Effective nucleon mass, incompressibility and third derivative of nuclear binding energy in the nonlinear relativistic mean field theory

H. Kouno, N. Kakuta, N. Noda, K. Koide, T. Mitsumori, A. Hasegawa
and
M. Nakano*

Department of Physics, Saga University, Saga 840, Japan

*University of Occupational and Environmental Health, Kitakyushu 807, Japan

ABSTRACT

We have studied the equations of state (EOS) of nuclear matter using the nonlinear $\sigma$-$\omega$ model. At the normal density, there is strong correlation among the effective nucleon mass $M_0^*$, incompressibility $K$ and third derivative $K'$ of binding energy. The $K'$ is small when $M_0^* \approx 0.6 \sim 0.8 M$ where $M$ is nucleon mass. The results are compared with the empirical analysis of the giant isoscalar monopole resonances data. It is difficult to fit the data when $K \lesssim 200\text{MeV}$. Furthermore, to fit the data, the EOS becomes very soft even in the higher density region because of small value of $K'$, when we assume incompressibility $K \lesssim 250\text{MeV}$. 
I. INTRODUCTION

Nuclear matter has been studied in the framework of quantum hadrodynamics (QHD). The meson mean-field model for nuclear matter [1] has made successful results to account for the saturation properties at the normal density. The original Hartree calculation [1] and the Hartree-Fock calculation [2] give larger incompressibility $K$ than the empirical value [3][4][5]. This quantity has an important role in describing the features of nuclear matter and neutron matter at high densities[6], and is calculated in the many improved methods [7]. The correlation between $K$ and effective nucleon mass $M^*_0$ at the normal density $\rho_0$ are pointed out in ref. [8].

On the other hand, there are two ways to get the empirical value from the experiments of the giant isoscalar monopole resonance (GIMR). The first is to use a microscopic random phase approximation[3]. Those calculations give $K \sim 210$ MeV. However, those calculations are based on non-relativistic theory and it is difficult to compare the result directly with the result of the relativistic QHD. The second way to use a semi-empirical expansion of nucleus incompressibility coefficient $K_A$ in $A^{-1/3}$[4][5]. Using this method, Pearson pointed out [4] that there is strong correlation between $K$ and the Coulomb coefficient $K_c$ in the $K_A$ expansion and there is uncertainty in determining $K (= 120 \sim 351$ MeV). Similar observations are done by Shlomo and Youngblood[5].

In this paper, we studies the equations of state (EOS) of nuclear matter, using the nonlinear $\sigma - \omega$ model [9][10][8] and compare the results with the GIMR data empirically, under the assumption of scaling model [3][4]. This paper is organized as follows. In section 2, we review the general formalism of nonlinear $\sigma - \omega$ model briefly. In section 3, the expression for effective nucleon mass, incompressibility and the third derivative of nuclear binding energy are shown. Numerical results and comparison with the empirical data are shown in section 4. Section 5 is devoted to summary and discussions.

II. FORMALISM

We use the nonlinear relativistic mean field theory [9][10][8] based on the $\sigma - \omega$ model. The lagrangian density consists of three fields, the nucleon $\psi$, the scalar $\sigma$-meson $\phi$ and the vector $\omega$-meson $V_\mu$, i.e.,

$$L = \bar{\psi} (i\gamma_\mu \partial^\mu - M) \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_s^2 \phi^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu$$

$$+ g_s \bar{\psi} \gamma_\mu \phi - g_\omega \bar{\psi} \gamma_\mu \gamma_5 \psi V^\mu - U(\phi) \quad , \quad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad (2.1)$$

where $m_s$, $m_v$, $g_s$ and $g_\omega$ are $\sigma$-meson mass, $\omega$-meson mass, $\sigma$-nucleon coupling and
\(\omega\)-nucleon coupling respectively. The \(U(\phi)\) is a nonlinear potential and we assume it has the following quartic polynomial in the scalar field \(\phi\).

\[
U(\phi) = \frac{1}{3}b\phi^3 + \frac{1}{4}c\phi^4, \tag{2.2}
\]

Strictly speaking, the coefficient \(c\) in the eq.(2.2) should be positive to assure the nuclear matter to be bounded at lower density. We will allow this coefficient to be a free parameter as in the ref. [8] and determine its value from a phenomenological fit in which the physics around the normal density is described appropriately.

