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Heavy Meson Decay Constants to $1/m_Q$ ¹

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Abstract

We introduce the vector meson contributions in heavy-light chiral perturbation theory (HLCPT) and construct the lagrangian and current to the order $1/m_Q$. With this, to the order $1/\Lambda_{cbb}^2$ (Λ_{cbb} is the chiral symmetry breaking scale) we calculate corrections to f_D and f_B arising from coupled-channel effects to the order $1/m_c$ and $1/m_b$. At the tree level in HLCPT, using the relativistic B-S equation with the kernel containing a confinement term and a gluon exchange term in a covariant generalization of the Coulomb gauge we calculate $f_D^{(0)}$ and $f_B^{(0)}$ when $m_Q \rightarrow \infty$ as well as the $1/m_Q$ corrections. HLCPT and the heavy quark effective theory (HQET) are matched at the scale Λ_{cbb} . Adding the perturbative and nonperturbative contributions we give the values for f_D and f_B respectively. It is found that our result is in agreement with the relation $f_D = f_B$ from lattice calculations. We also argue that there is correction to the $1/m_Q$ expansion formula for f_M from the region $p > m_Q$ which is likely of the next-to-leading order in $\ln \frac{m_Q}{\Lambda_{cbb}}$.

I. Introduction

The physics involving heavy quarks are currently the subject of intense interest. It has been shown when the quark masses approaches to infinity, new spin-flavor symmetries, which are not exhibited explicitly in QCD, are induced [1,2]. These symmetries are very powerful in reducing the number of weak decay form factors of heavy mesons and baryons [3].

However, to get the complete physics one still has to face the basic problem, the nonperturbative effects of QCD, although the new spin-flavor symmetry may greatly simplify the heavy flavor processes. For instance, the Isgur-Wise function has to be dealt with by nonperturbative methods [4]. Another simple example is the decay constant of heavy pseudoscalar mesons which are not well understood. The contributions to f_M (M denotes pseudoscalar mesons) come from both perturbative and nonperturbative aspects. In the framework of HQET the perturbative contribution can be explicitly obtained, leaving the nonperturbative form factors as unknown parameters which have to be treated by nonperturbative methods. The direct way to deal with the nonperturbative effects is lattice calculation which give the result $f_D \approx f_B$ [5]. This is in contradiction to the behaviour $f_P \sim 1/\sqrt{M_P}$. Calculations of QCD sum rules also support the lattice result [6]. Chiral perturbation theory is another very useful tool constructed by observing that in another limit of quark mass ($m_q \rightarrow 0$) QCD possesses the symmetry $SU(3)_L \times SU(3)_R$ which is spontaneously broken to $SU(3)_V$ [7]. Goldstones produced by the chiral symmetry breaking interact with the heavy mesons or baryons in HLCPT [7]. In this direction there have been a lot of works [8-13]. In ref. [9] the authors calculated the ratio f_{D_s}/f_D in HLCPT. Later on Falk [10] considered the effects of excited states on the chiral loop corrections to the decay constants. He also included the mass splitting terms since their effects are comparable with those from the Goldstones' masses. In these calculations there are no arguments about the validity of the

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Goldstone masses.

In order to have more knowledge about the decay constants of heavy mesons one has to take both the $1/m_Q$ and the $1/\Lambda_{c,b}$ corrections into account. Furthermore, when considering heavy meson M in HLCPT we are working at the scale $\bar{\Lambda} = m_M - m_Q \sim 500\text{MeV}$ which is not very small compared to $\Lambda_{c,b}$. Therefore we have to consider the contributions from vector mesons such as ρ, ω and K^* . Another reason to take vector mesons into account is that the coupling constant associated with vector mesons is numerically large although vector meson contributions are formally at the order of $1/\Lambda_{c,b}^2$. In the present paper we will include the $1/m_Q$ and vector meson contributions to find their effects on the decay constants of D and B mesons. While including the contributions from the vector mesons we have to introduce another free coupling constants, g_0 , besides g (g and g_0 are associated with the interactions of Goldstones and vector light mesons with heavy mesons respectively). These two parameters are to be determined by experiments. It will be shown that the $1/m_Q$ terms in the lagragian, besides $\frac{1}{m_Q} \text{Tr} \bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}$ which is responsible for the mass splitting, either do not contribute to the order $1/\Lambda_{c,b}^2$ or do not introduce new coupling constants after imposing reparametrization invariance. The same situation happens in the case of $1/m_Q$ corrections in the current. These calculations are done at one loop level in HLCPT which correspond to the so-called "coupled-channel" effects in say, J/ψ decay processes. To determine the value of f_M including the $1/m_Q$ corrections at the tree level in HLCPT we will use the B-S equation which has been applied to the heavy quark case [14-17]. Hence we get the nonperturbative contributions from both the tree and loop levels in HLCPT. These contributions can not be obtained from perturbative calculations.

HQET and HLCPT can be applied beyond and below $\Lambda_{c,b}$ respectively. Therefore, these two theories should be matched at the scale $\Lambda_{c,b}$. By matching these two theories we can absorb the perturbative and renormalization group correc-

tions to currents in HQET into the tree level contributions in HLCPT. Then we obtain the values of f_M to $1/m_Q$ by adding the perturbative and nonperturbative contributions.

The remainder of this paper is organized as follows. In Sect. II we briefly review the treatment of f_M in the framework of HQET and HLCPT and discuss the matching problem of these two theories. Then in Sect. III we construct the general form of the lagrangian and current in HLCPT to the order $1/m_Q$ including the contributions from vector mesons. Then we extract the terms which contribute to the decay constants to $1/\Lambda_{c,b}^2$ and try to determine constants in the lagrangian and the current by applying reparametrization invariance. In Sect. IV we use the B-S equation in the heavy quark case to get the value of $f_D^{(0)}$ and $f_B^{(0)}$ as well as the $1/m_Q$ corrections arising at the tree level in HLCPT. In Sect. V utilizing the formulas developed in previous sections we give the numerical values of f_D and f_B . Sect. V is served for conclusion and discussions.

II. f_M in HQET, HLCPT and matching

In this section we briefly review how to deal with f_M in the framework of HQET and HLCPT respectively. Then we will discuss the matching problem in these two theories.

The pseudoscalar meson decay constant f_M is defined as

$$\langle 0 | \bar{q} \gamma_\mu \gamma_5 q | M \rangle = i f_M m_M v^\mu \quad (1)$$

where q represents light quark in the meson M and v^μ is the velocity of M . The current in the HQET to the order $1/m_Q$ reads [16]

$$L^\mu = \sum_{i=1}^2 C_i(\mu) J_i + \frac{1}{2m_Q} \sum_{j=1}^6 B_j(\mu) O_j + \frac{1}{2m_Q} \sum_{k=1}^4 A_k(\mu) T_k + O(1/m_Q^2)$$

(2)

where $J_i = \bar{q}\Gamma_i Q(\Gamma_1 = \gamma_\mu, \Gamma_2 = v_\mu \text{ for vector current})$ are the leading order currents in HQET. The dimension-4 operators O_j are the local currents in HQET the explicit expresion of which can be found in ref [18]. The nonlocal operators T_k are defined by

$$\begin{aligned} T_k &= i \int dy T\{J_i(0), O_{kin}(y)\}, k=1,2 \text{ for } i=1,2 \\ T_k &= i \int dy T\{J_i(0), O_{mag}(y)\}, k=3,4 \text{ for } i=1,2 \end{aligned} \quad (3)$$

where O_{kin} and O_{mag} are the two $1/m_Q$ terms in HQET lagrangian,

$$O_{kin} = \bar{h}_v(iD)^2 h_v, \quad O_{mag} = g_s \bar{h}_v \sigma_{\mu\nu} G^{\mu\nu} h_v$$

The Wilson coefficients in eq. (2) can be determined by renormalization group equation analysis. The reparamterazition invariance can simplify this process by offering some relations among these coefficients. We refer the reader to ref. [16] for their expressions.

