_n = 95 \text{ MeV}, \\ i\langle S[\bar{s}\gamma_\mu D_\nu s] \rangle_p &= i\langle S[\bar{s}\gamma_\mu D_\nu s] \rangle_n = 18 \text{ MeV}, \\ \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle_N &= -738 \text{ MeV}, \quad \langle \frac{\alpha_s}{\pi} S[G_{\mu 0} G^{\mu 0}] \rangle_N = -50 \text{ MeV}. \end{aligned}$$

Having understood how to obtain hadron-nucleon scattering lengths in the QCD sum rule, we will discuss its applications in the following two sections.

3. Pion-Nucleon and Kaon-Nucleon Scattering Lengths

Let us start with the pion-nucleon and kaon-nucleon channels. As the interpolating field for the pion and kaon, we adopt the axial-vector current,

$$A_\mu(x) = \bar{q}_1(x) \gamma_\mu \gamma_5 q_2(x), \quad (12)$$

which has a property required for the interpolating field,

$$\langle 0 | A_\mu(0) | \varphi(k) \rangle = i\sqrt{2} f_\varphi k_\mu, \quad (13)$$

with f_φ being the pion (kaon) decay constant. The spectral functions in the vacuum and in the nucleon are assumed to be saturated by the pion (kaon) pole terms:

$$\begin{aligned} \rho_0^\varphi(\omega) &= m_\varphi f_\varphi^2 \{ \delta(\omega - m_\varphi) - \delta(\omega + m_\varphi) \}, \\ \rho_N^\varphi(\omega) &= -\frac{1}{2} f_\varphi^2 \left[\delta'(\omega - m_\varphi) T_{\varphi N} - \delta(\omega - m_\varphi) \left(T'_{\varphi N} - \frac{3}{m_\varphi} T_{\varphi N} \right) \right. \\ &\quad \left. + \delta'(\omega + m_\varphi) T_{\bar{\varphi} N} + \delta(\omega + m_\varphi) \left(T'_{\bar{\varphi} N} - \frac{3}{m_\varphi} T_{\bar{\varphi} N} \right) \right]. \end{aligned} \quad (14)$$

$T_{(\varphi N)} = T(\pm m_\varphi, 0, m_\varphi^2, m_\varphi^2)$, $T'_{(\varphi N)} = \pm \frac{\partial}{\partial \omega} T(\omega, 0, \omega^2, \omega^2)|_{\omega=\pm m_\varphi}$ with the off-shell T -matrix defined by

$$T(\nu, t, q^2, q'^2) = -i \frac{(q^2 - m_\varphi^2)(q'^2 - m_\varphi^2)}{2f_\varphi^2 m_\varphi^4} \int d^4x e^{iqx} \langle N(p) | T(\partial^\mu A_\mu(x) \partial^\nu A_\nu^\dagger(0)) | N(p') \rangle, \quad (15)$$

where $\nu = \omega + t/4M_N$, $t = (q - q')^2$ and $q + p = q' + p'$.

The correlation functions in the OPE are given up to dimension six as follows:

$$\Pi_0(\omega) = \frac{3}{8\pi^2} (m_1 + m_2)^2 \ln(-\omega^2) - (m_1 + m_2) (\langle \bar{q}_1 q_1 \rangle_0 + \langle \bar{q}_2 q_2 \rangle_0) \frac{1}{\omega^2}$$

$$\begin{aligned}
& -(m_1 + m_2)^2 \left\{ -\frac{1}{8} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \right\rangle_N \right\} \frac{1}{\omega^4}, \\
\Pi_N(\omega) = & \left(\langle q_1^\dagger q_1 \rangle_N - \langle q_2^\dagger q_2 \rangle_N \right) \frac{1}{\omega} - (m_1 + m_2) \left(\langle \bar{q}_1 q_1 \rangle_N + \langle \bar{q}_2 q_2 \rangle_N \right) \frac{1}{\omega^2} \\
& + (m_1 + m_2)^2 \left(\langle q_1^\dagger q_1 \rangle_N - \langle q_2^\dagger q_2 \rangle_N \right) \frac{1}{\omega^3} \\
& - (m_1 + m_2)^2 \left\{ 2 \left(i \langle \mathcal{S}[\bar{q}_1 \gamma_0 D_0 q_1] \rangle_N + i \langle \mathcal{S}[\bar{q}_2 \gamma_0 D_0 q_2] \rangle_N \right) \right. \\
& \quad \left. - \frac{1}{8} \left\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \right\rangle_N - \frac{1}{2} \left\langle \frac{\alpha_s}{\pi} \mathcal{S}(G_{0\nu} G_0^\nu) \right\rangle_N \right\} \frac{1}{\omega^4}, \quad (16)
\end{aligned}$$

