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**Effective nucleon mass, incompressibility and
third derivative of nuclear binding energy in
the nonlinear relativistic mean field theory**

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ABSTRACT

We have studied the equations of state(EOS) of nuclear matter using the nonlinear σ - ω model. At the normal density, there is strong correlation among the effective nucleon mass M_0^* , incompressibility K and third derivative K' of binding energy. The K' is small when $M_0^* \approx 0.6 \sim 0.8M$ where M is nucleon mass. The results are compared with the empirical analysis of the giant isoscalar monopole resonances data. It is difficult to fit the data when $K \lesssim 200\text{MeV}$. Furthermore, to fit the data, the EOS becomes very soft even in the higher density region because of small value of K' , when we assume incompressibility $K \lesssim 250\text{MeV}$.

I. INTRODUCTION

Nuclear matter has been studied in the framework of quantum hadrodynamics(QHD). The meson mean-field model for nuclear matter [1] has made successful results to account for the saturation properties at the normal density. The original Hartree calculation[1] and the Hartree-Fock calculation [2] give larger incompressibility K than the empirical value [3][4][5]. This quantity has an important role in describing the features of nuclear matter and neutron matter at high densities[6], and is calculated in the many improved methods [7]. The correlation between K and effective nucleon mass M_0^* at the normal density ρ_0 are pointed out in ref. [8].

On the other hand, there are two ways to get the empirical value from the experiments of the giant isoscalar monopole resonance (GIMR). The first is to use a microscopic random phase approximation[3]. Those calculations give $K \sim 210\text{MeV}$. However, those calculations are based on non-relativistic theory and it is difficult to compare the result directly with the result of the relativistic QHD. The second way to use a semi-empirical expansion of nucleus incompressibility coefficient K_A in $A^{-1/3}$ [4][5]. Using this method, Pearson pointed out [4] that there is strong correlation between K and the Coulomb coefficient K_c in the K_A expansion and there is uncertainty in determining K ($= 120 \sim 351\text{MeV}$). Similar observations are done by Shlomo and Youngblood[5].

In this paper, we studies the equations of state(EOS)of nuclear matter, using the nonlinear $\sigma-\omega$ model[9][10][[8] and compare the results with the GIMR data empirically, under the assumption of scaling model[3][4]. This paper is organized as follows. In section 2, we review the general formalism of nonlinear $\sigma-\omega$ model briefly. In section 3, the expression for effective nucleon mass, incompressibility and the third derivative of nuclear binding energy are shown. Numerical results and comparison with the empirical data are shown in section 4. Section 5 is devoted to summary and discussions.

II. FORMALISM

We use the nonlinear relativistic mean field theory[9][10][[8] based on the $\sigma-\omega$ model. The lagrangian density consists of three fields, the nucleon ψ , the scalar σ -meson ϕ and the vector ω -meson V_μ , i.e.,

$$L = \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_s^2\phi^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_v^2V_\mu V^\mu \\ + g_s\bar{\psi}\psi\phi - g_v\bar{\psi}\gamma_\mu\psi V^\mu - U(\phi) \quad ; \quad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad (2.1)$$

where m_s , m_v , g_s and g_v are σ -meson mass, ω -meson mass, σ -nucleon coupling and

ω -nucleon coupling respectively. The $U(\phi)$ is a nonlinear potential and we assume it has the following quartic polynomial in the scalar field ϕ .

$$U(\phi) = \frac{1}{3}b\phi^3 + \frac{1}{4}c\phi^4, \quad (2.2).$$

Strictly speaking, the coefficient c in the eq.(2.2) should be positive to assure the nuclear matter to be bounded at lower density. We will allow this coefficient to be a free parameter as in the ref. [8] and determine its value from a phenomenological fit in which the physics around the normal density is described appropriately.

From the lagrangian (2.1), using the mean-field approximation, we get the field equation for the mesons in the symmetric nuclear matter.

$$m_s^2\phi_0 + b\phi_0^2 + c\phi_0^3 = g_s\rho_s \quad (2.3)$$

and

$$m_v^2V_0 = g_v\rho, \mathbf{V} = \mathbf{0}, \quad (2.4)$$

where ϕ_0 , ρ_s , V_0 and ρ are the ground-state expectation value of scalar field $\langle \phi \rangle$, scalar density $\langle \bar{\psi}\psi \rangle$, $\langle V_0 \rangle$ and baryon density $\langle \bar{\psi}\gamma_0\psi \rangle$ respectively.

