Abstract

In comparison with the WT chiral identity which is indispensable for renormalization theory, relations deduced from the non-linear chiral transformation have a totally different physical significance. We wish to show that non-linear chiral transformations are powerful tools to deduce useful integral equations for propagators. In contrast to the case of linear chiral transformations, identities derived from non-linear ones contain more involved radiative effects and are rich in physical content. To demonstrate this fact we apply the simplest non-linear chiral transformation to the Nambu-Jona-Lasinio model, and show how our identity is related to the Dyson-Schwinger equation and Bethe-Salpeter amplitudes of the Higgs and \( \pi \). Unlike equations obtained from the effective potential, our resultant equation is exact and can be used for events beyond the LEP energy.
1. Introduction

Over the past four years Tevatron and LEP experiments have stimulated investigations on models beyond the standard model. In addition to those in existence for many years (preon, technicolor, supersymmetry, etc.) a new idea has been proposed to explain the heaviness of the top mass by a still unknown Nambu-Jona-Lasinio interaction (NJL) [1], which itself is an effective interaction of also a yet unknown vector meson based upon the underlying philosophy named “Bootstraps” by Nambu [2]. When such a new model is applied, the Dyson-Schwinger (DS) equation plays a predominantly important role in estimating the top mass.

The Bethe-Salpeter (BS) equation was derived originally by way of perturbation by summing the series of repetitions of one loop. Schwinger instead has derived this equation by the path integral formalism [3]. Such a non-perturbative approach based on the path integral has been largely developed since then, and the procedure of the calculation is nowadays well formulated [4,5]. In this approach the introduction of external fields (sources), denoted by and conventionally, is vitally important. The radiative corrections appears through the sources and .

Meanwhile a new horizon is opened by Fujikawa for an aspect of the path integral [6]. The chiral anomaly, whose signature was found for the first time by Fukuda, Miyamota and Steinberger, and then was formulated by Adler, has been dealt with by the path integral formalism in a concise manner. The path integral measure is not invariant for the chiral transformation, although the Lagrangian is. As a consequence the Ward-Takahashi (WT) identity has an anomalous term in comparison to what is expected by simple inspection of the Lagrangian.

Fujikawa has developed further his method to deal with BRs symmetry in QCD and the gravitational interaction where the non-linear transformations take place [8]. Recently a more general kind of non-linear transformations, coherent transformation so to speak, was introduced and various new kinds of anomalous WT identities with higher order products of fields have been deduced [9,10]. Higher the order of non-linearity, more involved the radiative corrections are. Here again the use of external sources and is indispensable to introduce desired radiative effects into the identities. Therefore, if one carries out Schwinger’s procedure in combination with non-linear chiral transformations, one ought to obtain equations which are more general than the DS equations.

In the present article we wish to show that even the WT identity through a simplest non-linear transformation includes an integrated form of the DS equations together with the source equations, and gives a more general form of the relation than the DS equation alone. To do this we adopt NJL, since the DS equation of NJL has been dealt with in a number of articles [2,11-16] such that we can compare our result with theirs.

2. Procedure

We start from the following non-linear chiral transformation [16]

\[ \psi(x) \rightarrow \{\exp \alpha(x) \gamma_5 F(y, z) \psi(x)\} \]  \hspace{1cm} (1)

with

\[ F(y, z) = \bar{\psi}(y)\gamma_5 \psi(z) + \bar{\psi}(z)\gamma_5 \psi(y). \]  \hspace{1cm} (2)

The suffix + indicates the chronological order.

Here a non-local form is chosen in order to bypass obstacles due to the composite fields. Then the partition function relative to the NJL Lagrangian

\[ L = i\bar{\psi}\gamma\psi + g\{\bar{\psi}\psi^3 - (\bar{\psi}\gamma_5 \psi)^3\} \]  \hspace{1cm} (3)
changes by an infinitesimal amount as

\[ \int D\tilde{\omega} D\psi \delta \int (L + \delta L + \alpha A) dx \]  

in Fujikawa’s notation. The quantity \( \delta L = i\alpha F(y, z) \partial_\mu (\bar{\psi} \gamma_5 \gamma^\mu \psi) + O(\alpha^2) \) represents the variation of the Lagrangian whereas the \( A \) represents the variation of the path integral measure and, in addition to the well-known chiral anomaly \( A(y, z) = \int \varphi^+_\lambda(x) \gamma_5 \varphi_\lambda(x) d\lambda \), contains anomalies of other kinds of tensors, and is