where $\mathcal{S}[A_\mu B_\nu] = \frac{1}{2}(A_\mu B_\nu + A_\nu B_\mu) - \frac{1}{4}g_{\mu\nu}A_\lambda B^\lambda$.

From eqs. (14) and (16) we obtain

$$\begin{aligned}
2 \frac{m_\varphi^2}{M_B^2} f_\varphi^2 \exp\left(-\frac{m_\varphi^2}{M_B^2}\right) &= \frac{3}{8\pi^2} (m_1 + m_2)^2 - \frac{1}{M_B^2} \langle \mathcal{O}_2 \rangle_0 - \frac{(m_1 + m_2)^2}{8M_B^4} \langle \mathcal{O}_3 \rangle_0, \\
2 \frac{m_\varphi^2}{M_B^6} f_\varphi^2 T_{\varphi N} \exp\left(-\frac{m_\varphi^2}{M_B^2}\right) &= -\frac{m_\varphi^3}{M_B^2} \langle \mathcal{O}_1 \rangle_N + \frac{m_\varphi^2}{M_B^2} \langle \mathcal{O}_2 \rangle_N + \left(1 - \frac{m_\varphi^2}{M_B^2}\right) \langle \mathcal{O}_4 \rangle_N, \quad (17)
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{O}_1 &= q_1^\dagger q_1 - q_2^\dagger q_2 \\
\mathcal{O}_2 &= (m_1 + m_2) (\bar{q}_1 q_1 + \bar{q}_2 q_2) \\
\mathcal{O}_3 &= \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \\
\mathcal{O}_4 &= (m_1 + m_2)^2 \left[m_\varphi (q_1^\dagger q_1 - q_2^\dagger q_2) - \left\{ 2 (i \bar{q}_1 \mathcal{S}(\gamma_0 D_0) q_1 + i \bar{q}_2 \mathcal{S}(\gamma_0 D_0) q_2) \right. \right. \\
&\quad \left. \left. - \frac{1}{8} \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} - \frac{1}{2} \frac{\alpha_s}{\pi} \mathcal{S}(G_{0\nu} G_0^\nu) \right\} \right]. \quad (18)
\end{aligned}$$

We first discuss the results with the dimension-three and -four operators for the pion-nucleon channel. Neglecting higher order terms in m_π^2/M_B^2 , we obtain the T -matrices as

$$\begin{aligned}
T_{\pi N}^{(+)} &= \frac{(m_u + m_d) \langle \bar{u} u + \bar{d} d \rangle_N}{f_\pi^2} = \frac{\sigma_{\pi N}}{f_\pi^2}, \\
T_{\pi N}^{(-)} &= -\frac{m_\pi \langle u^\dagger u - d^\dagger d \rangle_p}{f_\pi^2} = -\frac{m_\pi}{2f_\pi^2}, \quad (19)
\end{aligned}$$

where $T_{\pi N}^{(\pm)} = \frac{1}{2}(T_{\pi-p} \pm T_{\pi+p}) = \frac{1}{2}(T_{\pi^+n} \pm T_{\pi^-n})$ and $\sigma_{\pi N}$ is the pion-nucleon sigma term.

The leading order term, the dimension-three operator, contributes to the isospin-odd component of the T -matrix and gives the Tomozawa-Weinberg term.^{7,8} The next-to-leading order term, the dimension-four operator, contributes to the isospin-even component of the T -matrix and gives the sigma term, which is the same as that obtained by using the PCAC and current algebra at the Weinberg point.¹²

It is interesting that the quark number in the nucleon determines the leading-order form of the T -matrix. It should be emphasized that in the present approach the T -matrix is obtained at the pion-nucleon threshold, $\nu = m_\pi$, $t = 0$, $k^2 = k'^2 = m_\pi^2$,

while in the PCAC-plus-current-algebra approach it is at the Weinberg point, i.e. $\nu = t = k^2 = k'^2 = 0$.

