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**Energy-Momentum Tensor
as a Source of the Gravitational Field
and the Effective Riemannian Space-Time**

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Abstract

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An assumption that the energy-momentum tensor is a source of the gravitational field naturally leads to the effective Riemannian space-time geometry.

Аннотация

Логунов А.А. Тензор энергии-импульса как источник гравитационного поля и эффективное риманово пространство-время: Препринт ИФВЭ 94-87. – Протвино, 1994. – 6 с., библиогр.: 1.

В работе показано: предположение, что источником гравитационного поля является тензор энергии-импульса материи естественно приводит к эффективной римановой геометрии пространства-времени.

If all physical fields including gravitational are considered in the Minkowski space with metric $\gamma_{\mu\nu}(x)$, then the energy-momentum tensor density will satisfy both local and integral conservation laws.

In an arbitrary coordinate system the local conservation law has the form

$$D_\mu t^{\mu\nu} = 0, \quad (1)$$

where D_μ is the covariant derivative in the Minkowski space. It is reduced to the standard partial derivative in Cartesian coordinates.

Let L be the scalar Lagrangian density of matter. Then the energy-momentum tensor density of matter, according to Hilbert, can be expressed through the Lagrangian scalar density as:

$$t^{\mu\nu} = -2 \frac{\delta L}{\delta \gamma_{\mu\nu}}, \quad (2)$$

where

$$\frac{\delta L}{\delta \gamma_{\mu\nu}} = \frac{\partial L}{\partial \gamma_{\mu\nu}} - \frac{\partial}{\partial x^\sigma} \left(\frac{\partial L}{\partial \frac{\partial \gamma_{\mu\nu}}{\partial x^\sigma}} \right). \quad (3)$$

As any form of matter is characterized by the corresponding energy-momentum tensor it is natural to admit that due to the universal character of gravity a source of the gravitational field is the energy-momentum tensor of matter. Before going to gravitational field equations we shall consider electrodynamics, where a conserved vector current j^ν is the source of the electromagnetic field, the field is of the vector character and is described by the vector potential A^ν .

Equations of electrodynamics in arbitrary coordinates have the form:

$$\begin{aligned} \gamma^{\alpha\beta} D_\alpha D_\beta A^\nu + \mu^2 A^\nu &= 4\pi j^\nu, \\ D_\nu A^\nu &= 0. \end{aligned} \quad (4)$$

Here we have introduced a photon mass μ for generality.

Therefore, if we admit that the source of the gravitational field is the conserved energy-momentum tensor density $t^{\mu\nu}$, then it is natural to suppose that the gravitational field is of the tensor nature and is described by the density of symmetric tensor $\tilde{\Phi}^{\mu\nu}$, where

$$\tilde{\Phi}^{\mu\nu} = \sqrt{-\gamma}\Phi^{\mu\nu}, \quad \gamma = \det \gamma_{\mu\nu}. \quad (5)$$

By full analogy with electrodynamics it could be assumed that equations of the gravitational field should have the following form:

$$\gamma^{\alpha\beta} D_\alpha D_\beta \tilde{\Phi}^{\mu\nu} + m^2 \tilde{\Phi}^{\mu\nu} = \lambda t^{\mu\nu} \quad (6)$$

$$D_\mu \tilde{\Phi}^{\mu\nu} = 0, \quad (7)$$

$$t^{\mu\nu} = t_g^{\mu\nu} + t_M^{\mu\nu}.$$

Here λ is a constant; m is the graviton mass; $t_g^{\mu\nu}$ is the energy-momentum tensor density of the gravitational field; $t_M^{\mu\nu}$ is the energy-momentum tensor density of the matter fields.

From equations (6) it follows that they should be also nonlinear for the proper gravitational field. This is an important distinction from electrodynamics, since the gravitational field is also a special kind of a source of the field. There arises the question: is it possible to get such a system of gravitational field equations from the least action principle? As is known from the least action principle, equations of the gravitational field and matter are obtained by varying the Lagrangian scalar density of matter over the field $\tilde{\Phi}^{\mu\nu}$ and matter fields Φ_A . This approach usually leads to equations

$$\frac{\delta L}{\delta \tilde{\Phi}^{\mu\nu}} = 0, \quad (8)$$

$$\frac{\delta L}{\delta \Phi_A} = 0. \quad (9)$$

Equations (8) differ from equations (6) first of all due to taking variational derivative over the field $\tilde{\Phi}^{\mu\nu}$, whereas in equations (6) the variational derivative over $\gamma_{\mu\nu}$ is present.

