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VIBRATION MODES OF A VECTOR SOLITON IN A NONLINEAR OPTICAL FIBER

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Abstract We analyze the dynamics of a vector soliton governed by a nearly integrable system of coupled nonlinear Schrödinger equations. Positing a Gaussian ansatz, we find a continuous family of solutions which approximate vector solitons with arbitrary polarization. We find three small vibrational eigenmodes, of which only two were previously known. At 45° polarization the new internal oscillation consists of antisymmetric oscillations of the soliton. At arbitrary polarization all three eigenmodes are given in an implicit form. We find, additionally, a threshold of the relative velocity of the two polarizations that leads to splitting of the arbitrarily polarized vector soliton.

Introduction

A pulse of light propagating in a monomode nonlinear optical fiber is described by the nonlinear Schrödinger (NLS) equation

$$u_x \pm \frac{1}{2}u_{tt} + |u|^2u = 0. \quad (1)$$

The NLS equation is completely integrable, and supports bright solitons in the case where the group velocity dispersion is negative (positive sign). The NLS equation's

complete integrability underlies the solitons' remarkable stability, due to which they are currently of intense interest as information bits in optical fiber communication systems.

Real life intrudes on this simple mathematical picture in the form of light's polarization. Propagation of light in a bimodal optical fiber is described by a pair of coupled NLS equations rather than a single NLS equation,

$$u_x + \frac{1}{2}u_{tt} + (|u|^2 + B|v|^2)u = 0, \quad (2a)$$

$$v_x + \frac{1}{2}v_{tt} + (B|u|^2 + |v|^2)v = 0. \quad (2b)$$

(We restrict ourselves henceforth to anomalous dispersion.) Typical material parameters of an optical fiber are $B = 2/3$ or $B = 2$ [Ref. 1], whereas Eqs. (2) are completely integrable only for the physically not very important case $B = 1$ [Ref. 2] (and the here irrelevant $B = 0$ and $B = -1$).

In spite of their nonintegrability Eqs. (2) will support, if not solitons, solitary waves,¹ which are somewhat misleadingly called vector solitons. The lack of integrability will cause the system to partake to some degree in the phenomena connected with chaotic systems, for example soliton-soliton collisions will give rise to radiation. Yet vector solitons do retain much of the stability of true solitons, especially in isolation. The greater complexity of the nonintegrable system has caused it to resist analytic study of, among other important things, vector solitons' internal dynamics.

Herein, in order to gain tractability with which we may take two steps forward, we first take one step back in making an oversimple model of the vector solitons. At the cost of some quantitative accuracy we gain the ability to analytically investigate the small vibration dynamics and splitting of the vector soliton in general cases which were previously only handleable by numerical simulation.

Vector solitons' stability justifies their approximation for many purposes by an ansatz whose general shape is maintained but whose parameters are allowed to vary dynamically. We choose a Gaussian ansatz, which captures the pulselike character of the few exactly known special cases and which numerical simulations have shown the general vector solitons to possess,¹ and which also facilitates our calculations:

$$u = \eta_u \exp \left[-\frac{1}{2} \left(\frac{t - y_u}{W_u} \right)^2 \right] \exp \left[i \left(\sigma_u + 2V_u(t - y_u) + \frac{b_u}{2W_u}(t - y_u)^2 \right) \right], \quad (3a)$$

$$v = \eta_v \exp \left[-\frac{1}{2} \left(\frac{t - y_v}{W_v} \right)^2 \right] \exp \left[i \left(\sigma_v + 2V_v(t - y_v) + \frac{b_v}{2W_v}(t - y_v)^2 \right) \right]. \quad (3b)$$

Note that there is an independent parameter for each polarization mode that describes the central position, pulse width, amplitude, phase, carrier frequency and frequency

chirp. These parameters' dynamic evolution under the obeisance of the system's governing equations (2) in effect includes in the model internal energy excitations, pulsation of the vector solitons' shape, and attraction between the two modes. Excluded from the model is radiation, asymmetries in the individual modes, and anything but very simple changes in the pulse shape.

Internal Vibrations

The ansatz, Eqs. (3), is substituted into the Lagrangian density from which the governing equations (2) are derived. The result is integrated over time (t) from $-\infty$ to $+\infty$. Taking the variation with respect to each of the parameters in the ansatz gives a model of the vector soliton in terms of a set of equations of motion of the ansatz parameters.