The calculated scattering lengths are tabulated in table 1, where the observed scattering lengths are also shown for comparison. The scattering lengths calculated with the dimension-three operator, the entries in the first column, are surprisingly close to the experimental values, which is exactly the same result as in refs. 7,8. The contribution of the dimension-four operator in the isospin-even component of the T -matrix, the entry in the second column, is much larger than the experimental value. This is similar to the fact that the T -matrix at the Weinberg point in the PCAC-plus-current-algebra approach is much larger than the experimental value at the pion-nucleon threshold. The latter difference is usually attributed to sources such as resonance and/or smooth background contributions, which cannot be determined by the symmetry argument alone. Therefore, a natural question is what cancels the sigma term contribution to the T -matrix at the pion-nucleon threshold in the present approach. There are two possibilities: one is higher order terms in the OPE and the other is the continuum contribution above the pion-nucleon threshold in the spectral function. We will come back to this question later.

We second discuss the results with the dimension-three and -four operators for the kaon-nucleon channel. Since $\Lambda(1405)$ exists below the $\bar{K}N$ threshold, we should take into account its contribution to the spectral function. This is done by the following replacement⁸:

$$\begin{aligned}
T_{K-p} &\rightarrow T_{K-p} - g_{\Lambda^*}^2 \frac{m_K^2}{m_K + M_N - M_{\Lambda^*}} \left(\frac{1}{M_{\Lambda^*} - M_N} \right)^2, \\
T'_{K-p} &\rightarrow T'_{K-p} + g_{\Lambda^*}^2 \frac{m_K}{(M_{\Lambda^*} - M_N)^2} \left(\frac{1}{M_{\Lambda^*} - M_N - m_K} \right)^2, \quad (20)
\end{aligned}$$

where g_{Λ^*} is the $\bar{K}N\Lambda(1405)$ coupling constant and M_{Λ^*} is the mass of $\Lambda(1405)$. In eq. (20) we neglected higher order terms in the binding energy, $m_K + M_N - M_{\Lambda^*}$. Having done this modification and neglecting higher order terms of m_K^2/M_B^2 , we obtain the T -matrices as

$$\begin{aligned}
T_{Kp}^{(+)} &= \frac{\sigma_{Kp}}{f_K^2} + \frac{g_{\Lambda^*}^2}{2} \frac{m_K^2}{m_K + M_N - M_{\Lambda^*}} \left(\frac{1}{M_{\Lambda^*} - M_N} \right)^2, \\
T_{Kn}^{(+)} &= \frac{\sigma_{Kn}}{f_K^2}, \\
T_{Kp}^{(-)} &= -\frac{m_K}{f_K^2} + \frac{g_{\Lambda^*}^2}{2} \frac{m_K^2}{m_K + M_N - M_{\Lambda^*}} \left(\frac{1}{M_{\Lambda^*} - M_N} \right)^2, \\
T_{Kn}^{(-)} &= -\frac{m_K}{2f_K^2}, \quad (21)
\end{aligned}$$

where $T_{KN}^{(\pm)} = \frac{1}{2}(T_{K-N} \pm T_{K+N})$ and σ_{KN} is the kaon-nucleon sigma term.

The results for the kaon-nucleon channel are similar to those for the pion-nucleon channel except for the contribution from $\Lambda(1405)$: the dimension-three operator gives the Tomozawa-Weinberg term to $T^{(-)}$ and the dimension-four operator gives the sigma term to $T^{(+)}$.

Table 1. Calculated and observed pion-nucleon and kaon-nucleon scattering lengths in the unit of fm. Experimental values are taken from ref.¹⁴ for the pion-nucleon channel and ref.¹⁵ for the kaon-nucleon channel. In the parentheses are shown the scattering lengths calculated without $\Lambda(1405)$ contribution. We showed only the errors in the calculated scattering lengths due to the $\bar{K}N\Lambda(1405)$ coupling constant.