The initial scalar density of the Lagrangian is

$$L = L_g(\gamma_{\mu\nu}, \tilde{\Phi}^{\mu\nu}) + L_M(\gamma_{\mu\nu}, \tilde{\Phi}^{\mu\nu}, \Phi_A). \quad (10)$$

Here L_g is the density of the gravitational field Lagrangian; L_M is the density of the matter fields Lagrangian; Φ_A are fields of matter.

To reduce equations (8) to equations (6) for the arbitrary form of matter it is natural to suppose that tensor densities $\tilde{\gamma}^{\mu\nu}$, $\tilde{\Phi}^{\mu\nu}$ enter the Lagrangian density through the density $\tilde{g}^{\mu\nu}$ in the following form

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\Phi}^{\mu\nu}, \quad \tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}. \quad (11)$$

Therefore, the concept of the effective Riemannian geometry appears here. In other words, gravitational field partially joins directly the density of the metric tensor $\tilde{\gamma}^{\mu\nu}$ of the Minkowski space.

Condition (11) leads to the following structure of the Lagrangian density

$$L = L_g(\gamma_{\mu\nu}, \tilde{g}^{\mu\nu}) + L_M(\gamma_{\mu\nu}, \tilde{g}^{\mu\nu}, \Phi_A). \quad (12)$$

Having in mind (8) this leads directly to

$$\frac{\delta L}{\delta \tilde{\Phi}^{\mu\nu}} = \frac{\delta L}{\delta \tilde{g}^{\mu\nu}} = 0, \quad (13)$$

$$\frac{\delta L}{\delta \gamma_{\mu\nu}} = \frac{\delta L}{\delta \tilde{g}^{\alpha\beta}} \cdot \frac{\delta \tilde{g}^{\alpha\beta}}{\delta \gamma_{\mu\nu}} + \frac{\delta^* L}{\delta \gamma_{\mu\nu}}. \quad (14)$$

A star in (14) denotes a variational derivative of the Lagrangian density L over the explicitly present metric $\gamma_{\mu\nu}$.

According to (2) expression (14) can be written in the form:

$$t^{\mu\nu} = -2 \frac{\delta L}{\delta \tilde{g}^{\alpha\beta}} \cdot \frac{\delta \tilde{g}^{\alpha\beta}}{\delta \gamma_{\mu\nu}} - 2 \frac{\delta^* L}{\delta \gamma_{\mu\nu}}. \quad (15)$$

Taking into account equation (13) we get

$$t^{\mu\nu} = -2 \frac{\delta^* L}{\delta \gamma_{\mu\nu}}. \quad (16)$$

Making a comparison of this expression with equation (6) we obtain

$$-2 \frac{\delta^* L}{\delta \gamma_{\mu\nu}} = \frac{1}{\lambda} [\gamma^{\alpha\beta} D_\alpha D_\beta \tilde{\Phi}^{\mu\nu} + m^2 \tilde{\Phi}^{\mu\nu}]. \quad (17)$$

As the matter field Φ_A does not enter the r.h.s. of (17) the variation of the Lagrangian density of matter L_M over the explicitly present tensor density $\gamma_{\mu\nu}$ should be equal to zero. But it means that the tensor $\gamma_{\mu\nu}$ does not explicitly enter the expression of the Lagrangian matter density L_M . Condition (17) then takes the form

$$-2 \frac{\delta^* L_g}{\delta \gamma_{\mu\nu}} = \frac{1}{\lambda} [\gamma^{\alpha\beta} D_\alpha D_\beta \tilde{\Phi}^{\mu\nu} + m^2 \tilde{\Phi}^{\mu\nu}]. \quad (18)$$

From these considerations it follows that the Lagrangian density should have the following form:

$$L = L_g(\gamma_{\mu\nu}, \tilde{g}^{\mu\nu}) + L_M(\tilde{g}^{\mu\nu}, \Phi_A). \quad (19)$$

Therefore, we can see from (19) that only the metric tensor of the effective Riemannian space will appear in the matter equation of motion. It is also evident from (19) that the interaction of gravitational field with matter is realized through joining the tensor density $\tilde{\Phi}^{\mu\nu}$ directly to the density $\tilde{\gamma}^{\mu\nu}$ in the matter Lagrangian. Such an interaction has the universal character and does not depend on the sort of matter. Earlier, in Ref. [1] we have formulated this statement as a principle of geometrization. Now we can see that this principle is a natural consequence from the statement that a source of the gravitational field is the energy-momentum tensor density of matter.