From the lagrangian (2.1), using the mean-field approximation, we get the field equation for the mesons in the symmetric nuclear matter.

\[
m_s^2\phi_0 + b\phi_0^2 + c\phi_0^3 = g_s\rho_s, \tag{2.3}
\]

and

\[
m_v^2V_0 = g_v\rho_v, \mathbf{V} = \mathbf{0}, \tag{2.4}
\]

where \(\phi_0, \rho_s, V_0\) and \(\rho\) are the ground-state expectation value of scalar field \(<\phi>\), scalar density \(<\bar{\psi}\psi>\), \(<V_0>\) and baryon density \(<\bar{\psi}\gamma_0\psi>\) respectively.

Using eq.(2.3), the effective mass of nucleon is obtained as

\[
M^* = M - g_s\phi_0 = M - \frac{g_s^2}{m_s^2}\rho_s + \Delta M, \tag{2.5}
\]

and

\[
\Delta M = C_s^2B\left(\frac{M - M^*}{M}\right)^2 + C_s^2C\left(\frac{M - M^*}{M^2}\right)^3, \tag{2.6}
\]

where

\[
C_s = \frac{g_sM}{m_s}, B = \frac{b}{g_s^3M}, C = \frac{c}{g_s^4}, \tag{2.7}
\]

as in ref. [8].
Under the mean-field approximation, the baryon density $\rho$, the scalar density $\rho_s$, the energy density $\epsilon$ and pressure $p$ of cold (zero-temperature) nuclear matter are given by

$$\rho = \frac{\lambda}{3\pi} k_F^3, \quad (2.8)$$

$$\rho_s = \frac{\lambda}{4\pi^2} M^*[k_F E_F^* - M^* \ln \left(\frac{k_F + E_F^*}{M^*}\right)], \quad (2.9)$$

$$\epsilon = \epsilon_f(k_F, M^*) + \frac{1}{2} C_v \frac{\rho_B^2}{M^2} + \frac{1}{2C^2_v} M^2 (M^* - M)^2 + U(\phi_0), \quad (2.10)$$

and

$$P = P_f(k_F, M^*) + \frac{1}{2} C_v \frac{\rho_B^2}{M^2} - \frac{1}{2C^2_v} M^2 (M^* - M)^2 - U(\phi_0), \quad (2.11)$$

where

$$E_F^* = \sqrt{k^2 + M^{*2}}, \quad (2.12)$$

$$\epsilon_f(k_F, M^*) = \frac{\lambda}{12\pi^2} \left[ 3k_f^2 (k_F^2 + M^{*2})^{1/2} + \frac{3}{2} M^{*2} k_F (k_F^2 + M^{*2})^{1/2} \right. \left. - \frac{3}{2} M^{*4} \ln \frac{k_F + (k_F^2 + M^{*2})^{1/2}}{M^*} \right], \quad (2.13)$$

$$P_f(k_F, M^*) = \frac{\lambda}{12\pi^2} \left[ k_f^2 (k_F^2 + M^{*2})^{1/2} - \frac{3}{2} M^{*2} k_F (k_F^2 + M^{*2})^{1/2} \right. \left. + \frac{3}{2} M^{*4} \ln \frac{k_F + (k_F^2 + M^{*2})^{1/2}}{M^*} \right], \quad (2.14)$$

and

$$C_v = \frac{g_v M}{m_v}, \quad (2.15)$$

respectively, and the degree of freedom $\lambda = 2$ in the nuclear matter. We remark that $\epsilon_f(k_F, M)$ and $P_f(k_F, M)$ are energy density and pressure of free nucleon gas respectively.

Using eqs.(2.5),(2.6) and (2.9), the effective nucleon mass $M^*$ is determined self-consistently. Inserting the $M^*$ into eqs.(2.10) and (2.11), we can get the energy density $\epsilon$ and the pressure $P$ of nuclear matter.
III. EFFECTIVE NUCLEON MASS, INCOMPRESSIBILITY AND THIRD DERIVATIVE OF BINDING ENERGY

The nucleon spectrum $E_N(k)$ with momentum $k$ is given by

$$E_N(k) = \sqrt{k^2 + M^*^2} + g_v V_0 = \sqrt{k^2 + M^*^2} + C_v \frac{\rho}{M^2}.$$  \hfill (3.1)


$$E_N(k_F) = \frac{\epsilon + P}{\rho}.$$  \hfill (3.2)