In HQET the matrix element on the left hand side of eq. (1) has the following form at the leading order in the $1/m_Q$ expansion,

$$\langle 0 | \bar{q} \Gamma_i h_v | M(v) \rangle = i \frac{F(\mu)}{2} \text{Tr}\{\Gamma_i \mathcal{M}(v)\}, \quad (4)$$

where $\mathcal{M}$ is the spin wave function in HQET and for pseudoscalar meson $\mathcal{M} = -\sqrt{m_M} \frac{(1+\not{v})}{2} \gamma_5$. $F(\mu)$ in the above equation can not be determined from the symmetry of HQET alone. From the equations of motion for both the heavy and the light quarks all the matrix elements of O_j do not introduce new form factors while the nonlocal terms in eq. (2) induce two new form facotrs. The dependence on

m_Q and the renormalization point μ of C_i in eq. (2) can be factorized so that to the order $1/m_Q$ [18],

$$f_M \sqrt{m_P} = \hat{C}(m_Q) F_{ren} \left\{ 1 - \frac{\bar{\Lambda}}{6m_Q} [b(\mu) + d_M B(\mu)] + \frac{1}{m_Q} [G_1(\mu) + 2d_M C_{mag}(\mu) G_2(\mu)] \right\}, \quad (5)$$

where $\hat{C}(m_Q) = \alpha_s(m_Q)^{-\frac{1}{2\beta_0}}$ is the coefficient after the extraction of the μ dependence of $C(\mu)$ ($C(\mu) = C_0(\mu) + \frac{C_1(\mu)}{4}$) and F_{ren} is also μ independent since the decay constant is a physical quantity. $d_M = -1$ and 3 for vector and axial vector current respectively. C_{mag} , $B(\mu)$ and $b(\mu)$ are known short-distance coefficients.

The form factors G_1 and G_2 in eq. (5) correspond to the nonlocal operators in the current and are defined as

$$\begin{aligned} &\langle 0 | \int dy T\{J_i(0), \mathcal{L}_1(y)\} | M(v) \rangle \\ &= i F(\mu) [G_1(\mu) + 2d_M C_{mag}(\mu) G_2(\mu)] \text{Tr}\{\Gamma_i \mathcal{M}(v)\}. \end{aligned} \quad (6)$$

The equation (5) can only be applied in the region $\mu \geq \Lambda_{cst}$.

The form factors $F(\mu)$, $G_1(\mu)$ and $G_2(\mu)$ which describe the nonperturbative effects can be dealt with in the framework of HLCPT. The weak current in HLCPT is

$$L_a^{Cv} = \frac{i\alpha}{2} \text{Tr}[\gamma^\nu (1 - \gamma_5) H_b \xi_{ba}^+] + \dots, \quad (7)$$

where the ellipsis denotes the contributions from higher dimensional operators in the expansion with respect to $1/\Lambda_{cst}$ and $1/m_Q$, α itself may contain a m_Q dependence, $a, b=1,2,3$ correspond to u, d, s quarks respectively and the superscript C denotes the quantity in HLCPT. In the above equation a convenient 4×4 Dirac matrix H_a is introduced to represent heavy mesons

$$H_i(v) = \frac{1+\not{v}}{2} (P_i^* \not{v}^\mu - \not{v}^\mu \gamma_5)$$

where P_a and $P_{a\mu}$ are the ground states with $J^P = (0^-, 1^-)$.

The matrix element to determine the decay constant in HLCPT is $\langle 0|L^{C\nu}|M(v)\rangle^C$ (we omitted the subscript a) which can be calculated below the scale Λ_{cbb} . Since the renormalization group formulae of HQET and HLCPT are applicable beyond and below Λ_{cbb} these two theories should be matched at the scale Λ_{cbb} . Hence we have

$$\langle 0|L^{C\nu}|M(v)\rangle^C|_{\mu'=\Lambda_{cbb}} = \langle 0|L^\nu|M(v)\rangle|_{\mu=\Lambda_{cbb}}. \quad (9)$$

where μ' and μ are the renormalization scales in HLCPT and HQET respectively. Obviously $\mu' \leq \Lambda_{cbb}$ and $\mu \geq \Lambda_{cbb}$. In practice the matching condition (9) is used at the same order in $1/m_Q$ expansion on both sides and at each order of $1/m_Q$ the left and right hand sides have to be calculated to the desired orders of $1/\Lambda_{cbb}$ and α_s respectively. The renormalization group running effects in HQET have been described above while unfortunately in HLCPT this is still unknown. In this paper we will neglect the running between the scales $\bar{\Lambda}$ and Λ_{cbb} and leave it to our later work.

Now we use the matching condition (9) to find the relation between the contributions in the two theories.

In the leading order in $1/\Lambda_{cbb}$ and $1/m_Q$ in HLCPT we obtain from eqs. (5) and (9) that the leading order coefficient in the current (9),

$$\alpha^{(0)} = \hat{C}(m_Q)F_{ren}^{(0)}, \quad (10)$$

where $F_{ren}^{(0)}$ is F_{ren} in the leading order in $1/\Lambda_{cbb}$.

There is, however, an $O(\frac{1}{m_Q})$ correction term from the bound state effect in the tree order current in HLCPT. It can be distinguished from $O(\frac{1}{m_Q})$ terms in the loop corrections which represent the coupled channel effect in terms of mesons. In HQET it is a part of the terms G_1 and G_2 in eq.(5). We denote this term by

$$\alpha^{(1)} = \hat{C}(m_Q)F_{ren}^{(0)}A_{BS}. \quad (11)$$

$F_{ren}^{(0)}$ and A_{BS} can be calculated from B-S equation. We do not attempt to spell out the matching condition in the $1/m_Q$ order. It is clear that within this order the correction terms b and B from hard-gluon effects and those from low energy quantum fluctuations in loop corrections in HLCPT should be added.

From the above discussions we can write f_M as

$$f_M = f_M^{(0)}(1 + A_{CC}^{(0)} + \frac{1}{m_Q}A_{BS} + \frac{1}{m_Q}A_{CC}^{(1)} + \frac{1}{m_Q}A_{HG}), \quad (12)$$

where

$$\sqrt{m_M}f_M^{(0)} = \hat{C}(m_Q)F_{ren}^{(0)}, \quad A_{HG} = -\frac{1}{6}\bar{\Lambda}(b + d_MB), \quad (13)$$

$A_{CC}^{(0)}$ and $A_{CC}^{(1)}$ are the coupled-channel corrections at the leading order and the next-to-leading order in $1/m_Q$. However, in calculations of loop corrections in the following we shall for convenience include in the heavy meson propagator the mass splitting between 1^- and 0^- heavy mesons due to the spin dependent terms in the lagrangian. That spin dependent term has no other contributions in our calculations. Therefore $A_{CC}^{(0)}$ in the following will also have a $1/m_Q$ term from the 1^- and 0^- heavy meson mass splitting. While calculating $A_{CC}^{(0)}$ and $A_{CC}^{(1)}$ we will restrict us to the corrections to order $1/\Lambda_{cbb}^2$. Although the correction of $O(\Lambda_{cbb}^{-3})$ is comparable with the correction from the order $O(m_Q\Lambda_{cbb}^{-2})$, it does not influence the ratio f_D/f_B significantly. In the present paper we will not consider the $O(\Lambda_{cbb}^{-3})$ corrections.

III. Lagrangian and current including vector meson contributions to $O(1/m_Q)$

III.1 LAGRANGIAN IN THE CASE $m_Q \rightarrow \infty$

The lagrangian for the heavy-light chiral perturbation theory in the limit $m_Q \rightarrow \infty$ has been discussed intensely in literature [8-13]. The basic idea to construct a theory describing the interactions between heavy and light hadrons is to use the fact that in the two opposite limits of quark masses, one to zero and one to infinity, QCD interactions have two symmetries: one is $SU(3)_L \times SU(3)_R$, the other one is spin symmetry $SU(2)_s$. The lagrangian for such a theory then should be invariant under $SU(3)_L \times SU(3)_R$, $SU(2)_s$, Lorentz and parity transformations. The light Goldstone bosons produced because of the chiral symmetry breaking is incorporated in the form $\xi = \exp(i\pi/f_\pi)$ with $f_\pi = 135\text{MeV}$ and π is the 3×3 matrix associated with pseudoscalar Goldstones,

$$\pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \quad (14)$$

Under $SU(3)_L \times SU(3)_R$ transformation

$$\xi \rightarrow L\xi U^\dagger = U\xi R^\dagger, \quad (15)$$

where $L \in SU(3)_L$, $R \in SU(3)_R$, and U is a nonlinear function of L , R and π .