Using eq.(2.3), the effective mass of nucleon is obtained as

$$M^* = M - g_s\phi_0 = M - \frac{g_s^2}{m_s^2}\rho_s + \Delta M, \quad (2.5)$$

and

$$\Delta M = C_s^2 B \frac{(M - M^*)^2}{M} + C_s^2 C \frac{(M - M^*)^3}{M^2}, \quad (2.6)$$

where

$$C_s = \frac{g_s M}{m_s}, B = \frac{b}{g_s^3 M}, C = \frac{c}{g_s^4}, \quad (2.7)$$

as in ref. [8].

Under the mean-field approximation, the baryon density ρ , the scalar density ρ_s , the energy density ϵ and pressure p of cold (zero-temperature) nuclear matter are given by

$$\rho = \frac{\lambda}{3\pi} k_F^3, \quad (2.8)$$

$$\rho_s = \frac{\lambda}{4\pi^2} M^* [k_F E_F^* - M^{*2} \ln(\frac{k_F + E_F^*}{M^*})], \quad (2.9)$$

$$\epsilon = \epsilon_f(k_F, M^*) + \frac{1}{2} C_v^2 \frac{\rho_B^2}{M^2} + \frac{1}{2C_s^2} M^2 (M^* - M)^2 + U(\phi_0), \quad (2.10)$$

and

$$P = P_f(k_F, M^*) + \frac{1}{2} C_v^2 \frac{\rho_B^2}{M} - \frac{1}{2C_s^2} M^2 (M^* - M)^2 - U(\phi_0), \quad (2.11)$$

where

$$E_F^* = \sqrt{k^2 + M^{*2}}, \quad (2.12)$$

$$\begin{aligned} \epsilon_f(k_F, M^*) = & \frac{\lambda}{12\pi^2} [3k_f^3 (k_F^2 + M^{*2})^{1/2} + \frac{3}{2} M^{*2} k_F (k_F^2 + M^{*2})^{1/2} \\ & - \frac{3}{2} M^4 \ln \frac{k_F + (k_F^2 + M^{*2})^{1/2}}{M^*}], \end{aligned} \quad (2.13)$$

$$\begin{aligned} P_f(k_F, M^*) = & \frac{\lambda}{12\pi^2} [k_f^3 (k_F^2 + M^{*2})^{1/2} - \frac{3}{2} M^{*2} k_F (k_F^2 + M^{*2})^{1/2} \\ & + \frac{3}{2} M^4 \ln \frac{k_F + (k_F^2 + M^{*2})^{1/2}}{M^*}], \end{aligned} \quad (2.14)$$

and

$$C_v = \frac{g_v M}{m_v}, \quad (2.15)$$

respectively, and the degree of freedom $\lambda = 2$ in the nuclear matter. We remark that $\epsilon_f(k_F, M)$ and $P_f(k_F, M)$ are energy density and pressure of free nucleon gas respectively.

Using eqs.(2.5),(2.6) and (2.9), the effective nucleon mass M^* is determined self-consistently. Inserting the M^* into eqs.(2.10) and (2.11), we can get the energy density ϵ and the pressure P of nuclear matter.

III. EFFECTIVE NUCLEON MASS, INCOMPRESSIBILITY AND THIRD DERIVATIVE OF BINDING ENERGY

The nucleon spectrum $E_N(k)$ with momentum k is given by

$$E_N(k) = \sqrt{k^2 + M^{*2}} + g_v V_0 = \sqrt{k^2 + M^{*2}} + C_v^2 \frac{\rho}{M^2}. \quad (3.1)$$

Because of the Hugenholtz-van Hove theorem[11]

$$E_N(k_F) = \frac{\epsilon + P}{\rho}. \quad (3.2)$$

Since $P = 0$ at the normal density ρ_0 , $E_N(k_{F0})$ reduces to[8]

$$E(k_{F0}) = e_0 = -a_1 + M, \quad (3.3)$$

where k_{F0} , e_0 and a_1 are the Fermi momentum, the energy density $e = \epsilon/\rho$ per baryon and the binding energy at the normal density ρ_0 respectively. From eqs.(3.1) and (3.3), we can determine the effective nucleon mass M_0^* at the normal density when C_v is given, i.e.

$$M_0^* = \sqrt{(e_0 - C_v^2 \frac{\rho_0}{M^2})^2 - k_{F0}^2}. \quad (3.4)$$

The M_0^* has no explicit dependence on g_s , b and c [8]. If we put $C_v^2 \rightarrow 0$ and $k_{F0} = 1.34\text{fm}^{-1}$, we get $M_0^* \simeq 0.94M$. This value gives the upper limit of M_0^* in this model.