Since $P = 0$ at the normal density $\rho_0$, $E_N(k_{F0})$ reduces to[8]

$$E(k_{F0}) = e_0 = -a_1 + M,$$  \hfill (3.3)

where $k_{F0}$, $e_0$ and $a_1$ are the Fermi momentum, the energy density $\epsilon = \epsilon/\rho$ per baryon and the binding energy at the normal density $\rho_0$ respectively. From eqs.(3.1) and (3.3), we can determined the effective nucleon mass $M_0^*$ at the normal density when $C_v$ is given:

$$M_0^* = \sqrt{(e_0 - C_v \frac{\rho_0}{M^2})^2 - k_{F0}^2}.$$  \hfill (3.4)

The $M_0^*$ has no explicit dependence on $g_s$, $b$ and $c[8]$. If we put $C_v^2 \to 0$ and $k_{F0} = 1.34\text{fm}^{-1}$, we get $M_0^* \simeq 0.94M$. This value gives the upper limit of $M_0^*$ in this model.

The incompressibility $K$ at the normal density is defined as

$$K = 9\rho_0^2 \frac{\partial^2 e}{\partial \rho^2} \big|_{\rho=\rho_0} = 9 \frac{\partial P}{\partial \rho} \big|_{\rho=\rho_0} = 9\rho_0 \frac{\partial \mu}{\partial \rho} \big|_{\rho=\rho_0},$$  \hfill (3.5)

where $\mu$ is baryonic chemical potential. From the eq.(3.2) and the thermodynamical identity $\mu = (\epsilon + P)/\rho$,

$$\mu = E_F^* + \frac{C_v^2}{M^2 \rho}.$$  \hfill (3.6)

From eqs.(3.5),(3.6) and $E_F^* = \sqrt{k_F^2 + M^*^2}$, we get

$$K = \rho_0 \left[ \frac{C_v^2}{M^2} + \frac{k_F}{E_F^*} \frac{\partial k_F}{\partial \rho} \big|_{\rho=\rho_0} + \frac{M^* \partial M^*}{E_F^*} \frac{\partial \rho}{\partial \rho} \right].$$  \hfill (3.7)
From eq.(2.8), we get
\[
\frac{\partial k_F}{\partial \rho} = \frac{\pi^2}{\lambda k_F^*}.
\] (3.8)

Defferentiating the eq.(2.5), we get
\[
[1 + \frac{C_v^2 \lambda}{2\pi^2 M^2} f - \frac{\partial(\Delta M)}{\partial M^*} \frac{\partial M^*}{\partial \rho_B}] \frac{\partial M^*}{\partial \rho_B} = - \frac{C_v^2 M^*}{M^2 E_F^*},
\] (3.9)

where
\[
f = k_F E_F^* + 2 \frac{k_F M^*}{M^*} - 3 M^* \ln \left( \frac{k_F E_F^*}{M^*} \right),
\] (3.10)

and
\[
\frac{\partial(\Delta M)}{\partial M^*} = -2 C_v B \frac{(M - M^*)}{M} - 3 C_v C \frac{(M - M^*)^2}{M^2}.
\] (3.11)

Inserting eq.(3.9) and (3.7), we get
\[
K = \rho_0 \left[ \frac{C_v^2}{M^2} + \frac{k_F}{E_F^*} \frac{\partial k_F}{\partial \rho_B} - \frac{C_v^2 M^*}{M^2 E_F^*} \right]_{\rho = \rho_0},
\] (3.12)

where
\[
g = [1 + \frac{C_v^2 \lambda}{2\pi^2 M^2} f - \frac{\partial(\Delta M)}{\partial M^*}]^{-1}.
\] (3.13)

From eq.(3.4), small $M_0^*$ needs large $C_v$. Therefore, from eq.(3.12), it seems that the smaller $M_0^*$ gives the larger $K$, if $g$ is positive. This observation is consist with the results obtained in ref. [8]. Using eq.(3.12), we can calculate $K$ if $M_0^*$ is determined.