From the ten original degrees of freedom there are four constants of integration. Two are the energies of the polarization modes, $M_u = \sqrt{\pi}\eta_u^2 W_u$, $M_v = \sqrt{\pi}\eta_v^2 W_v$; the third is the total momentum, $P = M_u V_u + M_v V_v$; and the fourth is the Hamiltonian, which we do not write here.

There exists for any values M_u and M_v (in effect, for any polarization and total energy) exactly one fixed point:

$$\Delta y = \Delta V = b_u = b_v = 0, \quad (4a)$$

$$W_u = \frac{\sqrt{2\pi}}{M_u} \left(1 + B \left(\frac{2}{1 + \alpha^2} \right)^{\frac{3}{2}} \frac{M_v}{M_u} \right)^{-1}, \quad (4b)$$

$$W_v = \frac{\sqrt{2\pi}}{M_v} \left(1 + B \left(\frac{2}{1 + \alpha^{-2}} \right)^{\frac{3}{2}} \frac{M_u}{M_v} \right)^{-1} = \alpha W_u, \quad (4c)$$

where α is defined (uniquely on the positive real axis) by the equation

$$B \left(\frac{2}{1 + \alpha^2} \right)^{\frac{3}{2}} \left(\alpha^4 - \frac{M_v}{M_u} \right) + \frac{M_v}{M_u} \alpha - 1 = 0. \quad (4d)$$

This fixed point is stable for all positive values of B and physical (positive) M_u and M_v . The polarization angle of the vector soliton may be considered to be $\tan^{-1}(M_u/M_v)$.

The widths of the two components of the vector soliton, which in the general case are given implicitly by Eqs. (4), may be expressed explicitly either when the energy is entirely in one polarization mode, which is trivial, or when the energy is split almost evenly between the modes. Near 45° polarization we find pulse widths

$$W_u = \frac{\sqrt{2\pi}}{(1+B)M} \left(1 - \frac{1-B}{1+4B} \frac{\delta M}{M} \right), \quad (5a)$$

$$W_v = \frac{\sqrt{2\pi}}{(1+B)M} \left(1 + \frac{1-B}{1+4B} \frac{\delta M}{M} \right), \quad (5b)$$

where $M_u \equiv M + \delta M$ and $M_v \equiv M - \delta M$, $\delta M/M \ll 1$.

Expanding and linearizing the equations of motion about the fixed point represented by Eqs. (5), we obtain three second order ODEs for the small variations of Δy , W_u and W_v . The equation for $\delta \Delta y$ (δ indicates a variation) separates from those for δW_u and δW_v and contains only one mode of oscillation. The equations for δW_u and δW_v couple only these degrees of freedom and contain two modes of oscillations. In general the modes are a mixture of symmetric oscillations, in which the polarization modes' widths dilate and contract together, and antisymmetric oscillations, in which one polarization mode expands while the other narrows. The eigenmodes and their frequencies are given by the eigenvectors and corresponding eigenvalues of the matrix

$$\begin{pmatrix} -\frac{1}{W_u^4} - \frac{6}{\sqrt{\pi}} B \frac{M_v W_u^2}{(W_u^2 + W_v^2)^{5/2}} & \frac{6}{\sqrt{\pi}} B \frac{M_v W_u W_v}{(W_u^2 + W_v^2)^{5/2}} \\ \frac{6}{\sqrt{\pi}} B \frac{M_u W_u W_v}{(W_u^2 + W_v^2)^{5/2}} & -\frac{1}{W_v^4} - \frac{6}{\sqrt{\pi}} B \frac{M_u W_v^2}{(W_u^2 + W_v^2)^{5/2}} \end{pmatrix}, \quad (6)$$

where the basis is the vector $\begin{pmatrix} \delta W_u \\ \delta W_v \end{pmatrix}$. Close to $M_u = M_v$, the frequencies and their corresponding eigenvectors may be found explicitly:

$$\omega_+ = \frac{1}{\pi} (1+B)^2 M^2 + O(\delta M^2), \quad (7a)$$

$$\begin{pmatrix} 1 - \frac{1}{2} \left[1 - \frac{(1+2B)(13B-8)}{3B(1+4B)} \right] \frac{\delta M}{M} \\ 1 + \frac{1}{2} \left[1 - \frac{(1+2B)(13B-8)}{3B(1+4B)} \right] \frac{\delta M}{M} \end{pmatrix}, \quad (7b)$$

and

$$\omega_- = \frac{1}{\pi} (1+B)^{3/2} (1+4B)^{1/2} M^2 + O(\delta M^2), \quad (8a)$$

$$\begin{pmatrix} 1 - \frac{1}{2} \left[1 + \frac{(1+2B)(13B-8)}{3B(1+4B)} \right] \frac{\delta M}{M} \\ -1 - \frac{1}{2} \left[1 + \frac{(1+2B)(13B-8)}{3B(1+4B)} \right] \frac{\delta M}{M} \end{pmatrix}. \quad (8b)$$

The third oscillation mode is that of the relative position of the two components of the vector soliton, Δy , with frequency

$$\omega_{\Delta y} = \frac{1}{\pi} B^{1/2} (1+B)^{3/2} M^2 + O(\delta M^2). \quad (9)$$

It is noteworthy that to first order in $\delta M/M$, the two dilation-contraction modes couple together, but neither couples to the relative position oscillation. We mention here a deficiency in our model: it freezes out radiation modes. From the simple unpolarized NLS equation we know that in fact the energy of the oscillation will be

gradually shaken off and the vector soliton will asymptotically approach a steady state as described by Eqs. (4).

The calculation of the second (“–”) vibrational mode, Eqs. (8), which rounds out the vibrational analysis, is to our knowledge a novel result. The other oscillation modes have been calculated previously, but only at the point $M_u = M_v$, not including the first order perturbations nor the more general nonsymmetric case. Finally, note that, *mutatis mutandis*, each of the symmetric parts of our results agree up to a multiplicative constant with previous results,³ which are based on a similar model but with symmetric hyperbolic secant pulses.

Dynamics of Escaping Solitons

The interaction terms $B|u|^2|v|^2$ in the governing equations (2) create an effective attraction between the pulses in the two polarization modes. In order for the two polarization mode energies to separate from each other, they must have initially at least the minimum value of the Hamiltonian for widely separated pulses. At the point where the separation Δy and the frequency chirps b_u and b_v are zero, and for widths whose values minimize the Hamiltonian, the minimum escape velocity $\Delta V \equiv V_u - V_v$ is

$$\Delta V_{\text{escape}}^2 = \left(\frac{1}{M_u} + \frac{1}{M_v} \right) \left[\frac{BM_u M_v}{2\sqrt{\pi(W_u^2 + W_v^2)}} - \frac{M_u}{8} \left(\frac{M_u}{\sqrt{2\pi}} - \frac{1}{W_u} \right)^2 - \frac{M_v}{8} \left(\frac{M_v}{\sqrt{2\pi}} - \frac{1}{W_v} \right)^2 \right], \quad (10a)$$

where W_u and W_v are taken from Eqs. (4). Near $M_u = M_v$ this gives

$$\Delta V_{\text{escape}}^2 = \frac{1}{2\pi} B \left(1 + \frac{1}{2} B \right) M^2 + O(\delta M^2). \quad (10b)$$

This is not a sufficient condition for the vector soliton to split, since the separating pulses may contain frequency chirps and/or nonequilibrium widths, which may subtract enough of the relative kinetic energy to reduce it to below the escape threshold. On the other hand, contributions to the Hamiltonian at the start may reduce the initial velocity needed for eventual separation. A vector soliton may hold together over one or more oscillations yet still split apart in the future if and when the system wanders into a region of phase space where Δy is large and most of the Hamiltonian comes from its kinetic part rather than its internal part. Thus the escape velocity (10) is neither the minimum velocity which insures escape nor the maximum below which the vector soliton remains in one piece, but may rather be viewed as a guidepost which marks a grey area between the two regions.

1. C. R. Menyuk, J. Opt. Soc. Am. B **5**, 392 (1988).
2. S. V. Manakov, Zh. Eksp. Teor. Fiz. **65**, 505 (1973) [Sov. Phys. JETP **38**, 248 (1974)].
3. T. Ueda and W. L. Kath, Phys. Rev. A **42**, 42 (1990).