The incompressibility K at the normal density is defined as

$$K = 9\rho_0^2 \frac{\partial^2 e}{\partial \rho^2} \Big|_{\rho=\rho_0} = 9 \frac{\partial P}{\partial \rho} \Big|_{\rho=\rho_0} = 9\rho_0 \frac{\partial \mu}{\partial \rho} \Big|_{\rho=\rho_0}, \quad (3.5)$$

where μ is baryonic chemical potential. From the eq.(3.2) and the thermodynamical identity $\mu = (\epsilon + P)/\rho$,

$$\mu = E_F^* + \frac{C_v^2}{M^2} \rho. \quad (3.6)$$

From eqs.(3.5),(3.6) and $E_F^* = \sqrt{k_F^2 + M^{*2}}$, we get

$$K = \rho_0 \left[\frac{C_v^2}{M^2} + \frac{k_F}{E_F^*} \frac{\partial k_F}{\partial \rho} + \frac{M^*}{E_F^*} \frac{\partial M^*}{\partial \rho} \right]_{\rho=\rho_0}. \quad (3.7)$$

From eq.(2.8), we get

$$\frac{\partial k_F}{\partial \rho} = \frac{\pi^2}{\lambda k_F^2}. \quad (3.8)$$

Defferentiating the eq.(2.5), we get

$$\left[1 + \frac{C_s^2 \lambda}{2\pi^2 M^2} f - \frac{\partial(\Delta M)}{\partial M^*}\right] \frac{\partial M^*}{\partial \rho_B} = -\frac{C_s^2 M^*}{M^2 E_F^*}, \quad (3.9)$$

where

$$f = k_F E_F^* + 2 \frac{k_F M^{*2}}{E_F^*} - 3 M^{*2} \ln\left(\frac{k_F + E_F^*}{M^*}\right), \quad (3.10)$$

and

$$\frac{\partial(\Delta M)}{\partial M^*} = -2C_s^2 B \frac{(M - M^*)}{M} - 3C_s^2 C \frac{(M - M^*)^2}{M^2}. \quad (3.11)$$

Inserting eq.(3.9) and (3.7), we get

$$K = \rho_0 \left[\frac{C_v^2}{M^2} + \frac{k_F}{E_F^*} \frac{\partial k_F}{\partial \rho_B} - \frac{C_s^2}{M^2} \frac{M^{*2}}{E_F^*} g \right]_{\rho=\rho_0}, \quad (3.12)$$

where

$$g = \left[1 + \frac{C_s^2 \lambda}{2\pi^2 M^2} f - \frac{\partial(\Delta M)}{\partial M^*}\right]^{-1}. \quad (3.13)$$

From eq.(3.4), small M_0^* needs large C_v . Therefore, from eq.(3.12), it seems that the smaller M_0^* gives the larger K , if g is positive. This observation is consist with the results obtained in ref. [8]. Using eq.(3.12), we can calculate K if M_0^* is determined.

We also defined the third derivative of nuclear binding energy as

$$K' = 3\rho_0 \frac{\partial^3 e}{\partial \rho} \Big|_{\rho=\rho_0} = 3\rho_0 \frac{\partial^2 P}{\partial \rho^2} - \frac{4}{3}K. \quad (3.14)$$

In this model, K' reduces to

$$K' = 3\rho_0 \left[\frac{C_v^2}{M^2} + \frac{\partial E_F^*}{\partial \rho} + \rho \frac{\partial^2 E_F^*}{\partial \rho^2} \right]_{\rho=\rho_0} - \frac{4}{3}K, \quad (3.15)$$

where

$$\frac{\partial E_F^*}{\partial \rho} = \frac{k_F}{E_F^*} \frac{\partial k_F}{\partial \rho} + \frac{M^*}{E_F^*} \frac{\partial M^*}{\partial \rho} \quad (3.16)$$