We also defined the third derivative of nuclear binding energy as
\[
K' = 3 \rho_0 \frac{\partial^3 \epsilon}{\partial \rho^3} \bigg|_{\rho = \rho_0} = 3 \rho_0 \frac{\partial^2 P}{\partial \rho^2} - \frac{4}{3} K.
\] (3.14)

In this model, $K'$ reduces to
\[
K' = 3 \rho_0 \left[ \frac{C_v^2}{M^2} + \frac{\partial E_F^*}{\partial \rho} + \rho \frac{\partial^2 E_F^*}{\partial \rho^2} \right]_{\rho = \rho_0} - \frac{4}{3} K,
\] (3.15)

where
\[
\frac{\partial E_F^*}{\partial \rho} = \frac{k_F}{E_F^*} \frac{\partial k_F}{\partial \rho} + \frac{M^* \partial M^*}{E_F^* \partial \rho}
\] (3.16)
and

\[
\frac{\partial^2 E_F^*}{\partial \rho^2} = \frac{1}{E_F^*} \left( \frac{\partial k_F}{\partial \rho} \right)^2 - \frac{k_F}{E_F^*} \frac{\partial E_F^*}{\partial \rho} \frac{\partial k_F}{\partial \rho} + \frac{k_F}{E_F^*} \frac{\partial^2 k_F}{\partial \rho^2} + \frac{1}{E_F^*} \left( \frac{\partial M^*}{\partial \rho} \right)^2 - \frac{M^*}{E_F^*} \frac{\partial E_F}{\partial \rho} \frac{\partial M^*}{\partial \rho} + \frac{M^*}{E_F^*} \frac{\partial^2 M^*}{\partial \rho^2}.
\]

(3.17)

Differentiating eq.(3.9), we get

\[
[1 + \frac{\lambda C_s^2 f}{2\pi^2 M^2} - \frac{\partial (\Delta M)}{\partial M^*}] \frac{\partial^2 M^*}{\partial \rho^2}
\]

\[
= - \frac{C_s^2}{M^2 E_F^*} (E_F^* \frac{\partial M^*}{\partial \rho} - M^* \frac{\partial E_F^*}{\partial \rho}) + \left[ \frac{\partial M^*}{\partial \rho} \frac{\partial^2 (\Delta M)}{\partial M^* \partial \rho^2} - \frac{\lambda C_s^2}{2\pi^2 M^2} \frac{\partial f}{\partial \rho} \frac{\partial M^*}{\partial \rho} \right],
\]

(3.18)

where

\[
\frac{\partial f}{\partial \rho} = (3k_F - 2E_F^* + 2 \frac{M^*}{E_F^*}) \frac{\partial k_F}{\partial \rho} + (4k_F - 3E_F^* - 2 \frac{k_F M^*}{E_F^*}) \frac{\partial E_F^*}{\partial \rho}
\]

\[
+ (3M^* + 4 \frac{k_F M^*}{E_F^*}) - 6M^* \ln \left( \frac{k_F + E_F^*}{M^*} \right) \frac{\partial M^*}{\partial \rho},
\]

(3.19)

and

\[
\frac{\partial^2 (\Delta M)}{\partial M^* \partial \rho^2} = 2C_s^2 B \frac{M^*}{M} - 6C_s^2 C \frac{M - M^*}{M^2}.
\]

(3.20)

Using the eqs.(3.9),(3.12) and (3.14)~(3.20), we can calculate the value of \(K'\).

**IV. NUMERICAL RESULTS**

In our calculations, we put \(M = 939\text{MeV}, \rho_0 = 0.16\text{fm}^{-3} \) and \(a_1 = 15.75\text{MeV}\). The other independent parameters \(C_s, C_v, B \) and \(C \) are determined phenomenologically. Besides the saturation conditions \(P(\rho_0) = 0 \) and \(\epsilon(\rho_0) = \epsilon_0 = -a_1 + M\), if two of three values \(M_0^*, K \) and \(K'\) are given, we can determine the four parameters of this model and calculate the rest one physical observable.

——

**Fig.1**

——

- 7 -
In Fig. 1, we show the $M_0^*-K'$ relation with fixed values of $K$. Because this model has the unphysical behavior in the cases with $K < 200$MeV [8], we only show the results with $K = 200 \sim 400$MeV. There is strong correlation between $M_0^*$ and $K'$. The $K'$ decreases as $M_0^*$ increases, if $K \gtrsim 250$MeV. There is a weak local minimum of $K'$ at $M_0^* \sim 0.70M$ when $K = 200$MeV. There is a cross points of the lines at $M_0^* \sim 0.75M$. We also remark, in any case of $K$, $|K'| \lesssim 100$MeV, when $M_0^* = 0.6 \sim 0.8M$. The $K'$ has little contribution for the stiffness of nuclear EOS when $M_0^* = 0.6 \sim 0.8M$. They are distinct feature of this model. The results of original Walecka model (WM)[1] and the full relativistic Hartree calculation (RHA) [12] also shown in Fig.1. These two points are above the line with $K = 400$MeV. Naturally, Walecka model is obtained when we put $B = 0$ and $C = 0$.