	dim. 3	$\leq$ dim. 4	$\leq$ dim. 6	Experiment
$a_{\pi N}^{(+)}$	0	-0.07	-0.07	-0.01
$a_{\pi N}^{(-)}$	0.11	0.11	0.12	0.13
$a_{Kp}^{(+)}$	-0.67 ± 0.54	-0.97 ± 0.54	-1.02 ± 0.54	-0.50
	(0)	(-0.29)	(-0.34)	
$a_{Kp}^{(-)}$	-0.28 ± 0.54	-0.28 ± 0.54	-0.22 ± 0.54	-0.17
	(0.39)	(0.39)	(0.45)	
$a_{K n}^{(+)}$	0	-0.26	-0.31	0.10
$a_{K n}^{(-)}$	0.20	0.20	0.23	0.27

The calculated and observed scattering lengths are tabulated in table 1. If we believe the experimental values for the kaon-nucleon scattering lengths, the original Tomozawa-Weinberg results, the entries in the first column for $a_{K n}^{(\pm)}$ and those in the parentheses in the first column for $a_{K n}^{(\pm)}$, are not so successful as in the pion-nucleon channel. In the present approach, the leading terms are those with the contribution from $\Lambda(1405)$ included. Though the error of the $\Lambda(1405)$ contribution due to the experimental uncertainty of the $\bar{K}N\Lambda(1405)$ coupling constant is quite large, the inclusion of the $\Lambda(1405)$ contribution seems to improve the agreement with the observed values. However, we should also keep in mind that the experimental situation concerning the K^-p scattering length is still controversial. Namely, the scattering lengths determined by the atomic experiment, ref. 16,17,18, differ in sign from those shown in table 1, which are determined by the scattering experiment. Reliable experimental determination of the scattering length as well as the $\bar{K}N\Lambda(1405)$ coupling constant is needed.

The calculated scattering lengths including higher order terms are shown in the third column of table 1, where the Borel mass is taken to be 1 GeV. The calculated scattering lengths, however, are almost constant in the range, $M_B^2 = 1 \text{ GeV}^2 \sim 1.5 \text{ GeV}^2$. In the pion-nucleon channel the effect of the dimension-five and six operators is very small. Also in the kaon-nucleon channel the effect is small but not so extreme as in the pion-nucleon channel. This is because the kaon mass is not so small as the pion mass.

We now come back to the question concerning the discrepancy in the isospin-even component of the T -matrix in the pion-nucleon channel. Since we have observed that the effect of the dimension-five and -six terms of the OPE is so small, it is unlikely that the difference is explained by higher order terms of the OPE. Therefore, we speculate that the continuum contribution near above the pion-nucleon threshold should be responsible for the discrepancy. This will be our future problem.

4. Nucleon-Nucleon and Hyperon-Nucleon scattering lengths

Let us turn to the nucleon-nucleon and hyperon-nucleon channels. The interpolating fields of the octet baryons are taken as²

$$\begin{aligned}
 \eta_p(x) &= \epsilon_{abc} \left(u^{Ta}(x) C \gamma_\mu u^b(x) \right) \gamma_5 \gamma^\mu d^c(x) \\
 \eta_\Lambda(x) &= \sqrt{\frac{2}{3}} \epsilon_{abc} \left\{ \left(u^{Ta}(x) C \gamma_\mu s^b(x) \right) \gamma_5 \gamma^\mu d^c(x) - \left(d^{Ta}(x) C \gamma_\mu s^b(x) \right) \gamma_5 \gamma^\mu u^c(x) \right\}, \\
 \eta_{\Sigma^+}(x) &= \epsilon_{abc} \left(u^{Ta}(x) C \gamma_\mu u^b(x) \right) \gamma_5 \gamma^\mu s^c(x), \\
 \eta_{\Xi^0}(x) &= -\epsilon_{abc} \left(s^{Ta}(x) C \gamma_\mu s^b(x) \right) \gamma_5 \gamma^\mu u^c(x),
 \end{aligned} \tag{22}$$

where we have chosen p , Σ^+ and Ξ^0 as representatives of isospin multiplets.