Under $SU(3)_L \times SU(3)_R$ $H_a \rightarrow H_b U_{ba}^\dagger$ while under $SU(2)_s$ $H_a \rightarrow S H_a$ ($S \in SU(2)_s$). Under Lorentz transformation $H_a \rightarrow D H_a D^{-1}$ where D is the 4×4 matrix of Lorentz group.

Defining

$$\bar{H}_a = \gamma_0 H_a^\dagger \gamma_0 \quad (16)$$

one can find $\bar{H}_a \rightarrow U_{ab} \bar{H}_b$, $\bar{H}_a \rightarrow \bar{H}_a S^{-1}$, and $\bar{H}_a \rightarrow D \bar{H}_a D^{-1}$.

The lagrangian for the interactions between heavy mesons and pseudoscalar Goldstones in the limit $m_Q \rightarrow \infty$ has been constructed in ref. [8] where this kind of interactions are introduced by means of the covariant derivative and the axial vector field with

$$\tilde{D}_{ab}^\mu = \delta_{ab} \partial^\mu + V_{ab}^\mu, \quad (17)$$

where V_{ab}^μ is the vector field

$$V_{ab}^\mu = \frac{1}{2}(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)_{ab} = \frac{1}{2f_\pi^2}[\pi, \partial^\mu \pi]_{ab} + O(\pi^4) \quad (18)$$

which transforms under $SU(3)_L \times SU(3)_R$ as $V^\mu \rightarrow UVVU^\dagger + U\partial^\mu U^\dagger$. The axial vector field

$$A_{ab}^\mu = \frac{1}{2}(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)_{ab} = -\frac{1}{f_\pi} \partial^\mu \pi_{ab} + O(\pi^3) \quad (19)$$

transforms as $A^\mu \rightarrow UA^\mu U^\dagger$ under $SU(3)_L \times SU(3)_R$.

The light vector mesons such as ρ , ω and K^* are incorporated by regarding them as gauge fields associated with $SU(3)_V$ which is assumed to be a local symmetry group [12, 20]. Hence the covariant derivative associated with the light vector mesons is introduced [12]

$$\begin{aligned} (D^\mu H)_a &= \partial^\mu H_a - ig_V H_b \rho_{ba}^\mu, \\ (D^\mu \bar{H})_a &= \partial^\mu \bar{H}_a + ig_V \rho_{ab}^\mu \bar{H}_b, \end{aligned} \quad (20)$$

where ρ^μ represent the vector meson field and g_V is the coupling constant associated with it. The value of g_V can be determined from the chiral lagrangian for vector mesons and $\rho\pi\pi$ scattering as $g_V = 4.1$ [12]. Under $SU(3)_L \times SU(3)_R$

$$g_V \rho_{ab}^\mu \rightarrow U g_V \rho_{ab}^\mu U^\dagger - iU \partial^\mu U^\dagger \quad (21)$$

Therefore, $(V_\mu - ig_V \rho_\mu) \rightarrow U(V_\mu - ig_V \rho_\mu)U^\dagger$. Then one can construct the lagrangian including the interactions between heavy mesons and Goldstones as well as vector mesons in the limit $m_Q \rightarrow \infty$ [12],

$$\begin{aligned} \mathcal{L}_v^{(0)} = & -Tr[\bar{H}_a i v_\mu (D^\mu H)_a] + g Tr(\bar{H}_a H_b A_{ba} \gamma_5) \\ & + ig_0 Tr[\bar{H}_a H_b v^\mu \tilde{V}_{\mu ba}], \end{aligned} \quad (22)$$

where we introduce the combination

$$\tilde{V}_\mu \equiv V_\mu - ig_V \rho_\mu, \quad (23)$$

to preserve the $SU(3)_L \times SU(3)_R$ symmetry. The vector mesons should appear through the covariant derivative and the combination (23).

It is noted that the term $Tr(\bar{H}_a H_b \tilde{V}_{ba})$ is just $-Tr(\bar{H}_a H_b v \cdot \tilde{V}_{ba})$. Hence this term is not written down. Furthermore since we are interested only in the corrections to $1/\Lambda_{c\bar{c}b}^2$ we did not include the terms with more derivatives. This subject will be examined later.

The weak current needed to calculate f_D and f_B at the quark level in the limit $m_Q \rightarrow \infty$ is $\bar{q}_a \gamma^\mu (1 - \gamma_5) Q$. As shown in (7) the lowest dimensional operator corresponding to it at the hadron level is $L_a^{C\nu(0)} = \frac{ig_0}{2} Tr[\gamma^\mu (1 - \gamma_5) H_a]$. The leading order value of the constant α can be determined from (10) as $\alpha^{(0)} = f_M^{(0)} \sqrt{m_M}$. The contribution to f_M from the g term at one loop order has been calculated in Refs.[10,11]. In III.4 we will calculate the correction of f_M from g_V term at one loop order which will be called $\bar{f}_M$.

III.2 LAGRANGIAN AT THE ORDER $O(1/m_Q)$

In this subsection we construct the lagrangian and weak current to the order $1/m_Q$. The basic requirement is that the lagrangian should be invariant under

$SU(3)_L \times SU(3)_R$, Lorentz transformation and parity. First we discuss the lagrangian.

To the order $1/m_Q$

$$\mathcal{L}_v = \mathcal{L}_v^{(0)} + \frac{1}{m_Q} \mathcal{L}_v^{(1)} \quad (24)$$

Hence $\mathcal{L}_v^{(1)}$ has dimension 5. In addition g and g_0 in $\mathcal{L}_v^{(0)}$ can have $1/m_Q$ corrections. In $\mathcal{L}_v^{(1)}$ we have operators containing one additional D^μ acting on H_a or violating $SU(2)_s$ symmetry.

The most general form for $\mathcal{L}_v^{(1)}$ is then

$$\begin{aligned} \mathcal{L}_v^{(1)} = & \lambda_1 Tr[\bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}] + \lambda_2 Tr[(D_\mu \bar{H})_a (D^\mu H)_a] \\ & + i\lambda_3 Tr[\bar{H}_a \tilde{D}^\mu H_b A_{ba} \gamma_5] + \lambda_4 Tr[\bar{H}_a \tilde{D}^\mu H_b \tilde{V}_{ba}] \\ & + \lambda_5 Tr[\bar{H}_a \tilde{D}^\mu H_b \tilde{V}_{\mu ba}] + i\lambda_6 Tr[\gamma^\mu \bar{H}_a \tilde{D}_\mu H_b A_{ba} \gamma_5] \\ & + \lambda_7 Tr[\gamma^\mu \bar{H}_a \tilde{D}_\mu H_b \tilde{V}_{ba}] + \lambda_8 Tr[\bar{H}_a \sigma^{\mu\nu} H_a \rho_{\mu\nu}] + \dots, \end{aligned} \quad (25)$$

where $\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - ig_V [\rho_\mu, \rho_\nu]$ (there is another possible term with $\rho_{\mu\nu}$ replaced by $[\tilde{V}^\mu, \tilde{V}^\nu]$, it will be shown that this term has no contribution to the decay constants to the order $1/\Lambda_{c\bar{c}b}^2$.) and the ellipsis denotes the contributions from the following two categories: (i) Terms with no covariant derivatives. Such terms must contain another A_μ or $\tilde{V}_\mu$ to take place D_μ . (ii) Terms with derivatives of A and $\tilde{V}$ which are suppressed by $1/\Lambda_{c\bar{c}b}$. It will be shown later that (i) and (ii) types do not contribute. Furthermore, in eq. (25) we did not write down the term $Tr[(D^\mu \bar{H})_a H_b A_{\mu ba} \gamma_5]$ since it vanishes because of the trace operator. As mentioned before, the contribution from $Tr(\bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu})$ which is responsible to the mass splitting of say, D and D^* , can be absorbed into the meson propagator.