and

$$\begin{aligned} \frac{\partial^2 E_F^*}{\partial \rho^2} &= \frac{1}{E_F^*} \left(\frac{\partial k_F}{\partial \rho} \right)^2 - \frac{k_F}{E_F^{*2}} \frac{\partial E_F^*}{\partial \rho} \frac{\partial k_F}{\partial \rho} \\ &+ \frac{k_F}{E_F^*} \frac{\partial^2 k_F}{\partial \rho^2} + \frac{1}{E_F^*} \left(\frac{\partial M^*}{\partial \rho} \right)^2 - \frac{M^*}{E_F^{*2}} \frac{\partial E_F^*}{\partial \rho} \frac{\partial M^*}{\partial \rho} + \frac{M^*}{E_F^*} \frac{\partial^2 M^*}{\partial \rho^2}. \end{aligned} \quad (3.17)$$

Differentiating eq.(3.9), we get

$$\begin{aligned} & \left[1 + \frac{\lambda C_s^2 f}{2\pi^2 M^2} - \frac{\partial(\Delta M)}{\partial M^*} \right] \frac{\partial^2 M^*}{\partial \rho^2} \\ &= -\frac{C_s^2}{M^2 E_F^{*2}} \left(E_F^* \frac{\partial M^*}{\partial \rho} - M^* \frac{\partial E_F^*}{\partial \rho} \right) + \left[\frac{\partial M^*}{\partial \rho} \frac{\partial^2(\Delta M)}{\partial M^{*2}} - \frac{\lambda C_s^2}{2\pi^2 M^2} \frac{\partial f}{\partial \rho} \right] \frac{\partial M^*}{\partial \rho}, \end{aligned} \quad (3.18)$$

where

$$\begin{aligned} \frac{\partial f}{\partial \rho} &= (3k_F - 2E_F^* + 2\frac{M^{*2}}{E_F^*}) \frac{\partial k_F}{\partial \rho} + (4k_F - 3E_F^* - 2\frac{k_F M^{*2}}{E_F^{*2}}) \frac{\partial E_F^*}{\partial \rho} \\ &+ (3M^* + 4\frac{k_F M^*}{E_F^*}) - 6M^{*2} \ln\left(\frac{k_F + E_F}{M^*}\right) \frac{\partial M^*}{\partial \rho}, \end{aligned} \quad (3.19)$$

and

$$\frac{\partial^2(\Delta M)}{\partial M^{*2}} = 2C_s^2 B \frac{M^*}{M} - 6C_s^2 C \frac{M - M^*}{M^2}. \quad (3.20)$$

Using the eqs.(3.9),(3.12) and (3.14)~(3.20), we can calculate the value of K' .

IV. NUMERICAL RESULTS

In our calculations, we put $M = 939\text{MeV}$, $\rho_0 = 0.16\text{fm}^{-3}$ and $a_1 = 15.75\text{MeV}$. The other independent parameters C_s , C_v , B and C are determined phenomenologically. Besides the saturation conditions $P(\rho_0) = 0$ and $e(\rho_0) = e_0 = -a_1 + M$, if two of three values M_0^* , K and K' are given, we can determine the four parameters of this model and calculate the rest one physical observable.

Fig.1

In Fig. 1, we show the $M_0^* - K'$ relation with fixed values of K . Because this model has the unphysical behavior in the cases with $K < 200\text{MeV}$ [8], we only show the results with $K = 200 \sim 400\text{MeV}$. There is strong correlation between M_0^* and K' . The K' decreases as M_0^* increases, if $K \gtrsim 250\text{MeV}$. There is a weak local minimum of K' at $M_0^* \sim 0.70M$ when $K = 200\text{MeV}$. There is a cross points of the lines at $M_0^* \sim 0.75M$. We also remark, in any case of K , $|K'| \lesssim 100\text{MeV}$, when $M_0^* = 0.6 \sim 0.8M$. The K' has little contribution for the stiffness of nuclear EOS when $M_0^* = 0.6 \sim 0.8M$. They are distinct feature of this model. The results of original Walecka model(WM)[1] and the full relativistic Hartree calculation(RHA)[12] also shown in Fig.1. These two points are above to the line with $K = 400\text{MeV}$. Naturally, Walecka model is obtained when we put $B = 0$ and $C = 0$.