As it has been shown earlier [1], the form of (19) permits to construct a gauge group and unambiguously find a Lagrangian density for the proper gravitational field:

$$L_g = \frac{1}{16\pi} \tilde{g}^{\mu\nu} (G_{\mu\nu}^\lambda G_{\lambda\sigma}^\sigma - G_{\mu\sigma}^\lambda G_{\nu\lambda}^\sigma) - \frac{m^2}{16\pi} (\frac{1}{2} \gamma_{\mu\nu} \tilde{g}^{\mu\nu} - \sqrt{-g} - \sqrt{-\gamma}). \quad (20)$$

Here

$$G_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\sigma} (D_\mu g_{\sigma\nu} + D_\nu g_{\sigma\mu} - D_\sigma g_{\mu\nu}) \quad (21)$$

Just using this expression for L_g it is easy to get convinced that it fulfils condition (18), if (7) is taken into account

$$-\frac{\delta^* L_g}{\delta \gamma_{\mu\nu}} = \frac{1}{16\pi} [\gamma^{\alpha\beta} D_\alpha D_\beta \tilde{\Phi}^{\mu\nu} + m^2 \tilde{\Phi}^{\mu\nu}]. \quad (22)$$

The constant λ in the equations is defined through the correspondence with Newton law.

The main conclusion we have arrived at is that the identification of the energy-momentum tensor of matter with a source of the gravitational field naturally leads to the appearance of the effective Riemannian geometry of space-time. Therefore, the Riemannian geometry of space-time is a straightforward consequence of the assumption that the source of the gravitational field is a conserved energy-momentum tensor of matter.

Defining the density of the energy-momentum tensor in the Riemannian space as

$$T^{\mu\nu} = -2 \frac{\delta L_M}{\delta g_{\mu\nu}}. \quad (23)$$

and taking into account (20) we get the complete system of equations for the gravitational field and matter in the form:

$$(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R) + \frac{m^2}{2}[g^{\mu\nu} + (g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}g^{\mu\nu}g^{\alpha\beta})\gamma_{\alpha\beta}] = \frac{8\pi}{\sqrt{-g}}T^{\mu\nu}, \quad (24)$$

$$D_\mu \tilde{g}^{\mu\nu} = 0. \quad (25)$$

It is noteworthy that the metric tensor of the Minkowski space enters the system of equations (24). The introduction of the graviton mass is of principal importance for this theory since only with its help it is possible to construct a theory of gravitational field in the Minkowski space.

This theory describes uniquely all the known observational facts in the Solar system. The theory predicts the presence of large "hidden mass" of matter in the Universe. There are no "black holes" in this theory.

From equations (24) and (25) it is possible to derive matter equations of motion in the form

$$\nabla_\mu T^{\mu\nu} = 0, \quad (26)$$

here ∇_μ is the covariant derivative in the Riemannian space with the metric $g_{\mu\nu}$.

From equations (26) it directly follows that test bodies move along geodesic lines of the effective Riemannian space-time. But this motion can not be considered as free due to the fact, that in the Minkowski space it occurs with acceleration under the action of the gravitational field. The last point is especially evident if we write the geodesic line equation in the form

$$\frac{DU^\nu}{d\sigma} = G_{\alpha\beta}^\rho U^\alpha U^\beta (\delta_\rho^\nu - U^\nu U_\rho). \quad (27)$$

Here

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu, \quad U^\nu = \frac{dx^\nu}{d\sigma},$$

$\frac{D}{d\sigma}$ is a covariant derivative in the Minkowski space. A free motion in the Minkowski space is described by the equation

$$\frac{DU^\nu}{d\sigma} = 0.$$

Therefore, equation (27) describes a motion of the test body under the action of the force F^ν and this force is of the vector nature

$$F^\nu = G_{\alpha\beta}^\rho U^\alpha U^\beta (\delta_\rho^\nu - U^\nu U_\rho).$$

According to equation (27) the charged test body in its motion along a geodesic line of the Riemannian space will emit the electromagnetic field. Therefore, in this theory an acceleration has the absolute meaning since there exist inertial systems of reference which are directly related to a distribution of matter as we have shown before.

References

- [1] Logunov A.A., Mestvirishvili M.A. Relativistic Theory of Gravitation. M.: Nauka, 1989 (in Russian).
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