Now we demonstrate that to the order $1/\Lambda_{c\bar{c}b}^2$ the terms in eq. (25) either do not contribute to the decay constants of 0^- heavy mesons or do not introduce new

coupling constant by reparameterization invariance. As a first step we write out $\mathcal{L}_v^{(0)}$ in eq. (20) explicitly.

$$\begin{aligned} \mathcal{L}_v^{(0)} = & -2P_{a\mu}^{*+}(iv \cdot DP^{*\mu})_a + 2P_a^+(iv \cdot DP)_a - \frac{2g}{f_\pi}(P_{a\mu}^{*+}P_b\partial^\mu M_{ba} \\ & + P_a^+P_{b\mu}^*\partial^\mu M_{ba}) + 2ig_0(P_{a\mu}^{*+}P_b^{*\mu} - P_a^+P_b)v \cdot \tilde{V}_{ba}. \end{aligned} \quad (26)$$

From eq. (26) we can see that Goldstones couple to heavy mesons in the manner $PP^*\pi$ while for vector mesons we have $PP\rho$ and $P^*P^*\rho$ vertices. The equation of motion can be found from eq. (26) immediately

$$\begin{aligned} i(v \cdot DP)_a &= \frac{g}{f_\pi}P_{b\mu}^*\partial^\mu M_{ba} + ig_0P_bv \cdot \tilde{V}_{ba} + O(1/m_Q), \\ i(v \cdot DP^{*\mu})_a &= -\frac{g}{f_\pi}P_b\partial^\mu M_{ba} + ig_0P_b^{*\mu}v \cdot \tilde{V}_{ba} + O(1/m_Q). \end{aligned} \quad (27)$$

Since we are working to the order $1/m_Q$ the lagragian $\mathcal{L}_v^{(1)}$ can only contract with the leading order current $L_a^{C\mu(0)}$ in eq. (7) when we calculate the decay constants of heavy mesons. After trace operation we have

$$L_a^{C\mu(0)} = 2(P_b^{*\mu} - v^\mu P_b)\xi_{ba}^+. \quad (28)$$

First we consider the λ_2 term in eq. (25). This term is kinetic energy term and the constant λ_2 should be determined from reparameterization invariance. In the same way of ref. [19] we can easily find that $\lambda_2 = -\frac{1}{2}$. The λ_2 term will contract with $L_a^{C\mu(0)}$. For each line of Goldstones π there is a suppression $1/\Lambda_{c\pi}^2$. On the other hand, since vector mesons appear in the covariant derivative g_V is formally considered as of the order $1/\Lambda_{c\pi}$. Therefore, for each vector meson line there is also a $1/\Lambda_{c\pi}^2$ suppression. Bearing it in mind we can easily show that the part $D_\mu P_{a\nu}^{*+}D^\mu P_a^{*\nu}$ does not contribute to the 0^- meson decay constant to the order $1/\Lambda_{c\pi}^2$. Using the same method we can see that type (i) either only affects mass renormalization or do not contribute to the order $1/\Lambda_{c\pi}^2$.

The $\partial^\mu P \partial_\mu P$ and $PP\rho\rho$ parts from the kinetic term only affect the mass renormalization of the meson to $O(1/\Lambda_{c\pi}^2)$ and can be absorbed into the physical mass splitting Δ . Hence only $PP\rho$ vertex (with one derivative on P) may contribute. The contribution from this term will be called $f_M^{(a)}$ afterwards.

Now we demonstrate that the λ_3 term does not contribute to f_D to $1/\Lambda_{c\pi}^2$. Using the fact $\frac{1+\not{v}}{2}\gamma_\mu\frac{1+\not{v}}{2} \sim v_\mu$ we have $\bar{H}_a(DH)_b \sim \bar{H}_a(v \cdot DH)_b$. Hence the λ_3 term is proportional to

$$\frac{1}{f_\pi}[P_{a\nu}^{*+}(v \cdot DP)_b\partial^\nu M_{ba} + P_a^+(v \cdot DP)_b\partial^\nu M_{ba}]. \quad (29)$$

Substituting the equation of motion (27) we can easily find that to the order $1/\Lambda_{c\pi}^2$ the λ_3 term does not contribute. In the same way it can be shown that the λ_4 term can be ignored.

Obviously in the λ_5 term the $P^*P^*\tilde{V}$ part does not contribute. Hence we are left with the part $P_a^+D_\mu P_b\tilde{V}_{ba}$. We can also show that in λ_7 term only the part $P_a^+D_\mu P_b\tilde{V}_{ba}$ contributes by using the factor $(g_{\mu\nu} - v_\mu v_\nu)$ in the P^* propagator and partial integral technique. This term combines with the corresponding term in the g_0 term at $O(1)$ to get the reparameterization invariant form. Hence the coefficient of this term is $2g_0$ from reparameterization invariant requirement. From this term we have $f_M^{(b)}$.

After the trace operation and the analysis similar to that in the above discussion we can see that only two parts in λ_6 term contribute to $1/\Lambda_{c\pi}^2$. They are $2(D_\mu P_a^+P_b^{*\mu} + D_\mu P_a^{*\mu}P_b)v \cdot A_{ba}$. This two parts can be combined with the corresponding parts in the g term in $\mathcal{L}_v^{(0)}$ to obtain the reparameterization invariant form. Since under the velocity change $v \rightarrow v + q/m_Q$, $P_a^{*\mu} \equiv P_a^{*\mu} - v^\mu \frac{D \cdot P_a^*}{m_Q}$ picks up the phase e^{iqx} , we can determine the coefficient of these two parts. After using partial integral technique these two parts can be written as

$$\frac{2ig}{m_Q}(D_\mu P_a^{\star\mu+}P_b - P_a^+D_\mu P_b^{\star\mu})v \cdot A_{ba}, \quad (30)$$

which leads to $f_M^{(c)}$.

The vertices with at least one P^* in the λ_8 term give no contribution to the order $1/\Lambda_{c\bar{c}b}^2$ because of the factor $(g_{\mu\nu} - v_\mu v_\nu)$. The vertex with two P vanishes after the straightforward trace calculation. Obviously the term with $\rho_{\mu\nu}$ replaced by $[\tilde{V}^\mu, \tilde{V}^\nu]$ can be neglected.

At each order of $1/m_Q$ we can expand the lagrangian with respect to $1/\Lambda_{c\bar{c}b}$. For instance we have the term $Tr[\bar{H}_a H_b \sigma_{\mu\nu} \rho_{ba}^{\mu\nu}]$. In the calculation of the decay constant besides the factor $1/\Lambda_{c\bar{c}b}$ there is another suppression of $1/\Lambda_{c\bar{c}b}^2$ arising from the line of Goldstone or vector meson lines. Hence the lagrangian at $1/\Lambda_{c\bar{c}b}$ will not be considered in the present work.

In summary while calculating the decay constant to the order $1/m_Q \Lambda_{c\bar{c}b}^2$ we need not to introduce new coupling constant in the lagrangian at $1/m_Q$.

III.3 CURRENT AT THE ORDER $O(1/m_Q)$

The current in HLCPT to the order $O(1/m_Q)$ can be written as

$$L_a^{C\nu} = L_a^{C(0)\nu} + \frac{1}{m_Q} L_a^{C(1)\nu}, \quad (31)$$

where $L_a^{C(0)\nu}$ has been given in eq. (7). At $O(1/m_Q)$

$$L_a^{C(1)\nu} = b Tr[\gamma^\nu (1 - \gamma_5)(\gamma_\alpha D^\alpha H)_b] \xi_{ba}^+ + \dots, \quad (32)$$

where the ellipsis denotes the terms which are suppressed by $1/\Lambda_{c\bar{c}b}$. It can be shown easily that the change of the position of γ_α in the above equation does not lead to new terms after trace operation.

After the trace operation

$$L_a^{C(1)\nu} = -2(v^\nu D \cdot P_b^* + D^\nu P_b) \xi_{ba}^+, \quad (33)$$

where we have ignored the terms which do not contribute to the decay constant to $1/\Lambda_{c\bar{c}b}^2$ by using the equation of motion and the factor $(g_{\mu\nu} - v_\mu v_\nu)$. Eq. (33) should be combined with eq. (28) to meet the reparameterization invariant requirement. Thus we have

$$b = f_M^{(0)} \sqrt{m_M}. \quad (34)$$

Therefore the $1/m_Q$ current do not induce new parameters when we are calculating f_M to $1/\Lambda_{c\bar{c}b}^2$.