Fig.2

Waldhauser et al.[8] found unphysical behavior in the case in which C has a negative value and K is small($\lesssim 200\text{MeV}$), especially in the high density region. In fig. 2, we show the relation between M_0^* and C with fixed value of K . The condition $C \geq 0$ gives $M_0^*/M \geq 0.63(0.67, 0.72, 0.78, 0.84)$ when $K = 400(350, 300, 250, 200)\text{MeV}$. The smaller value of K needs the larger value of M_0^* for the condition.

Fig.3(a),(b)

In fig.3(a), we show the $K-K'$ relations with the fixed value of M_0^* . The K' increases as K increases, in the cases of $M_0^* \leq 0.6M$. On the other hand, K' decreases as K increases, in the cases of $M_0^* \geq 0.9M$. The K' is not well determined empirically. However, in the framework of scaling model K' is given by [3][4]

$$K' = \frac{-1}{9} \left(\frac{5r_0}{3g_e^2} K_c + 8 \right) K, \quad (4.1)$$

where g_e and K_c are proton charge and the Coulomb coefficient of the leptodermous expansion[3]. In fig.3(a), we also show the values K' which are obtained by using the relation (4.1) and the empirical values of K and K_c in the table 3 of reference [4]. (We remark that our definition of K' is different from that in ref. [4].) There is uncertainty in determining K empirically[4][5]. In the conclusion of the referece [4], K is considered to be $120 \sim 351\text{MeV}$. To be consistent with these values, $M_0^* \gtrsim 0.6\text{MeV}$ in our model.

On the other hand, if we assume $M_0^* \approx 0.6 \sim 0.9M$, K must have value $250 \sim 350 \text{ MeV}$ to be consistent with the data. In fig. 3(b), we also show the K - K_c relations obtained by using the relations (4.1) and our calculations results. The K_c is negative when $200 \text{ MeV} \lesssim K \lesssim 400 \text{ MeV}$ and $M_0^* \lesssim 0.9M$. Especially, $K_c \approx -4 \sim -8 \text{ MeV}$, when $200 \text{ MeV} \lesssim K \lesssim 400 \text{ MeV}$ and $M_0^*/M \approx 0.6 \sim 0.8$. It is difficult to realize $K_c > 0$ in this model.

Table I(a),(b),(c)

Conversely, by using the relation (4.1) and the empirical values in ref. [4] we can determine the values of M_0^* , C_s , C_v , B and C , and can calculate the other physical observables. The results are summarized in table I. Because there are experimental error bars of K_c , we use the mean value in table I(a), the upper bound in table(b) and the lower bound in table (c). In any case, there is a strong correlation between K and M_0^* . The M_0^* decreases as K increases. We could not find the parameters sets which realized $K \leq 150 \text{ MeV}$ and $K = 200 \text{ MeV}$ with $K_c = 2.577$ and $2.577 + 2.06 \text{ MeV}$ in the table 3 of ref. [4], because these sets of K and K_c need too large $M_0^* (\geq 0.94M)$.

Using the parameters sets in table 1(a), we have calculated the binding energy in nuclear matter. The results are shown in fig. 4. The EOS with the smaller incompressibility K is much softer because it has also the larger M_0^* and the smaller K' . The EOS is very soft when we assume $K \lesssim 250 \text{ MeV}$. In fig. 5 and fig. 6, we also show the density dependences of the effective nucleon mass M^* and sound velocity v_s , which is defined by

$$v_s^2 = \frac{\partial p}{\partial \epsilon}, \quad (4.2)$$

in each EOS. With $K = 250 \text{ MeV}$, M^* hardly decreases as ρ increases. On the other hand, with $K = 400 \text{ MeV}$, M^* rapidly damps as ρ increases. The v_s^2 hardly increases when $K = 250 \text{ MeV}$. Though the EOS 3 and 4 have the negative value of C , they have correct causal limit ($v_s^2 < 1$).

Fig.4

Fig.5

Fig.6

Fig.7

In fig. 7, to describe the effects of the error bar of K_c , we show the binding energy obtained by using the EOS in table I(b) and I(c). From this figure, we see uncertainties of K_c have much effects on EOS. The uncertainty is especially large when $K \approx 250 \sim 300 \text{ MeV}$. This is because M_0^* change rapidly in the region $K' = 0 \sim -200 \text{ MeV}$ as is shown in fig.1. The other physical quantities, e.g., the effective nucleon mass and the sound velocity, have similar uncertainties. However, these uncertainties are not large enough to make the results meaningless. We also done the same calculations using $\rho_0 = 0.15$ and 0.17 fm^{-3} and found that the EOS hardly change when we vary ρ_0 .