Fig.2

Waldhauser et al. [8] found unphysical behavior in the case in which $C$ has a negative value and $K$ is small ($\lesssim 200$MeV), especially in the high density region. In fig. 2, we show the relation between $M_0^*$ and $C$ with fixed value of $K$. The condition $C \geq 0$ gives $M_0^*/M \geq 0.63(0.67, 0.72, 0.78, 0.84)$ when $K = 400(350, 300, 250, 200)$MeV. The smaller value of $K$ needs the larger value of $M_0^*$ for the condition.

Fig.3(a),(b)

In fig.3(a), we show the $K$-$K'$ relations with the fixed value of $M_0^*$. The $K'$ increases as $K$ increases, in the cases of $M_0^* \leq 0.6M$. On the other hand, $K'$ decreases as $K$ increases, in the cases of $M_0^* \geq 0.9M$. The $K'$ is not well determined empirically. However, in the framework of scaling model $K'$ is given by [3][4]

\[ K' = \frac{-1}{9} \left( \frac{5r_0}{3g_e^2}K_e + 8 \right)K, \]  \hfill (4.1)

where $g_e$ and $K_e$ are proton charge and the Coulomb coefficient of the leptodermous expansion[3]. In fig.3(a), we also show the values $K'$ which are obtained by using the relation (4.1) and the empirical values of $K$ and $K_e$ in the table 3 of reference [4]. ( We remark that our definition of $K'$ is different from that in ref. [4].) There is uncertainty in determining $K$ empirically[4][5]. In the conclusion of the reference [4], $K$ is considered to be $120 \sim 351$MeV. To be consistent with these values, $M_0^* \gtrsim 0.6$MeV in our model.
On the other hand, if we assume $M_0^* \approx 0.6 \sim 0.9M$, $K$ must have value 250\textendash350MeV to be consistent with the data. In fig. 3(b), we also show the $K$-$K_c$ relations obtained by using the relations (4.1) and our calculations results. The $K_c$ is negative when $200\text{MeV} \lesssim K \lesssim 400\text{MeV}$ and $M_0^* \lesssim 0.9M$. Especially, $K_c \approx -4 \sim -8\text{MeV}$, when $200\text{MeV} \lesssim K \lesssim 400\text{MeV}$ and $M_0^*/M \approx 0.6 \sim 0.8$. It is difficult to realize $K_c > 0$ in this model.

Table I(a),(b),(c)

Conversely, by using the relation (4.1) and the empirical values in ref. [4] we can determine the values of $M_0^*$, $C_s$, $C_v$, $B$ and $C$, and can calculate the other physical observables. The results are summarized in table I. Because there are experimental error bars of $K_c$, we use the mean value in table I(a), the upper bound in table(b) and the lower bound in table (c). In any case, there is a strong correlation between $K$ and $M_0^*$. The $M_0^*$ decreases as $K$ increases. We could not find the parameters sets which realized $K \leq 150\text{MeV}$ and $K = 200\text{MeV}$ with $K_c = 2.577$ and $2.577 + 2.06\text{MeV}$ in the table 3 of ref. [4], because these sets of $K$ and $K_c$ need too large $M_0^*(\geq 0.94M)$.

Using the parameters sets in table 1(a), we have calculated the binding energy in nuclear matter. The results are shown in fig. 4. The EOS with the smaller incompressibility $K$ is much softer because it has also the larger $M_0^*$ and the smaller $K'$. The EOS is very soft when we assume $K \lesssim 250\text{MeV}$. In fig. 5 and fig. 6, we also show the density dependences of the effective nucleon mass $M^*$ and sound velocity $v_s$ which is defined by

$$v_s^2 = \frac{\partial p}{\partial \epsilon},$$

(4.2)

in each EOS. With $K = 250\text{MeV}, M^*$ hardly decreases as $\rho$ increases. On the other hand, with $K = 400\text{MeV}, M^*$ rapidly damps as $\rho$ increases. The $v_s^2$ hardly increases when $K = 250\text{MeV}$. Though the EOS 3 and 4 have the negative value of $C$, they have correct causal limit ($v_s^2 < 1$).