III.4 EXPLICIT CALCULATIONS OF COUPLED CHANEL CONTRIBUTIONS TO f_M AT $1/m_Q \Lambda_{c\bar{c}b}^2$

In this subsection we will apply the lagrangian and current to $O(1/m_Q)$ to calculate the corrections to f_M from the coupled chanel effects, i.e., A_{CC} in eq. (12). The $1/m_Q$ corrections fall into two categories: (i) the $O(1)$ currents contract with the $1/m_Q$ lagrangian and (ii) the $1/m_Q$ currents contract with the $O(1)$ lagrangian. As analyzed before in the first category we have $f_M^{(a)}$, $f_M^{(b)}$ and $f_M^{(c)}$ respectively. In these calculations we will meet the following integrals

$$\begin{aligned} I &= \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m^2)(v \cdot q + \Delta)}, \\ I^{\mu\nu} &= \int \frac{d^4 q}{(2\pi)^4} \frac{q^\mu q^\nu}{(q^2 - m^2)(v \cdot q + \Delta)}, \\ I^{\mu\nu\sigma} &= \int \frac{d^4 q}{(2\pi)^4} \frac{q^\mu q^\nu q^\sigma}{(q^2 - m^2)(v \cdot q + \Delta)}. \end{aligned} \quad (35)$$

These integrals are carried out using the dimensional regularization and the dependence on $\log m^2/\mu^2$ (μ is the renormalization scale) have been given explicitly

by Falk [11]. In the following we give their complete expressions in the $\overline{MS}$ renormalization scheme.

$$I = -\frac{i}{16\pi^2} [2\Delta \ln \frac{m^2}{\mu^2} + 4\Delta F(\frac{m}{\Delta}) + 2\pi\sqrt{m^2 - \Delta^2} - 4\Delta], \quad (36)$$

$$\begin{aligned} I^{\mu\nu} = & -\frac{i}{16\pi^2} \{ [\Delta(m^2 - \frac{2}{3}\Delta^2) \ln \frac{m^2}{\mu^2} + \frac{4}{3}\Delta(m^2 - \Delta^2) F(\frac{m}{\Delta}) \\ & + \Delta(\frac{10}{9}\Delta^2 - \frac{4}{3}m^2) + \frac{2\pi}{3}(m^2 - \Delta^2)^{\frac{1}{2}}] g^{\mu\nu} \\ & + [\Delta(\frac{4}{3}\Delta^2 - m^2) \ln \frac{m^2}{\mu^2} - \frac{2}{3}\Delta(m^2 - 4\Delta^2) F(\frac{m}{\Delta}) \\ & + \frac{1}{3}\Delta(5m^2 - \frac{26}{3}\Delta^2) + \frac{\pi}{3}\sqrt{m^2 - \Delta^2}(4\Delta^2 - m^2)] v^\mu v^\nu \}, \end{aligned} \quad (37)$$

$$I^{\mu\nu\sigma} |_{g^{\mu\sigma}v^\nu} = -\frac{i}{64\pi^2} (m^4 \ln \frac{m^2}{\mu^2} - \frac{3}{2}m^4 - 2m^2\Delta^2 - \Delta^4) - \Delta I^{\mu\sigma} |_{g^{\mu\sigma}}, \quad (38)$$

where in eq. (38) we only give the result for the Lorentz structure which will be used below. In eqs. (36-38)

$$F(\frac{m}{\Delta}) = \sqrt{(\frac{m}{\Delta})^2 - 1} \arctg \frac{1}{\sqrt{(\frac{m}{\Delta})^2 - 1}}. \quad (39)$$

In the above three equations μ is the scale introduced by renormalization. The dependence on μ will be canceled by the counterterms at higher order in HLCPT. Since the terms associated with $\ln m$ dominate the corrections when the light quark masses are small and can not be absorbed by renormalization [9, 10], i.e., they are independent of the renormalization scheme, in the following calculations for the decay constants we only keep these terms.

First we calculate $\bar{f}_M$ which is from the g_V term in the lagrangian at $O(1)$ and $L_a^{C(0)\nu}$. The Feynman diagram corresponding to $\bar{f}_M$ is shown in Fig. (1). While calculating the decay constant we have to extract the contributions from the wave function renormalization and mass renormalization can be absorbed into Δ and the redefinition of the heavy meson fields.

The vector mesons are expressed in the form of the following matrix [20]

$$\rho = \begin{pmatrix} \frac{1}{\sqrt{2}}(\rho^0 + \omega) & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}(\rho^0 - \omega) & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \quad (40)$$

Using the equations (36) and (37) we have the following equation after straightforward calculations

$$\bar{f}_M = -\frac{3}{32\pi^2} (g_0 - 1)^2 g_V^2 f_M^{(0)} \sum_m A_m \ln \frac{m^2}{\mu^2}, \quad (41)$$

where the sum is over ρ, ω and K^* and $A_\rho = \frac{3}{2}$, $A_\omega = \frac{1}{2}$, and $A_{K^*} = 1$.

Next we calculate $f_M^{(a)}$. We can select ∂_μ from one D_μ and ρ_μ from the other D_μ . The Feynman diagram for $f_M^{(a)}$ which contributes to wave function renormalization is also Fig. (1). It can be shown that there is no $\ln m$ dependence in $f_M^{(a)}$. Hence $f_M^{(a)} = 0$. In the same way $f_M^{(b)} = 0$.

The Feynman diagram for $f_M^{(c)}$ is shown in Fig. (2). Using the equations (36), (37) and (38) we have

$$f_M^{(c)} = -\frac{3g^2}{32\pi^2 m_M f^2} f_M^{(0)} \sum_m A_m \Delta (m^2 - \frac{4}{3}\Delta^2) \ln \frac{m^2}{\mu^2}, \quad (42)$$

where the sum is over π, K and η and $A_\pi = \frac{3}{2}$, $A_K = 1$, and $A_\eta = \frac{1}{6}$.

Finally we examine the corrections from $L_a^{C(1)\nu}$ in eq. (33). $L_a^{C(1)\nu}$ can contract with both the g and the g_0 terms in the lagrangian at $O(1)$. They are denoted as $f_M^{(d)}$ and $f_M^{(e)}$ respectively. Obviously we can only select ∂_μ from D_μ . The Feynman diagrams for $f_M^{(d)}$ and $f_M^{(e)}$ are shown in Fig. (3) and Fig. (4) respectively. It can be obtained that

$$f_M^{(d)} = \frac{3g}{16\pi^2 m_M f^2} f_M^{(0)} \sum_m A_m \Delta (m^2 - \frac{2}{3}\Delta^2) \ln \frac{m^2}{\mu^2}, \quad (43)$$

and $f_M^{(*)} = 0$.

With the above results we can give the expression for A_{CC} in eq. (12). Obviously

$$A_{CC}^{(1)} f_M^{(0)} = \sum_{i=a}^6 f_M^{(i)}. \quad (44)$$

$A_{CC}^{(0)}$ include not only $\bar{f}_M$ but also the Goldstones' loop effects. The latter have been calculated by Falk [11] where he retains the logarithmic terms. In our calculations the expression for the vertex contribution f'_M remains the same as Falk's while for f''_M which is from the wave function renormalization we use the following expression

$$f''_M = \frac{-3g^2}{32\pi^2 f^2} f_M^{(0)} \sum_m A_m(m^2 - 2\Delta^2) \ln \frac{m^2}{\mu^2} \quad (45)$$

where again the sum is over π, K and η . Hence we have

$$A_{CC}^{(0)} f_M^{(0)} = f'_M + f''_M + \bar{f}_M. \quad (46)$$

Substituting the value of masses and mass splittings we can obtain the coupled-channel contributions to the decay constants of D and B which depend on the parameters g and g_0 .

IV. Tree level in HLCPT: B-S equation approach

IV.1 REVIEW OF THE $1/m_Q$ EXPANSION IN B-S EQUATION

The $1/m_Q$ expansion can also be carried out systematically in Bethe-Salpeter equation [14-17]. The B-S wave functions of heavy mesons with arbitrary spin-parity have simple forms in the limit $m_Q \rightarrow \infty$ [16]. With some suitable models

for the kernel, the B-S wave functions to the order $1/m_Q$ can be obtained without difficulty.