In table I, we also show the symmetry energy which is given by

$$a_4 = \frac{k_{F0}^2}{6\sqrt{k_{F0}^2 + M_0^{*2}}} \quad (4.3)$$

in this model. Equation (4.3) is the same as in the original Walecka model [1]. The values in the table are smaller than the empirical one ($\sim 33 \text{ MeV}$) [13]. However, if the ρ meson effects are taken into account, eq.(4.3) is modified as [14]

$$a_4 = \frac{k_{F0}^2}{6\sqrt{k_{F0}^2 + M_0^{*2}}} + \frac{g_\rho^2}{12\pi^2 m_\rho^2} k_{F0}^3 \quad (4.4)$$

In mean field approximation, inclusion of ρ meson does not affect the saturation condition and the second term in eq.(4.4) gives positive contribution to a_4 . In this point, our results are not inconsistent with the empirical value. Nevertheless, it need large value of $g_\rho^2 (\gtrsim 60)$ to fit the empirical value for the small $K (\lesssim 250 \text{ MeV})$, because small K has large M_0^* in eq.(4.4).

V. SUMMARY AND DISCUSSIONS

We have studied the effective nucleon mass M_0^* at ρ_0 , incompressibility K and the third derivative K' of binding energy in nuclear matter, using the nonlinear $\sigma - \omega$ model. The results are summarized as follows.

- (1) There is strong correlation between K and K' , when we fix M_0^* . The K' decreases as M_0^* increases, if $K \gtrsim 250\text{MeV}$. There is a weak local minimum of K' at $M_0^* \sim 0.70M$ when $K = 200\text{MeV}$ and there are cross points of the lines at $M_0^* \sim 0.75M$.
- (2) When we assume $K = 200 \sim 400\text{MeV}$, $|K'| \lesssim 100\text{MeV}$, when $M_0^* = 0.6 \sim 0.8M$. In these case, the K' has little contribution to the stiffness of nuclear EOS.
- (3) The smaller value of K needs the larger value of M_0^* for the condition $C \geq 0$.
- (4) The K - K' relations much depends on the value of M_0^* . The K' increases as K increases, in the cases of $M_0^* \leq 0.6M$. On the other hand, K' decreases as K increases, in the cases of $M_0^* \geq 0.9M$.
- (5) The K_c is negative when $200\text{MeV} \lesssim K \lesssim 400\text{MeV}$ and $M_0^* \lesssim 0.9M$.
- (6) The parameters sets are determined to fit the empirical value of K and K_c in the data of the GIMR[4]. It is difficult to fit the data with $K \lesssim 200\text{MeV}$.
- (7) We have negative value of C when we put $K \gtrsim 350\text{MeV}$ to fit the data. This fact may indicate the unphysical behavior as pointed out in ref. [8], although we had the correct causal limit ($v_s^2 < 1$).
- (8) The EOS with $K \lesssim 250\text{MeV}$ which are obtained in (2) is very soft, because they have large M_0^* and small K' besides small K .
- (9) The uncertainty of K_c has much effects on the EOS, especially when $K \approx 250 \sim 300\text{MeV}$. In this case, more accurate measurements are desirable.

In this paper we almostly restrict ourselves on the discussion of K and K_c . We have used the symmetric nuclear EOS for analysis. The other coefficients in K_A expansion must be studied in this model, or in the extended version of this model. Very recently, Von-Eiff et al.[15] analyzed GIMR data using the nonlinear σ - ω - ρ model with the NL1 [16] and the NL-SH [17] parameters sets, and found these parameters set are disagreement with experiment. More extended model may be need for the accurate fit to the data.

One simple way to extend the model within the framework of mean field theory is to add more higher terms of ϕ to the lagrangian (2.1). However, we remark that such a modification does not change the M_0^* - C_v relation (3.4) and may not change M_0^* - K (3.7) relation drastically. Modification of vector meson interaction might be needed.

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Table and Figure Captions

Table I(a),I(b),I(c)

Parameter sets fitted for the empirical value of K and K_c in the table 3 in ref. [4]. The K , K_c , K' and a_4 are shown in MeV. (a) Results obtained by using the mean value of K_c in the table. (b) Results obtained by using the upper bound for K_c in the table. (c) Results obtained by using the lower bound for K_c in the table.