The spectral functions in the vacuum and in the nucleon are assumed to be saturated by the nucleon (hyperon) pole terms:

$$\begin{aligned}
 \rho_0^B(\omega) &= \lambda^2 \delta(\omega - M_B) P_+ - \lambda^2 \delta(\omega + M_B) P_-, \\
 \rho_N^B(\omega) &= \lambda^2 \left\{ -T^+ \delta'(\omega - M_B) + \delta \lambda^+ \delta(\omega - M_B) \right\} P_+ \\
 &\quad + \lambda^2 \left\{ -T^- \delta'(\omega + M_B) - \delta \lambda^- \delta(\omega + M_B) \right\} P_-,
 \end{aligned} \tag{23}$$

where λ is the coupling strength of the interpolating field to the octet baryon and $P_\pm = (1 \pm \gamma^0)/2$. The continuum part is neglected for simplicity. In the spin-triplet nucleon-nucleon channel, the contribution of the deuteron to the spectral function has to be taken into account. In the nonrelativistic approximation it amounts to the following replacement:

$$a_{NN}^3 \rightarrow \tilde{a}_{NN}^3 = a_{NN}^3 + 2\pi^2 M_N B_D |\tilde{f}_D(0)|^2, \tag{24}$$

where B_D is the binding energy and $\tilde{f}_D(\mathbf{p})$ is the relative wave function of the deuteron in momentum space.

We first discuss the results for the nucleon-nucleon channel. The correlation functions in the OPE are given up to dimension six as follows:

$$\begin{aligned}
 \Pi_0^p(q) &= \frac{1}{4\pi^4} \gamma_\mu q^\mu \left[q^4 \ln(-q^2) \frac{1}{16} + \ln(-q^2) \pi^2 \left\{ m_d \langle \bar{d}d \rangle_0 + \frac{1}{8} \left(\frac{\alpha_s}{\pi} G^2 \right)_0 \right\} \right. \\
 &\quad \left. + \frac{1}{q^2} \frac{8}{3} (\pi^2 \langle \bar{u}u \rangle_0)^2 \right] \\
 &\quad + \frac{1}{4\pi^4} \left[q^4 \ln(-q^2) \frac{1}{8} m_d + q^2 \ln(-q^2) \left(-\pi^2 \langle \bar{d}d \rangle_0 \right) \right], \\
 \Pi_N^p(q) &= \frac{1}{4\pi^4} \gamma^\mu \left[q^2 \ln(-q^2) \pi^2 \left\{ -\frac{7}{3} \langle \bar{u} \gamma_\mu u \rangle_N - \frac{1}{3} \langle \bar{d} \gamma_\mu d \rangle_N \right\} \right. \\
 &\quad \left. + q_\mu q^\nu \ln(-q^2) \pi^2 \left\{ -\frac{2}{3} \langle \bar{u} \gamma_\nu u \rangle_N - \frac{2}{3} \langle \bar{d} \gamma_\nu d \rangle_N \right\} \right]
 \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{4\pi^4} \left[q^2 \ln(-q^2) (-\pi^2 \langle \bar{d}d \rangle_N) \right] \\
& + \frac{1}{4\pi^4} \gamma_\mu q^\mu \left[\ln(-q^2) \pi^2 \left\{ m_d \langle \bar{d}d \rangle_N + \frac{1}{8} \langle \frac{\alpha_s}{\pi} G^2 \rangle_N \right\} + \frac{1}{q^2} \frac{16}{3} \pi^4 \langle \bar{u}u \rangle_0 \langle \bar{u}u \rangle_N \right] \\
& + \frac{1}{4\pi^4} \gamma_\mu q^\nu \left[\ln(-q^2) \pi^2 \left\{ \frac{16}{3} i \langle S[\bar{u}\gamma_\mu D_\nu u] \rangle_N + \frac{4}{3} i \langle S[\bar{d}\gamma_\mu D_\nu d] \rangle_N \right\} \right] \\
& + \frac{1}{4\pi^4} q^\mu \left[\ln(-q^2) \pi^2 \left\{ -2m_d \langle \bar{u}\gamma_\mu u \rangle_N + 2i \langle \bar{d}D_\mu d \rangle_N \right\} \right. \\
& \quad \left. + \frac{1}{q^2} \frac{16}{3} \pi^4 \langle \bar{d}d \rangle_0 \langle \bar{u}\gamma_\mu u \rangle_N \right]. \tag{25}
\end{aligned}$$