Let m_1 and m_2 be the masses of the two quarks in the meson, $\mu_1 = \frac{m_1}{m_1+m_2}$, $\mu_2 = \frac{m_2}{m_1+m_2}$, P and p be the total and relative momentum of the two quarks. B-S equation for a meson has the following form

$$\chi_P(p) = S_1(\mu_1 P + p) \int \frac{d^4 q}{(2\pi)^4} G(P, p, q) \chi_P(q) S_2(-\mu_2 P + p), \quad (47)$$

with $P = m_M v$. In the following we will use the variables $p_t = v \cdot p - \mu_2 m_M$, $p_t = p - (v \cdot p)v$.

In the heavy meson case, we consider the limit $m_1 = m_Q \gg m_2 = m, m_Q \gg p_t, p_t = O(\Lambda_{QCD})$. In this limit, the residue of the r.h.s. of (47) at the pole of the heavy quark propagator S_1 in the upper-half p_t plane is of the order of $1/m_Q^2$. Therefore, we can expand S_1 as

$$S_1(\mu_1 P + p) = \frac{1 + \not{p}}{2(p_t + E_0 + m + i\epsilon)} - \frac{E_1(1 + \not{p})}{2(p_t + E_0 + m + i\epsilon)^2} + \frac{p_t^2(1 + \not{p})}{4m_Q(p_t + E_0 + m + i\epsilon)^2} - \frac{1 - \not{p}}{4m_Q} + \frac{\not{p}_t}{2m_Q(p_t + E_0 + m + i\epsilon)} + O\left(\frac{1}{m_Q^2}\right), \quad (48)$$

where $p_t^2 = -p_{t\mu} p_t^\mu$, $M - m_Q - m = E_0 + E_1 + O(\frac{1}{m_Q^2})$, E_0 and E_1 are the first two terms in the expansion of the binding energy.

The kernel G can also be expanded as

$$G = G_0 + \frac{1}{m_Q} G_1 + O\left(\frac{1}{m_Q^2}\right), \quad (49)$$

$$G_0 = G_+ + G_-,$$

where the vertex on the heavy quark line of G_+ and G_- commutes and anti-commutes with $\not{p}$ respectively.

Substituting the expansion of S_1 and G into (47) we obtain the expansion

$$\chi(p) = \chi_0 + \chi_1^+ + \chi_1^- + O(m_Q^{-2}), \quad (50)$$

where χ_0 is the leading order wave function and χ_1^+ and χ_1^- are the first order corrections satisfying $\not{p}\chi_1^\pm = \pm\chi_1^\pm$. They satisfy the following equations

$$\begin{aligned} \chi_0(p) &= -\frac{1+\not{p}}{2(p_t + E_0 + m + i\epsilon)} \int \frac{d^4q}{(2\pi)^4} G_+(P, p, q) \chi_0(q) \\ &\times \frac{p_t \not{p} + \not{p}_t + m}{p_t^2 - w_p^2 + i\epsilon}, \end{aligned} \quad (51)$$

$$\begin{aligned} \chi_1^+(p) &= -\frac{1+\not{p}}{2(p_t + E_0 + m + i\epsilon)} \int \frac{d^4q}{(2\pi)^4} G_+(P, p, q) \chi_1^+(q) \\ &\times \frac{p_t \not{p} + \not{p}_t + m}{p_t^2 - w_p^2 + i\epsilon} + J[\chi_0], \end{aligned} \quad (52)$$

$$\begin{aligned} \chi_1^-(p) &= \frac{1-\not{p}}{4m_Q} \int \frac{d^4q}{(2\pi)^4} G_-(P, p, q) \chi_1^-(q) \times \frac{p_t \not{p} + \not{p}_t + m}{p_t^2 - w_p^2 + i\epsilon} \\ &+ \frac{\not{p}_t}{2m_Q} \chi_0(p), \end{aligned} \quad (53)$$

where

$$\begin{aligned} J[\chi_0] &= \frac{p_t^2}{2m_Q(p_t + E_0 + m + i\epsilon)} \chi_0 - \frac{E_1}{p_t + E_0 + m + i\epsilon} \chi_0 \\ &- \frac{1+\not{p}}{2} \frac{\not{p}_t}{2m_Q(p_t + E_0 + m + i\epsilon)} \int \frac{d^4q}{(2\pi)^4} G_-(P, p, q) \chi_0(q) \\ &\times \frac{p_t \not{p} + \not{p}_t + m}{p_t^2 - w_p^2 + i\epsilon} - \frac{1+\not{p}}{4m_Q(p_t + E_0 + m + i\epsilon)} \int \frac{d^4q}{(2\pi)^4} G_-(P, p, q) \not{p}_t \chi_0(q) \\ &\times \frac{p_t \not{p} + \not{p}_t + m}{p_t^2 - w_p^2 + i\epsilon} + \dots, \end{aligned} \quad (54)$$

where $w_p = \sqrt{p_t^2 + m^2}$. The ellipsis in the above equation represents terms containing G_1 and the product of two G_- 's.

It has been shown in [14] that if G_0 contains only the Dirac matrices I and γ_μ on the heavy quark line the solution χ_0 of the equation (52) for 0^- and 1^- mesons are degenerate in energy E_0 and have the following form

$$\chi_0^{0-} = \frac{1+\not{p}}{2\sqrt{2}} \gamma_5 (\phi_1 - \not{p}_t \phi_2), \quad (55)$$

$$\chi_0^{1-} = \frac{1+\not{p}}{2\sqrt{2}} \not{p} (\phi_1 - \not{p}_t \phi_2). \quad (56)$$

This is a special case of the general results obtained in [16] for wave functions of heavy mesons with arbitrary spin and parity. The assumption made in obtaining the results stated above is equivalent to the condition that the $m_Q \rightarrow \infty$ limit the interaction is independent of the spin of the heavy quark.

We assume that the kernel is the sum of a static scalar confinement potential and a gluon exchange term in the Coulomb gauge in the rest frame of the meson. Written in a covariant form the kernel is $G = G_+ + G_-$, where

$$\begin{aligned} -iG_+ &= I \otimes IV_1 + \not{p} \otimes \not{p} V_2, \\ V_1 &= \frac{8\pi\kappa}{[(p_t - q_t)^2 + \delta^2]^2} - (2\pi)^3 \delta^3(p_t - q_t) \int \frac{d^3k}{(2\pi)^3} \frac{8\pi\kappa}{(k^2 + \delta^2)^2}, \\ V_2 &= -\frac{16\pi}{3} \frac{\alpha_{eff}}{(p_t - q_t)^2 + \delta^2}. \end{aligned} \quad (57)$$

$$-iG_- = -\frac{16\pi}{3} \frac{\alpha_{eff}}{(p - q)^2 + i\epsilon} [\gamma_t^\mu \otimes \gamma_{t\mu} + \frac{(\not{p}_t - \not{q}_t) \otimes (\not{p}_t - \not{q}_t)}{(p_t - q_t)^2}], \quad (58)$$

where $\gamma_t^\mu = \gamma_\mu - \not{p} v^\mu$ and the limit δ approaching zero is taken after solving the equations numerically. We approximately take the running coupling constant α_{eff} to be

$$\begin{aligned} \alpha_{eff}(p_t, q_t) &= \frac{2\pi}{\beta_0 \ln \frac{Max(p_t, q_t)}{\Lambda_{QCD}}}, \quad \text{when } \alpha_{eff} < 1, \\ \alpha_{eff}(p_t, q_t) &= 1, \quad \text{otherwise,} \end{aligned} \quad (59)$$

where $\beta_0 = \frac{25}{3}$.