Fig.4
In fig. 7, to describe the effects of the error bar of $K_c$, we show the binding energy obtained by using the EOS in table I(b) and I(c). From this figure, we see uncertainties of $K_c$ have much effects on EOS. The uncertainty is especially large when $K \approx 250 \sim 300$MeV. This is because $M_0^*$ change rapidly in the region $K' = 0 \sim -200$MeV as is shown in fig.1. The other physical quantities, e.g., the effective nucleon mass and the sound velocity, have similar uncertainties. However, these uncertainties are not large enough to make the results meaningless. We also done the same calculations using $\rho_0 = 0.15$ and 0.17fm$^{-3}$ and found that the EOS hardly change when we vary $\rho_0$.

In table I, we also show the symmetry energy which is given by

$$a_4 = \frac{k_{F_0}^2}{6\sqrt{k_{F_0}^2 + M_0^{*2}}}$$

in this model. Equation (4.3) is the same as in the original Walecka model [1]. The values in the table are smaller than the empirical one($\sim 33$MeV) [13]. However, if the $\rho$ meson effects are taken into account, eq.(4.3) is modified as [14]

$$a_4 = \frac{k_{F_0}^2}{6\sqrt{k_{F_0}^2 + M_0^{*2}}} + \frac{g_\rho^2}{12\pi^2 m_\rho^2 k_{F_0}^3}$$

In mean field approximation, inclusion of $\rho$ meson does not affect the saturation condition and the second term in eq.(4.4) gives positive contribution to $a_4$. In this point, our results are not inconsistent with the empirical value. Nevertheless, it need large value of $g_\rho^2(\gtrsim 60)$ to fit the empirical value for the small $K(\lesssim 250$MeV), because small $K$ has large $M_0^{*}$ in eq.(4.4).
V. SUMMARY AND DISCUSSIONS

We have studied the effective nucleon mass \( M_0^* \) at \( \rho_0 \), incompressibility \( K \) and the third derivative \( K' \) of binding energy in nuclear matter, using the nonlinear \( \sigma - \omega \) model. The results are summarized as follows.

(1) There is strong correlation between \( K \) and \( K' \), when we fix \( M_0^* \). The \( K' \) decreases as \( M_0^* \) increases, if \( K \gtrsim 250 \text{MeV} \). There is a weak local minimum of \( K' \) at \( M_0^* \approx 0.70M \) when \( K = 200 \text{MeV} \) and there are cross points of the lines at \( M_0^* \approx 0.75M \).

(2) When we assume \( K = 200 \sim 400 \text{MeV} \), \( |K'| \lesssim 100 \text{MeV} \), when \( M_0^* = 0.6 \sim 0.8M \). In these case, the \( K' \) has little contribution to the stiffness of nuclear EOS.

(3) The smaller value of \( K \) needs the larger value of \( M_0^* \) for the condition \( C \geq 0 \).

(4) The \( K-K' \) relations much depends on the value of \( M_0^* \). The \( K' \) increases as \( K \) increases, in the cases of \( M_0^* \leq 0.6M \). On the other hand, \( K' \) decreases as \( K \) increases, in the cases of \( M_0^* \geq 0.9M \).

(5) The \( K_c \) is negative when \( 200 \text{MeV} \lesssim K \lesssim 400 \text{MeV} \) and \( M_0^* \lesssim 0.9M \).

(6) The parameters sets are determined to fit the empirical value of \( K \) and \( K_c \) in the data of the GIMR[4]. It is difficult to fit the date with \( K \approx 200 \text{MeV} \).

(7) We have negative value of \( C \) when we put \( K \gtrsim 350 \text{MeV} \) to fit the data. This fact may indicate the unphysical behavior as pointed out in ref. [8], although we had the correct causal limit\( (v^2 \lesssim 1) \).

(8) The EOS with \( K \lesssim 250 \text{MeV} \) which are obtained in (2) is very soft, because they have large \( M_0^* \) and small \( K' \) besides small \( K \).

(9) The uncertainty of \( K_c \) has much effects on the EOS, especially when \( K \approx 250 \sim 300 \text{MeV} \). In this case, more accurate measurements are desirable.