By substituting eqs. (23) and (25) into eq. (9) we obtain

$$\begin{aligned}
a_{pp}^1 &= \frac{4}{\pi} M_p \frac{A_p M_B^4 - M_p (B_p M_B^2 + C_p)}{M_B^6 + \pi^2 (8m_d \langle \bar{d}d \rangle_0 + \langle \frac{\alpha_s}{\pi} G^2 \rangle_0) M_B^2 + \frac{64}{3} (\pi^2 \langle \bar{u}u \rangle_0)^2}, \\
\frac{1}{4} a_{pn}^1 + \frac{3}{4} \bar{a}_{pn}^3 &= \frac{4}{\pi} \frac{M_p M_n}{M_p + M_n} \frac{A_n M_B^4 - M_p (B_n M_B^2 + C_n)}{M_B^6 + \pi^2 (8m_d \langle \bar{d}d \rangle_0 + \langle \frac{\alpha_s}{\pi} G^2 \rangle_0) M_B^2 + \frac{64}{3} (\pi^2 \langle \bar{u}u \rangle_0)^2}, \tag{26}
\end{aligned}$$

where

$$\begin{aligned}
A_N &= \pi^2 \langle \bar{d}d \rangle_N + 3\pi^2 \langle u^\dagger u \rangle_N + \pi^2 \langle d^\dagger d \rangle_N, \\
B_N &= m_d \pi^2 (2 \langle u^\dagger u \rangle_N - \langle \bar{d}d \rangle_N) - 2\pi^2 i \langle \bar{d}D_0 d \rangle_N - \frac{1}{8} \pi^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle_N \\
&\quad - \frac{4}{3} \pi^2 (4i \langle S[\bar{u}\gamma_0 D_0 u] \rangle_N + i \langle S[\bar{d}\gamma_0 D_0 d] \rangle_N), \\
C_N &= -\frac{16}{3} \pi^4 (\langle \bar{d}d \rangle_0 \langle u^\dagger u \rangle_N + \langle \bar{u}u \rangle_0 \langle \bar{u}u \rangle_N). \tag{27}
\end{aligned}$$

We first concentrate on the leading order terms. The results are

$$\begin{aligned}
a_{NN}^1 &= \frac{4\pi}{M_B^2} M_N (\langle \bar{d}d \rangle_p + 3 \langle u^\dagger u \rangle_p + \langle d^\dagger d \rangle_p) = 23.2 \text{ fm}, \\
\bar{a}_{NN}^3 &= \frac{4\pi}{3M_B^2} M_N (2 \langle \bar{d}d \rangle_n + 6 \langle u^\dagger u \rangle_n + 2 \langle d^\dagger d \rangle_n - \langle \bar{d}d \rangle_p - 3 \langle u^\dagger u \rangle_p - \langle d^\dagger d \rangle_n) \\
&= 5.4 \text{ fm}. \tag{28}
\end{aligned}$$

Experimental scattering lengths are $a_{NN}^1(\text{exp}) = 23.7 \text{ fm}$ and $a_{NN}^3(\text{exp}) = -5.4 \text{ fm}$. We evaluated the deuteron pole contribution by employing the Paris potential¹⁹ and found that $2\pi^2 M_N B_D |\hat{f}(0)|^2 = 9.4 \text{ fm}$. If this contribution is added to $a_{NN}^3(\text{exp})$, we obtain $\bar{a}_{NN}^3(\text{exp}) = 4.0 \text{ fm}$.

The calculated scattering lengths are surprisingly close to these values. It is also interesting that the scalar and vector densities of quarks in the nucleon induce attraction between two nucleons.

We next take into account higher order terms. The calculated scattering lengths vary little in the region from $M_B^2 \approx 1 \text{ GeV}^2$ to 1.5 GeV^2 . By taking the maximum values, we obtain $a_{NN}^1 = 11.6 \text{ fm}$ and $\bar{a}_{NN}^3 = 2.8 \text{ fm}$.

The results are in qualitative agreement with the experimental values. Even though the leading-order results are closer to the experimental values than the full results, we do not take it seriously because of crude approximations used in the present calculation. It should be also noted that a_{NN}^1 is sensitive to the change of the interaction strength since there is almost a bound state in the spin-singlet nucleon-nucleon channel.

