Abstract

We demonstrate that the final state $p$-$n$ interaction in the reaction of electrodissociation of the deuteron at large $Q^2$ in a static external field leads to the apparent reacceleration of neutrons. The shift of the neutron velocity from the velocity of the deuteron beam is related to the quantum-mechanical forward-backward asymmetry of the missing momentum distribution in the $^2H(\epsilon, \epsilon'p)n$ scattering.

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In the recent experiment [1] an intriguing velocity shift between the neutrons and $^9Li$ fragments in the Coulomb dissociation of $^{11}Li$ nuclei was observed. The observation was qualitatively interpreted as an effect of classical deceleration of $^{11}Li$ and acceleration of $^9Li$ in the Coulomb field and led to the discussion of the Coulomb reacceleration as a clock for nuclear reactions [1,2]. The subsequent quantum-mechanical treatment of the reacceleration effect did not support this classical interpretation, though ([2,3], see also [4-6]).

In this paper we address the contribution of final state interaction (FSI) between the fragments of electrodisassociation into the shift of the average velocity of fragments from the beam velocity. We consider the shift of the velocity of neutrons in the simplest case of the Coulomb dissociation of deuterons at large momentum transfer squared $Q^2$. The deuteron is particularly suited for our investigation as it is the simplest example for the mechanism of reacceleration by FSI and as there is a realistic wave function calculated from the Bonn potential [7] available and FSI in $^2H(e,e'p)n$ is well understood [8]. The neutron does not interact with the Coulomb field, and the shift $\Delta v_z$ of the average velocity of neutrons from the velocity of the deuteron beam only can come from FSI between the neutron and the proton. We consider the case of weak Coulomb field which is treated to the first order, and relate the velocity shift $\Delta v_z$ to the quantum-mechanical FSI effect [8,9] of the forward-backward asymmetry of the missing momentum distribution $W(p_m)$ in the related $^2H(e,e'p)n$ reaction. For the large $Q^2$, we derive a simple relationship between the velocity shift $\Delta v_z$ and the neutron-proton forward scattering amplitude. Depending on the real part of the $n-p$ scattering amplitude, both the apparent deceleration and acceleration of neutrons are possible.

The kinematics of the $^2H(e,e'p)n$ reaction is usually described in the laboratory frame in which the deuteron is at rest, the virtual photon has the 4-momentum $q = (\nu, \vec{q})$ and $Q^2 = -q^2$, the struck proton is detected with the momentum $\vec{p}$ and the spectator neutron carries the missing momentum $\vec{p}_m = \vec{q} - \vec{p}$. The same process can also be viewed in the Breit frame in which $\nu = 0$ and $q = (0, 0, 0, -\sqrt{Q^2})$, often used in the parton model description of deep inelastic lepton-nucleon scattering. In the Breit frame, the beam of
deuterons with velocity $\vec{v}_D$ along the $z$-axis dissociates in a static external Coulomb field. In the laboratory frame in the nonrelativistic case, $\vec{v}_D = 0$ and $\Delta\vec{v}_{lab}^{\text{lab}} = \vec{v}_n - \vec{v}_D = \vec{v}_n = \frac{\vec{p}_n}{m_n}$. The average velocity shift $\Delta v_z$ equals in the nonrelativistic case in the Breit frame

$$\Delta v_z = \langle \Delta v_z^{\text{Breit}} \rangle = -\frac{1}{m_n} \langle p_{m,z}^{lab} \rangle$$

and vanishes unless the missing momentum distribution $W(\vec{p}_m)$ has a forward-backward asymmetry.

In the relativistic case, the velocity shift depends on the reference frame, and it is more convenient to consider the shift of the average rapidity of neutrons $y = \frac{1}{2} \log \frac{1 + v_z}{1 - v_z}$ from the rapidity $y_D$ of deuterons, $\Delta y = \langle y_n \rangle - y_D$, which is the same in all reference frames. As $v_D = 0$ in the laboratory frame, $y_{D,lab} = 0$, and

$$\Delta y = -\langle y_{lab}^{lab} \rangle = -\frac{1}{2} \left( \log \frac{m_n + p_{m,z}}{m_n - p_{m,z}} \right) \approx -\frac{1}{m_n} \langle p_{m,z}^{lab} \rangle = \Delta v_z,$$  

where we have assumed $E_{lab}^{lab} \approx m_n$.