In this paper we almost restrict ourselves on the discussion of \( K \) and \( K_c \). We have used the symmetric nuclear EOS for analysis. The other coefficients in \( K_A \) expansion must be studied in this model, or in the extended version of this model. Very recently, Von-Eiff et al.[15] analyzed GIMR data using the nonlinear \( \sigma-\omega-\rho \) model with the NL1 [16] and the NL-SH [17] parameters sets, and found these parameters set are disagreement with experiment. More extended model may be need for the accurate fit to the data.

One simple way to extend the model within the framework of mean field theory is to add more higher terms of \( \phi \) to the lagrangian (2.1). However, we remark that such a modification does not change the \( M_0^*-C_v \) relation (3.4) and may not change \( M_0^*-K \) (3.7) relation drastically. Modification of vector meson interaction might be needed.
Acknowledgment: Authors are grateful to Prof. Kohmura and Prof. Kumano for useful discussions, and to the members of nuclear theorist group in Kyushyu district in Japan for their continuous encouragement. The authors also gratefully acknowledge the computing time granted by the Research Center for Nuclear Physics (RCNP).

References

Table and Figure Captions

Table I(a), I(b), I(c)
Parameter sets fitted for the empirical value of $K$ and $K_c$ in the table 3 in ref. [4]. The $K$, $K_c$, $K'$ and $a_4$ are shown in MeV. (a) Results obtained by using the mean value of $K_c$ in the table. (b) Results obtained by using the upper bound for $K_c$ in the table. (c) Results obtained by using the lower bound for $K_c$ in the table.

Fig. 1 The $M_0' - K'$ relations. The solid line, the dotted line, the dashed line, the dashed-dotted line and the bold solid line are results with $K = 200, 250, 300, 350$ and 400MeV respectively. The solid circle and solid square are results of the original Walecka model(WM) and the RHA respectively.

Fig. 2 The $M_0' - C$ relations. The solid line, the dotted line, the dashed line, the dashed-dotted line and the bold solid line are results with $K = 200, 250, 300, 350$ and 400MeV respectively.

Fig. 3 (a) The $K - K'$ relations. The solid line, the dotted line, the dashed line, the dashed-dotted line and the bold solid line are results with $M_0'/M = 0.5, 0.6, 0.7, 0.8$ and 0.9 respectively. The solid circles are results obtained by using the empirical values of $K$ and $K_c$ in the table 3 in ref. [4].
(b) The $K - K_c$ relations. The solid line, the dotted line, the dashed line, the dashed-dotted line and the bold solid line are results with $M_0'/M = 0.5, 0.6, 0.7, 0.8$ and 0.9 respectively. The solid circles are the empirical values of $K$ and $K_c$ in the table 3 in ref. [4].

Fig. 4 The $\rho$ dependence of the binding energy. The solid line, the dotted line, the dashed line and the dashed-dotted line are results of EOS1,2,3 and 4 in table I(a) respectively.

Fig. 5 The $\rho$ dependence of the effective nucleon mass $M'$. The solid line, the dotted line, the dashed line and the dashed-dotted line are results of EOS1,2,3 and 4 in table I(a) respectively.
Fig. 6 The $\rho$ dependence of the sound velocity $v_s$. The solid line, the dotted line, the dashed line and the dashed-dotted line are results of EOS1, 2, 3 and 4 in table I(a) respectively.

Fig. 7 The $\rho$ dependence of the binding energy. The solid line, the dotted line, the dashed line and the dashed-dotted line are results of EOS5, 6, 7 and 8 in table I(b) respectively. The solid circles, the bold solid line, the bold dotted line, the bold dashed line and the bold dashed-dotted line are results of EOS9, 10, 11, 12 and 13 in table I(c) respectively.
Table I (a)