Fig. 1 The $M_0^* - K'$ relations. The solid line, the dotted line, the dashed line, the dashed-dotted line and the bold solid line are results with $K = 200, 250, 300, 350$ and 400MeV respectively. The solid circle and solid square are results of the original Walecka model(WM) and the RHA respectively.

Fig. 2 The $M_0^* - C$ relations. The solid line, the dotted line, the dashed line, the dashed-dotted line and the bold solid line are results with $K = 200, 250, 300, 350$ and 400MeV respectively.

Fig. 3 (a) The $K - K'$ relations. The solid line, the dotted line, the dashed line, the dashed-dotted line and the bold solid line are results with $M_0^*/M = 0.5, 0.6, 0.7, 0.8$ and 0.9 respectively. The solid circles are results obtained by using the empirical values of K and K_c in the table 3 in ref. [4].

(b) The $K - K_c$ relations. The solid line, the dotted line, the dashed line, the dashed-dotted line and the bold solid line are results with $M_0^*/M = 0.5, 0.6, 0.7, 0.8$ and 0.9 respectively. The solid circles are the empirical values of K and K_c in the table 3 in ref. [4].

Fig. 4 The ρ dependence of the binding energy. The solid line, the dotted line, the dashed line and the dashed-dotted line are results of EOS1,2,3 and 4 in table I(a) respectively.

Fig. 5 The ρ dependence of the effective nucleon mass M^* . The solid line, the dotted line, the dashed line and the dashed-dotted line are results of EOS1,2,3 and 4 in table I(a) respectively.

Fig. 6 The ρ dependence of the sound velocity v_s . The solid line, the dotted line, the dashed line and the dashed-dotted line are results of EOS1,2,3 and 4 in table I(a) respectively.

Fig. 7 The ρ dependence of the binding energy. The solid line, the dotted line, the dashed line and the dashed-dotted line are results of EOS5,6,7 and 8 in table I(b) respectively. The solid circles, the bold solid line, the bold dotted line, the bold dashed line and the bold dashed-dotted line are results of EOS9,10,11,12 and 13 in table I(c) respectively.

Table I (a)

EOS	1	2	3	4
K	250	300	350	400
K_c	-0.7065	-3.990	-7.274	-10.56
K'	-196	-90.5	63.5	266
M_0^*/M	0.91	0.83	0.60	0.51
a^4	12.8	14.0	18.4	20.9
C_s^2	44.02	153.0	312.8	373.0
C_v^2	20.20	72.62	214.8	269.4
B	0.5145	4.617×10^{-3}	-1.397×10^{-3}	-1.149×10^{-3}
C	4.983	9.223×10^{-2}	-1.480×10^{-3}	-1.832×10^{-3}

Table I (b)

EOS	5	6	7	8
K	250	300	350	400
K_c	-0.7065+2.06	-3.990+2.06	-7.274+2.06	-10.56+2.06
K'	-272.0	-181.5	-42.55	144.8
M_0^*/M	0.93	0.88	0.75	0.56
a^4	12.5	13.2	15.2	19.5
C_s^2	3.618	77.82	205.8	335.6
C_v^2	4.759	37.64	121.7	239.8
B	12.55	0.1402	-3.473×10^{-4}	-9.838×10^{-4}
C	133.3	1.181	1.327×10^{-2}	-1.297×10^{-3}

Table I (c)

EOS	9	10	11	12	13
K	200	250	300	350	400
K_c	2.557-2.06	-0.7065-2.06	-3.990-2.06	-7.274-2.06	-10.56-2.06
K'	-193.0	-120.4	0.430	169.6	387.3
M_0^*/M	0.93	0.88	0.66	0.54	0.48
a^4	12.6	13.3	17.2	20.2	21.9
C_s^2	18.48	111.3	284.6	360.6	397.7
C_v^2	7.169	40.96	182.9	254.9	287.4
B	2.100	4.044×10^{-2}	-2.348×10^{-3}	-1.383×10^{-3}	-1.270×10^{-3}
C	23.63	0.5344	-2.044×10^{-3}	-2.059×10^{-3}	-2.078×10^{-3}

Fig.1

Fig.2

Fig.3.(a)

Fig.3.(b)

Fig.4

Fig.5

Fig.6

Fig.7