Now we focus on the calculation of the average missing momentum $\vec{p}_m$ in the laboratory frame, where the deuteron is initially at rest. For the sake of simplicity, we neglect the magnetic interaction of nucleons and the spin dependence of the proton-neutron scattering amplitude. Then, for the unpolarized deuterons, one finds the missing momentum distribution [8]

$$W(\vec{p}_m) = \frac{1}{4\pi(2\pi)^3} \int d^3\hat{r} d^3\hat{r}' \exp[i\vec{p}_m \cdot (\vec{r}' - \vec{r})] S(\vec{r}) S'(\vec{r}') \left[ \frac{u(r)}{r} \frac{u'(r')}{r'} + \frac{1}{2} \frac{w(r)}{r} \frac{w'(r')}{r'} \left( \frac{(\vec{r} \cdot \vec{r}')^2}{(rr')^2} - 1 \right) \right],$$

where $u/r$ and $w/r$ are the radial wave functions of the S and D wave states of the deuteron, with the conventional normalization $\int dr (u^2 + w^2) = 1$, and $S(\vec{r})$ is the FSI operator. In this paper we consider the case of large $Q^2$ and high kinetic energy of the struck proton $T_{kin} \approx Q^2/2m_p$, such that FSI can be described by the Glauber theory [10]. Defining the transverse and longitudinal components $\vec{r} \equiv (\vec{b} + z\vec{q})$, where $\vec{b}$ and $\vec{q}$ are orthogonal, we can write

$$S(\vec{r}) = 1 - \theta(z) \Gamma(\vec{b}),$$

where $\theta(z)$ and $\Gamma(\vec{b})$ are the Fermi distribution and the scattering amplitude, respectively.
where $\Gamma(\tilde{b})$ is the profile function of the proton-neutron scattering. The Glauber theory representation (3) assumes predominantly forward scattering [10], and in the nucleon-nucleon and nucleon-nucleus scattering it was shown to hold at $T_{\text{kin}} \gtrsim 0.5 \text{ GeV}$, i.e., at $Q^2 \gtrsim 1 \text{GeV}^2$ (for a comprehensive review see [11]). At high energy, the profile function can conveniently be parameterized as

$$\Gamma(\tilde{b}) \equiv \frac{\sigma_{\text{tot}}(1 - i\rho)}{4\pi b_o^2} \exp \left[-\frac{\tilde{b}^2}{2b_o^2}\right] = (1 - i\rho)\Gamma_a(b),$$

(5)

where $\rho$ is the ratio of the real to imaginary part of the forward elastic scattering amplitude, and $b_o^2$ is the diffraction slope of elastic scattering, $d\sigma_{\text{el}}/dt \propto \exp(-b_o^2|t|)$, where $t$ is the square of the momentum transfer.

Introducing new variables $\tilde{r} = \tilde{r}' - \tilde{r}$ and $\tilde{R} = \frac{1}{2}(\tilde{r}' + \tilde{r})$ and integrating by parts, we can write

$$\int d^3\tilde{p}_m\tilde{p}_mW(\tilde{p}_m) = \frac{i}{4\pi} \int d^3\tilde{R}\frac{\partial}{\partial \tilde{s}} \left\{ S(\tilde{r})S^1(\tilde{r}') \left[ \frac{u(r)u(r')}{r - r'} + \frac{1}{2} \frac{w(r)w(r')}{r - r'} \left( 3\left( r \cdot r' \right)^2 - 1 \right) \right] \right\} \tilde{s} = 0$$

$$= \hat{q}\int d^3\tilde{b}\Gamma_a(b)\frac{1}{4\pi b_o^2}[v(b)^2 + w(b)^2],$$

(6)

where $\hat{q}$ is the unit vector in z direction. Here we made an explicit use of equation (3) for the FSI operator. Because of the attenuation of the flux of protons by FSI, $\int d^3\tilde{p}_mW(\tilde{p}_m) < 1$, but this departure from unity is small, $\sim 7\%$ [8,12], and to this accuracy

$$\langle \tilde{p}_m \rangle = \frac{\int d^3\tilde{p}_m\tilde{p}_mW(\tilde{p}_m)}{\int d^3\tilde{p}_mW(\tilde{p}_m)} \approx \int d^3\tilde{p}_m\tilde{p}_mW(\tilde{p}_m).$$