Since in Coulomb gauge the kernel $\dot{G}_+$ in (57) is independent of p_t and q_t , we can obtain the 3-dimensional equation

$$\tilde{\chi}_0(p_t) = -\frac{i}{E_0 + m - w_p} \int \frac{d^3 q}{(2\pi)^3} G_+(P, p, q) \tilde{\chi}_0(q_t) \frac{w_p \not{p} - \not{p}_t - m}{2w_p}, \quad (60)$$

where $\tilde{\chi}_0(p_t) = \int \frac{dp_z}{(2\pi)^3} \chi_0(p)$. It can be obtained from (60) that the two components of $\tilde{\chi}_0$ satisfy the relation

$$\tilde{\phi}_1(p_t) = -(m + w_p) \tilde{\phi}_2(p_t), \quad (61)$$

and $\tilde{\phi}_1$ satisfies the equation

$$\begin{aligned} \tilde{\phi}_1(p_t) &= -\frac{w_p + m}{2w_p(E_0 + m - w_p)} \{A_-^0[\tilde{\phi}_1] + (w_p - m)A_+^1[\frac{\tilde{\phi}_1}{w_q + m}]\}, \\ A_\pm^j[f] &= \frac{1}{|p_t|^j} \int \frac{d^3 q_t}{(2\pi)^3} |q_t|^j P_j(\cos\theta)(V_1 \pm V_2)f(q_t). \end{aligned} \quad (62)$$

The condition for the existence of solution of inhomogeneous equation (53) determining the value of E_1 for the $O(1/m_Q)$ correction of the binding energies. The formula for E_1 can be found in [17] where the bound state energy for $0^-, 1^-$ and low-lying excited states of D, D_s, B and B_s series were calculated. It is found that good agreement with the experimental data can be obtained with the following values of parameters

$$\begin{aligned} m_c &= 1.58 \text{ GeV}, & m_b &= 5.024 \text{ GeV}, & m_u &= 0.35 \text{ GeV}, \\ m_s &= 0.465 \text{ GeV}, & \Lambda_{QCD} &= 0.38 \text{ GeV}, & \kappa &= 0.2 \text{ GeV}^2, \end{aligned} \quad (63)$$

which are quite close to the values determined from studies on quarkonium system and other considerations.

IV.2 THE DECAY CONSTANTS OF 0^- HEAVY MESONS OBTAINED FROM $1/m_Q$ EXPANSION IN B-S EQUATION

The decay constants f_D and f_B have been calculated with B-S equation in [21] where instantaneous approximation is used and the running of α_s is neglected. We shall use the formalism of $1/m_Q$ expansion described above to calculate f_M . The reason for this is two-fold. First, we want to make contact with HQET. Second, beyond the instantaneous approximation it is difficult to solve the B-S equation without the $1/m_Q$ expansion.

The decay constants of 0^- heavy mesons are obtained from the B-S equation with the formula

$$\begin{aligned} f_M^{BS} &= \frac{1}{\sqrt{m_M}} \frac{-i}{N} \int \frac{d^4 p}{(2\pi)^4} \text{tr}(\chi \gamma_5 \not{p}) \\ &= \frac{1}{N \sqrt{m_M}} \int \frac{d^3 p_t}{(2\pi)^3} \text{tr}(\tilde{\chi} \gamma_5 \not{p}), \end{aligned} \quad (64)$$

where N is the normalization constant determined by the requirement that the Isgur-Wise function of the meson M is equal to one at zero momentum transfer. For $p_t < m_Q$, we can use the wave function obtained from the equations (50-53). With the values of parameters shown in (63) we obtain the result

$$\begin{aligned} f_{D^*}^{BS} &= 0.233 + \frac{1}{m_c}(-0.090) = 0.181(\text{GeV}), \\ f_{B^*}^{BS} &= 0.199 + \frac{1}{m_b}(-0.248) = 0.149(\text{GeV}), \end{aligned} \quad (65)$$

where we have cut off the integration for f_M at $p_t = m_Q$. The above results can not be directly used to obtain parameters in HLCPT partly because the cut-off momentum is different for B and D .

To extract the parameter $F_{ren}^{(0)}$ in HLCPT from our numerical calculations it is necessary to analyze the asymptotic behaviour of the B-S wave function. For large p_t , the confinement term can be neglected. The equation (62) is reduced to

$$\begin{aligned}\tilde{\phi}_1(p_t) = & -\frac{4}{3\pi} \frac{w_p + m}{2w_p(E_0 + m - w_p)} \int dq_t \alpha_{eff}(p_t, q_t) \left[\frac{q_t}{p_t} \ln \frac{|p_t + q_t|}{|p_t - q_t|} \right. \\ & \left. + \frac{q_t^2 w_p - m}{p_t^2 w_p + m} \left(1 - \frac{p_t^2 + q_t^2}{2p_t q_t} \ln \frac{|p_t + q_t|}{|p_t - q_t|} \right) \right] \tilde{\phi}_1(q_t).\end{aligned}\quad (66)$$

It can be shown that for $p_t \gg \Lambda_{QCD}$ the integral on the r.h.s. of the above equation is dominated by the contribution of the region $q_t \sim \lambda p_t, \lambda \ll 1$. By equating the coefficients of the leading log term of both sides of eq. (66) we find

$$\tilde{\phi}_1(p_t) \sim \frac{1}{p_t^3} \left(\ln \frac{p_t}{\Lambda_{QCD}} \right)^{-a}, \quad (67)$$

with $a = \frac{17}{25}$. This leads to the behavior

$$f_{M<}^{BS} \sim \left(\ln \frac{m_Q}{\Lambda_{QCD}} \right)^{\frac{1}{25}} \left[1 + O \left(\frac{1}{\ln \frac{m_Q}{\Lambda_{QCD}}} \right) \right], \quad (68)$$

for f_M^{BS} at the leading order in $1/m_Q$.

Notice that, the contribution of hard gluons with momentum $q_t > \mu \sim \Lambda_{csb}$ is a part of corrections included in the renormalization group formula of HQET. To determine the parameter $F_{ren}^{(0)}$ in HLCPT, this factor should be separated out. The asymptotic behavior of $f_{M<}^{BS}(m_Q)$ in (68) does not agree with the renormalization group result in HQET which gives the behavior $[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}]^{-\frac{1}{25}}$ in the leading log order. This can be understood since we are not using the same approximation as the one loop renormalization group analysis. In fact, from the diagram for the correction to the weak current vertex $\gamma_5 \not{p}$ from the Coulomb propagator in Fig.5, we obtain the vertex renormalization constant Z_1 and its contribution to the anomalous dimension of the weak current in HQET

$$\begin{aligned}Z_1 &= 1 + \frac{4\alpha_s}{3\pi} \frac{1}{\epsilon} + \text{finite term}, \\ \gamma_1 &= -\frac{16}{3}.\end{aligned}\quad (69)$$

The contribution of this term to the renormalization group factor of the weak current just gives the behavior in (68).

From the above discussions it is clear that $F_{ren}^{(0)}$ in HLCPT can be identified as

$$F_{ren}^{(0)} = \sqrt{m_M} (\alpha(m_Q))^{\frac{1}{25}} \times (\text{leading log term in } f_{M<}^{BS}). \quad (70)$$

Fitting our numerical data from the solution of B-S equation with the formula

$$\tilde{\phi}_1(p_t) = \frac{c_1}{p_t^3 + a} [\beta_0 \frac{\alpha_{eff}(p_t)}{2\pi}]^{\frac{1}{25}} [1 + c_2 \alpha_{eff}(p_t)], \quad (71)$$

we find that the set of parameters $c_1 = 0.124 GeV^3$, $a = 0.34 GeV^3$, and $c_2 = 2.5$ works well. Hence we obtain from eq. (70) and our numerical data the contribution to $F_{ren}^{(0)}$ at the leading order in $1/m_Q$

$$F_{ren}^{(0)} = \frac{1}{N} \frac{25}{8\sqrt{2}\pi^2} c_1 = 0.281 GeV^{\frac{1}{2}}. \quad (72)$$

Combining the above equation with eq. (13) we find that

$$f_D^{(0)} = 0.260 GeV, \quad f_B^{(0)} = 0.170 GeV. \quad (73)$$

We can extract the leading log term in the $1/m_Q$ correction to f_M^{BS} by separating the term in $\tilde{\chi}_1^+$ and $\tilde{\chi}_1^-$ behaving as $1/p_t^3$ asymptotically. From the structure of the equations (52) and (53) we can approximately fit the numerical data by

$$\tilde{\phi}_1^+ = \frac{d_1 p_t + d_2}{p_t^3 + a'} [\beta_0 \frac{\alpha_{eff}(p_t)}{2\pi}]^{\frac{1}{25}} [1 + d_3 \alpha_{eff}(p_t)]. \quad (74)$$

where

$$\tilde{\chi}_1^+ = \frac{1 + \not{p}}{2\sqrt{2}} (\tilde{\phi}_1^+ - \not{p}_t \tilde{\phi}_2^+).$$

The fitting for $\tilde{\chi}_1^+$ can be directly obtained from $\tilde{\phi}_1(p_t)$ and (61) while it is a little subtle for $\tilde{\chi}_1^+$ which changes sign at low momentum. Since it is difficult to fit

$\tilde{\chi}_1^+$ well with the numerical result from B-S equation in the whole momentum range from zero to m_Q we try to give a good fitting in the region from 1 GeV to m_Q . This does not directly affect the contribution from hard gluons, i.e., the leading log behaviour of $\tilde{\chi}_1^+$. Numerically the contribution from the region $p_t < 1\text{GeV}$ is small. It is found that the following parameters can be used: $a' = 0, d_1 = -0.3\text{GeV}^2, d_2 = -0.3\text{GeV}^3, d_3 = -1.2$.