We next discuss the results for the hyperon-nucleon channels. The leading order results are as follows:

$$\begin{aligned}
a_{pp}^1 &= \frac{8\pi}{s} \frac{M_N M_N}{M_N + M_N} \{ \langle \bar{d}d \rangle_p + 3 \langle u^\dagger u \rangle_p + \langle d^\dagger d \rangle_p \} = 23.2 \text{ fm}, \\
\bar{a}_{pn} &= \frac{4\pi}{s} \frac{M_N M_N}{M_N + M_N} \{ \langle \bar{d}d \rangle_n + 3 \langle u^\dagger u \rangle_n + \langle d^\dagger d \rangle_n \} = 9.8 \text{ fm}, \\
\bar{a}_{\Lambda p} &= \frac{4\pi}{s - 4m_s^2} \frac{M_\Lambda M_N}{M_\Lambda + M_N} \left\{ \frac{1}{3} (2 \langle \bar{u}u \rangle_p + 2 \langle \bar{d}d \rangle_p - \langle \bar{s}s \rangle_p) \right. \\
&\quad \left. + \frac{1}{6} (7 \langle u^\dagger u \rangle_p + 7 \langle d^\dagger d \rangle_p + 10 \langle s^\dagger s \rangle_p) \right\} = 10.9 \text{ fm}, \\
\bar{a}_{\Sigma^+ p} &= \frac{4\pi}{s - 4m_s^2} \frac{M_\Sigma M_N}{M_\Sigma + M_N} \{ \langle \bar{s}s \rangle_p + 3 \langle u^\dagger u \rangle_p + \langle s^\dagger s \rangle_p \} = 9.4 \text{ fm}, \\
\bar{a}_{\Sigma^+ n} &= \frac{4\pi}{s - 4m_s^2} \frac{M_\Sigma M_N}{M_\Sigma + M_N} \{ \langle \bar{s}s \rangle_n + 3 \langle u^\dagger u \rangle_n + \langle s^\dagger s \rangle_n \} = 5.0 \text{ fm}, \\
\bar{a}_{\Xi^0 p} &= \frac{4\pi}{s - 6m_s^2} \frac{M_\Xi M_N}{M_\Xi + M_N} \{ \langle \bar{u}u \rangle_p + \langle u^\dagger u \rangle_p + 3 \langle s^\dagger s \rangle_p \} = 9.0 \text{ fm}, \\
\bar{a}_{\Xi^0 n} &= \frac{4\pi}{s - 6m_s^2} \frac{M_\Xi M_N}{M_\Xi + M_N} \{ \langle \bar{u}u \rangle_n + \langle u^\dagger u \rangle_n + 3 \langle s^\dagger s \rangle_n \} = 6.5 \text{ fm}, \tag{29}
\end{aligned}$$

where $\bar{a}_{B_1 B_2} \equiv (3a_{B_1 B_2}^3 + a_{B_1 B_2}^1)/4$, ($\bar{a}_{pn} \equiv (3\bar{a}_{pn}^3 + a_{pn}^1)/4$), and the nucleon-nucleon scattering lengths are also shown for comparison.

From the above results, one notices that the hyperon-nucleon scattering lengths are smaller than the nucleon-nucleon scattering lengths:

$$\begin{aligned}
a_{pp}^1 &> \bar{a}_{\Lambda p}, \bar{a}_{\Sigma^+ p}, \bar{a}_{\Xi^0 p}, \\
\bar{a}_{pn} &> \bar{a}_{\Sigma^+ n}, \bar{a}_{\Xi^0 n}.
\end{aligned}$$

Comparing the hyperon-proton and hyperon-neutron scattering lengths, one also notices the following inequalities:

$$\begin{aligned}
\bar{a}_{\Sigma^+ p} &> \bar{a}_{\Sigma^+ n}, \\
\bar{a}_{\Xi^0 p} &> \bar{a}_{\Xi^0 n}.
\end{aligned}$$

These inequalities can be understood by the quark densities in the parentheses of the expressions for the scattering lengths, eq. (29), (and a factor 2 in the expression for a_{pp}^1). The former inequalities are due to

$$\begin{aligned}
\langle u^\dagger u \rangle_N, \langle d^\dagger d \rangle_N &> \langle s^\dagger s \rangle_N = 0, \\
\langle \bar{u}u \rangle_N, \langle \bar{d}d \rangle_N &> \langle \bar{s}s \rangle_N = 0,
\end{aligned}$$