<table>
<thead>
<tr>
<th>EOS</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>250</td>
<td>300</td>
<td>350</td>
<td>400</td>
</tr>
<tr>
<td>$K_c$</td>
<td>-0.7065</td>
<td>-3.990</td>
<td>-7.274</td>
<td>-10.56</td>
</tr>
<tr>
<td>K'</td>
<td>-196</td>
<td>-90.5</td>
<td>63.5</td>
<td>266</td>
</tr>
<tr>
<td>$M_0^*/M$</td>
<td>0.91</td>
<td>0.83</td>
<td>0.60</td>
<td>0.51</td>
</tr>
<tr>
<td>$a^4$</td>
<td>12.8</td>
<td>14.0</td>
<td>18.4</td>
<td>20.9</td>
</tr>
<tr>
<td>$C_s^2$</td>
<td>44.02</td>
<td>153.0</td>
<td>312.8</td>
<td>373.0</td>
</tr>
<tr>
<td>$C_v^2$</td>
<td>20.20</td>
<td>72.62</td>
<td>214.8</td>
<td>269.4</td>
</tr>
<tr>
<td>B</td>
<td>0.5145</td>
<td>$4.617 \times 10^{-3}$</td>
<td>$-1.397 \times 10^{-3}$</td>
<td>$-1.149 \times 10^{-3}$</td>
</tr>
<tr>
<td>C</td>
<td>4.983</td>
<td>$9.223 \times 10^{-2}$</td>
<td>$-1.480 \times 10^{-3}$</td>
<td>$-1.832 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Table I (b)

<table>
<thead>
<tr>
<th>EOS</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>250</td>
<td>300</td>
<td>350</td>
<td>400</td>
</tr>
<tr>
<td>$K_c$</td>
<td>-0.7065+2.06</td>
<td>-3.990+2.06</td>
<td>-7.274+2.06</td>
<td>-10.56+2.06</td>
</tr>
<tr>
<td>K'</td>
<td>-272.0</td>
<td>-181.5</td>
<td>-42.55</td>
<td>144.8</td>
</tr>
<tr>
<td>$M_0^*/M$</td>
<td>0.93</td>
<td>0.88</td>
<td>0.75</td>
<td>0.56</td>
</tr>
<tr>
<td>$a^4$</td>
<td>12.5</td>
<td>13.2</td>
<td>15.2</td>
<td>19.5</td>
</tr>
<tr>
<td>$C_s^2$</td>
<td>3.618</td>
<td>77.82</td>
<td>205.8</td>
<td>335.6</td>
</tr>
<tr>
<td>$C_v^2$</td>
<td>4.759</td>
<td>37.64</td>
<td>121.7</td>
<td>239.8</td>
</tr>
<tr>
<td>B</td>
<td>12.55</td>
<td>0.1402</td>
<td>$-3.473 \times 10^{-4}$</td>
<td>$-9.838 \times 10^{-4}$</td>
</tr>
<tr>
<td>C</td>
<td>133.3</td>
<td>1.181</td>
<td>$1.327 \times 10^{-2}$</td>
<td>$-1.297 \times 10^{-3}$</td>
</tr>
<tr>
<td>EOS</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>-----</td>
<td>-----</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>K</td>
<td>200</td>
<td>250</td>
<td>300</td>
<td>350</td>
</tr>
<tr>
<td>$K_c$</td>
<td>2.557-2.06</td>
<td>-0.7065-2.06</td>
<td>-3.990-2.06</td>
<td>-7.274-2.06</td>
</tr>
<tr>
<td>K'</td>
<td>-193.0</td>
<td>-120.4</td>
<td>0.430</td>
<td>169.6</td>
</tr>
<tr>
<td>$M_5^*/M$</td>
<td>0.93</td>
<td>0.88</td>
<td>0.66</td>
<td>0.54</td>
</tr>
<tr>
<td>$a^4$</td>
<td>12.6</td>
<td>13.3</td>
<td>17.2</td>
<td>20.2</td>
</tr>
<tr>
<td>$C^2$</td>
<td>18.48</td>
<td>111.3</td>
<td>284.6</td>
<td>360.6</td>
</tr>
<tr>
<td>$C^2_v$</td>
<td>7.169</td>
<td>40.96</td>
<td>182.9</td>
<td>254.9</td>
</tr>
<tr>
<td>B</td>
<td>2.100</td>
<td>$4.044 \times 10^{-2}$</td>
<td>$-2.348 \times 10^{-3}$</td>
<td>$-1.383 \times 10^{-3}$</td>
</tr>
<tr>
<td>C</td>
<td>23.63</td>
<td>0.5344</td>
<td>$-2.044 \times 10^{-3}$</td>
<td>$-2.059 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Fig. 1
Fig. 3.(a)
$K_c$ [MeV]

$K$ [MeV]

Fig. 3.(b)
Fig. 5
Fig. 6
Fig. 7