Therefore, after removing the factor already taken into account in hard gluon corrections we obtain

$$F_{ren}^{(0)} A_{BS} = \frac{1}{N} \frac{25}{8\sqrt{2}\pi^2} (d_2 - 3a'd_1) - \frac{m_u}{2} F_{ren}^{(0)} = -0.160\text{GeV}^{\frac{1}{2}}, \quad (75)$$

which leads to the contributions to f_D and f_B as -0.092GeV and -0.019GeV respectively in eq. (5).

IV.3 A PROBLEM OF THE $1/m_Q$ EXPANSION

The above analysis also reveals that there is a shortcoming in the $1/m_Q$ expansion analysis for f_M . Since the integral for f_M does not converge quickly below $p_t = m_Q$ one must consider the contribution to f_M from $p_t > m_Q$. We emphasize that this is not only a problem for the B-S approach but also for HQET when applied to f_M . This is because for large p the contribution from single gluon exchanges dominates and the B-S approach used by us is good in this region.

Since f_M is a physical quantity the integral over $\text{tr}(\chi_M \gamma_5 \not{p})$ must converge. We have not been able to obtain the asymptotic behavior of χ_M when $p_t \gg m_Q$ in the Coulomb gauge used by us. This can be done in the Feymann Gauge for one gluon exchange kernel. The asymptotic equations are almost the same as that for the π meson. Differences between these two cases do not affect the leading order analysis. In [22,23] these asymptotic equations were investigated analytically and numerically. The leading terms in the four dimension wave function $\text{tr}(\chi_M \gamma_5 \not{p})$ are

of the order $[(p^2)^2 \ln p^2 / \Lambda_{QCD}^2]^{-1}$. But they cancel each other. It can be seen from these analysis that the remaining term is at most $[p^2 \ln p^2 / \Lambda_{QCD}^2]^{-2}$. Therefore the integral for f_M converge. The contribution from $|p^2| \geq m_Q^2$ is at most of the order $c(\ln \frac{m_Q^2}{\Lambda_{QCD}^2})^{-1}$. The constant c contains a factor depending on m_Q . This dependence is from the initial value of the wave function at $p_t \sim m_Q$ which can be extracted from the wave function in the region $p_t \leq m_Q$.

In general, if

$$\begin{aligned} \text{tr}(\chi_M \gamma_5 \not{p}) &\sim \frac{c}{p^4} (\ln \frac{p}{\Lambda_{QCD}})^{-(1+\alpha)}, & p_t > m_Q, \\ \text{tr}(\tilde{\chi}_M \gamma_5 \not{p}) &\sim \frac{c'}{p^3} (\ln \frac{p}{\Lambda_{QCD}})^{\beta}, & p_t < m_Q, \end{aligned} \quad (76)$$

where c' is m_Q independent. From matching the values of the wave function at $p_t \sim m_Q$ we obtain $c \sim c'(\ln \frac{m_Q}{\Lambda_{QCD}})^{1+\alpha+\beta}$. Therefore, in the leading log order the integral in the two regions are of the order $(\ln \frac{m_Q}{\Lambda_{QCD}})^{\beta}$ and $(\ln \frac{m_Q}{\Lambda_{QCD}})^{1+\beta}$ respectively. We conclude that the $1/m_Q$ expansion formula (12) has additional corrections from the region $p > m_Q$, which is of the next-to-leading order in the power of $\ln \frac{m_Q}{\Lambda_{QCD}}$.

IV. Numerical Results

In the numerical evaluations we have two coupling parameters g and g_0 the values of which are not known exactly. For g we have the upper bound $g^2 < 0.5$ from the experimental upper limit of D^{*+} decay rate. There have been several approaches to deal with the value of g which give different results [16, 24]. In our evaluations we let g change from 0.4 to 0.7.

Comparing to g , g_0 is more arbitrary. $g_0 - 1$ is associated with the vertices $PP\rho$ and $P^*P^*\rho$. This kind of interaction has to be considered when the energy scale is not very low comparing to Λ_{csb} . It is not easy to give an exact value of g_0 . However, if the scattering of π and the matter field Π is dominated by ρ exchange the g_0

term should be small since this term violates the so-called ρ coupling universality [20]. Hence g_0 should be near zero and in our calculations we let it change from 0 to 0.5.

From eq. (12) we can obtain the numerical results for f_D and f_B with the help of the analysis in the previous sections. In the calculations we use $\mu = 1\text{GeV}$, $\Delta_{DD^*} = 0.140\text{GeV}$, and $\Delta_{BB^*} = 0.046\text{GeV}$. Our numerical calculations show that each of the correction terms from B-S approach, coupled-channel effects, and the hard gluon contribution make f_D approach to f_B and the final results are shown in Figs. 6 and 7 respectively. It can be seen that in the ranges of g and g_0 $f_D = f_B$ holds very well. This supports Lattice result. The values for both f_D and f_B are around 200MeV which is also in agreement with the Lattice calculations [5]. From Figs. 6 and 7 we can also see that the dependence of f_D and f_B on g and g_0 is not sensitive: the change is about twenty percent.

V. Summary and Discussions

In this paper we introduce the vector meson contributions in HLCPT based on the considerations that the scale about 500MeV at which we are working is not very small compared with $\Lambda_{c\bar{c}b}$ and the coupling constant associated with vector mesons g_V is numerically not small. Then we construct the lagrangian and current to the order $1/m_Q$ in order to consider the corrections which will affect both the values of f_D , f_B and the relation f_D/f_B . Then we calculate the corrections to the decay constants to the order $1/\Lambda_{c\bar{c}b}^2$ from coupled-channel contributions and bound state effects in the tree order in HLCPT in addition to the known hard gluon contributions. From the matching condition of HLCPT and HQET at the chiral symmetry breaking scale $\Lambda_{c\bar{c}b}$ these three parts are added together to give the total values of f_D and f_B respectively. It is found that in the expected ranges of g and g_0 the relation $f_D = f_B$ holds very well. The absolute values for f_D and

f_B are about 200MeV. This is in agreement with lattice calculations. Although the coupling constants g and g_0 are not known exactly, it is found that in the reasonable ranges of g and g_0 our numerical results change only by about twenty percent.

As in HQET, our calculations are restricted to the region $p_t < m_Q$. It is argued that the correction from the region $p_t > m_Q$ will be at the next-to-leading order in $\ln \frac{m_Q}{\Lambda_{c\bar{c}b}}$.

In this paper we did not include the corrections from the order $\Lambda_{c\bar{c}b}^{-3}$ which may be comparable with the $1/m_Q \Lambda_{c\bar{c}b}^2$ order corrections. This will be more difficult since we have to introduce some new free parameters for which we have not found a way to reduce the number of them. However, they will not affect the ratio f_D/f_B significantly.

In our calculations for the coupled-channel effects $\mu = 1\text{GeV}$ is chosen and we only keep the $\ln m^2$ terms which are renormalization scheme independent and have logarithmic enhancement. In principle the renormalization scale dependence should be canceled from higher order counterterms in the chiral perturbation theory. This will affect the finite terms besides $\ln m^2$ and this needs further research.

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Figure Captions

Fig.1-Fig.4 The diagrams contributing to f_M . The cross denotes the axial vector current vertex and $P(P^*)$ can be $D, B(D^*, B^*)$. (Fig.1) and (Fig.2) are the wave function renormalization correctons from ρ and π loops respectively while (Fig.3) and (Fig.4) are the vertex corrections from π and ρ loops.

Fig.5 The diagram for the correction to the weak current vertex from the Coulomb propagator.

Fig.6-Fig.7 The plots for f_D and f_B where the dependence on g and g_0 are shown.

Fig.1

Fig.2

Fig.3

Fig.4

Fig.5

Fig.6

Fig.7