(7)

At $T_{\text{kin}} \gtrsim 0.5 \text{ GeV}$, the radius of n-p FSI is small. Typically, $b_o \sim 0.5 \text{ fm}$ [11,13], and we have a strong inequality $b_o^2 \ll R_D^2$, where $R_D \sim 2 \text{ fm}$ is the deuteron radius. This leads to a simple estimate

$$\Delta v_z \sim -\rho\frac{\sigma_{\text{tot}}(np)}{4\pi m_n R_D^3},$$

(8)

which is not sensitive to $b_o$.

This apparent reacceleration of neutrons is the purely quantum-mechanical effect of an interference between the plane wave and FSI components of the electrodissoication amplitude [8]. We emphasize that it is not the effect of higher orders in an external Coulomb
The sign of the velocity shift only depends on the sign of the real part of the forward proton-neutron scattering amplitude. The Glauber approximation considered here is applicable when many partial waves contribute to the $n - p$ FSI. The opposite limiting case of small $Q^2$ and very low excitation energies, when FSI effects can be modelled by the zero-range interaction which takes place only in the $S$-wave, was considered in [3]. In Ref. [3] a similar conclusion that the sign of the reacceleration effect depends on the sign of the scattering phase, was reached. With $\sigma_{tot}(np) \sim 40 \text{mb}$, $\rho \sim -0.4$ and $b_0 \sim 0.5 \text{fm}$, typical of the $N-N$ interaction at $T_{kin} \gtrsim 1.0 \text{GeV}$ [10,12,13], equation (7) gives the shift of the velocity of neutrons $\Delta v_z \sim 0.0026$, which is very close to the estimate (8). The estimate (8) makes it obvious that $\Delta v_z$ is very small because of the deuteron being a dilute system, which makes the $n - p$ FSI weak. The results of numerical calculations with the realistic Bonn wave function [7] are presented in Fig. 1. The parameters $\sigma_{tot}, \rho$ and $b_0$ have been taken from [13,14]. The $Q^2$ dependence of $\langle \Delta v_z \rangle$ is due to the energy dependence of $\rho$ as given by dispersion relation calculations as reported in [14].

To summarize, the purpose of this note has been a derivation of the FSI generated effect of the apparent reacceleration of neutrons in the electrodissoociation of fast deuterons in a static external field. We related the apparent reacceleration to the quantum-mechanical effect of the FSI induced forward-backward asymmetry of the missing momentum distribution in the related $^2H(\epsilon, \epsilon'p)n$ scattering. The quantal physics of the apparent reacceleration by FSI is simple and both the acceleration and deceleration effects are possible depending on the sign of the real part of the $p-n$ forward scattering amplitude. The forward-backward asymmetry of $W(\vec{p}_m)$ and $\langle \vec{p}_{m,z} \rangle$ can, of course, be directly measured in the traditional $^2H(\epsilon, \epsilon'p)n$ experiments. The measurement of the velocity shift $\langle \Delta v_z \rangle$ in the Coulomb dissociation of relativistic deuterons is not feasible, at least at the large $Q^2$ of interest for the present formalism. The point is that for the finite, and large, size of nuclei the electromagnetic dissociation of deuterons will be masked by diffraction dissociation $dA \rightarrow (pn)A,(pn)A^*$, induced by strong nuclear interaction [6,15].

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References


Figure captions:

Fig. 1 The shift of the average velocity of neutrons $\Delta v_z$ from the velocity of deuterons in the electrodisintegration of deuterons vs. $Q^2$. 