Table 2. Calculated and observed nucleon-nucleon and hyperon-nucleon scattering lengths in the unit of fm.

	dim. 3	$\leq$ dim. 6	Experiment
$a_{NN}^{(I=1)}$	23.2	11.6	23.7
$a_{NN}^{(I=0)}$	-6.6	-4.0	-5.4
$\bar{a}_{\Lambda N}$	10.9	6.5	—
$\bar{a}_{\Sigma N}^{(I=\frac{3}{2})}$	9.4	3.8	—
$\bar{a}_{\Sigma N}^{(I=\frac{1}{2})}$	2.8	0.6	—
$\bar{a}_{\Xi N}^{(I=1)}$	9.0	7.7	—
$\bar{a}_{\Xi N}^{(I=0)}$	4.0	2.8	—

and the latter inequalities are due to

$$\begin{aligned}\langle u^\dagger u \rangle_p &> \langle u^\dagger u \rangle_n, \\ \langle \bar{u}u \rangle_p &> \langle \bar{u}u \rangle_n.\end{aligned}$$

This implies that the number of quarks (and antiquarks) in the nucleon determines the strengths of its interaction with other baryons. It is extremely interesting to see if above inequalities among scattering lengths really hold in nature.

Experimentally, the Λ -nucleon interaction is known to be weaker than the nucleon-nucleon interaction. The scattering length, however, is not experimentally determined yet. The interactions of other hyperons and the nucleon are even less known. Experimental progress in this field is highly requested.

5. Summary

To summarize, I reported recent progress of the QCD sum rule in its application to the calculation of hadron-nucleon scattering lengths. I explained how to calculate scattering lengths in the QCD sum rule and then presented the results of its applications first for the pion-nucleon and kaon-nucleon channels and secondly for the nucleon-nucleon and hyperon-nucleon channels. It was found that the quark densities in the nucleon determines the hadron-nucleon scattering lengths in the leading order and that the calculated scattering lengths are in qualitative agreement with experimentally known scattering lengths.

In conclusion the results are promising. But, clearly much work has to be done in order to confirm the present results and clarify existing discrepancies.

Acknowledgement

I would like to thank Y. Kondo and Y. Nishino for the collaboration on which this talk is based.

I also would like to take this opportunity to express my sincere gratitude to Professor Akito Arima, who accepted me as a graduate student in his group and had great influences on me throughout my career in physics.

References

1. M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys.* **B147** (1979) 385.
2. L. J. Reinders, H. Rubinstein and S. Yazaki, *Phys. Rep.* **127** (1985) 1, and references therein.
3. B. L. Ioffe, *Nucl. Phys.* **B188** (1981) 349
4. M. Gell-Mann, R. J. Oakes and B. Renner, *Phys. Rev.* **175** (1968) 2195.
5. Y. Kondo and O. Morimatsu, *Phys. Rev. Lett.* (1982) 2015 and to be published.
6. Y. Kondo, O. Morimatsu and Y. Nishino, INS-Rep.-1059, RCNP Preprint 73.
7. Y. Tomozawa, *Nuovo Cimento* **46A** (1966) 707.
8. S. Weinberg, *Phys. Rev. Lett.* **17** (1966) 616.
9. E.D. Drukarev and E.M. Levin, *Nucl. Phys.* **A511**, 679 (1990), **A516**, 715(E) (1990)
10. T. Hatsuda and S.H. Lee, *Phys. Rev. C* **46** (1992) R34
11. X. Jin, T.D. Cohen, R.J. Furnstahl and D.K. Griegel, *Phys. Rev. C* **47** (1993) 2882.
12. T. P. Cheng and R. Dashen, *Phys. Rev. Lett.* **26** (1971) 594.
13. O. Dumbrajs et al., *Nucl. Phys.* **B216** (1983) 277.
14. R. Koch and E. Pietarinen, *Nucl. Phys.* **A336** (1980) 331.
15. A. D. Martin, *Nucl. Phys.* **B179** (1981) 33.
16. J. D. Davis et al., *Phys. Lett.* **B83** (1979) 55,
17. M. Izycki et al., *Z. Phys.* **A297** (1980) 11,
18. P. M. Bird et al., *Nucl. Phys.* **A404** (1983) 482.
19. M. Lacombe et al., *Phys. Rev. C* **21**, 861 (1980).