\[ A = \int d\lambda \{ (\bar{\psi}(y) \gamma_5 \chi(z))(\varphi^+_\lambda(x) \gamma_5 \varphi_\lambda(x)) \]
\[ + \sum_a \frac{1}{N_a} (\bar{\psi}(y) J_a \psi(x))(\varphi^+_\lambda(x) J_a \varphi(z)) \]
\[ + \sum_a \frac{1}{N_a} (\bar{\psi}(x) J_a \psi(z))(\varphi^+_\lambda(y) J_a \varphi(x)) \]
\[ + (y \leftrightarrow z) \]  

arranged in chronological order. The \( J_a \) runs over all possible tensors, 1, \( \gamma_\mu \),

\[ \frac{1}{2} (\gamma_\mu \gamma_5 - \gamma_5 \gamma_\mu) \gamma_5 \gamma_5 \]  

and \( \gamma_5 \). The second and third terms are due to the variation of \( F(y, z) \) and \( \bar{\psi}(x) \) and \( \psi(x) \) and are equal to 1/4 in our case. The \( \varphi_\lambda \) are a complete orthogonal set of functionals introduced by Fujikawa \([6,8]\). The equations for \( \varphi_\lambda \) have never been given explicitly for the case of BRS or for the gravitation because they are rather complicated.

In our case it is just

\[ \gamma^0 \mathcal{P} \varphi_\lambda = \lambda \varphi_\lambda \]  

with

\[ \mathcal{P} = i\bar{\psi} + 2g \{ \bar{\psi}(x) \psi(x) - \gamma_5 \bar{\psi}(x) \gamma_5 \psi(x) \} \]  

The simplest WT identity can be obtained by applying \( \delta/\delta \alpha \) on \( Z \), but does not contain sufficient radiative effects for our purpose. Therefore we introduce sources and apply \( \delta/\delta \eta \delta/\delta \tilde{\eta} \delta/\delta \alpha \) on the expression of the generating functional \( Z \),

\[ Z = \int D\tilde{\omega} D\psi e^{\int (L + \tilde{\omega} \psi \psi)} dx \]  

transformed by means of (1). The WT identity thus obtained does contain sufficient ingredients but this time it is full of terms irrelevant for our purposes.

To extract only the terms necessary for us we construct a device. We first apply \( \mathcal{P} \) and \( \mathcal{P} \) as

\[ \mathcal{P}^{\chi}(\omega) \mathcal{P}^{a d}(u) \left[ \delta \tilde{\eta}(\omega) \delta \tilde{\eta}(u) \right] \frac{\delta Z}{\delta \alpha} \right|_{\alpha=0} = 0 \]  

where \( \mathcal{P} = \gamma^0 \mathcal{P} \gamma^0 \). The \( \mathcal{P}(\omega) \delta/\delta \tilde{\eta}(\omega) \) corresponds to \( \delta/\delta \tilde{\omega}(\omega) \) in Schwinger’s procedure for deducing the DS equation, (see eq.(10.13) of [5]). If one integrates over \( u \), one gets a generalized form of the DS equation.

The \( \mathcal{P}(\omega) \delta/\delta \eta(u) \), together with \( \mathcal{P}(\omega) \delta/\delta \tilde{\eta}(\omega) \), have been used also to derive the BS equation (see eq.10.24 of [5]). If we had started from a transformation of a higher non-locality and non-linearity differing by additional factor \( \{ (\bar{\psi}(y) \psi(z)) \}^2 - (\bar{\psi}(y) \gamma_5 \psi(z))^2 + (y \leftrightarrow z) \), the above identity (9) would have contained a generalized form of the BS equation.
Now the eq. (9) itself is already too complicated and contains equations other than what we seek. However, there is an alternative way to obtain a simple relation which the quark mass has to satisfy. When we apply $\delta^c(\omega)$ and $\delta^d(u)$ on $(\delta/\delta \bar{\psi}(\omega) \delta/\eta_u(\omega) \delta/\alpha(x))Z$ in eq. (9), pairs $\bar{\psi}(x) \psi(\omega)$ and $\bar{\psi}(u) \psi(x)$ are reduced to $\delta$-functions. When integrated over $u$ and $\omega$, eq. (9) becomes very simplified. All the irrelevant components drop out when one takes the trace due to $\gamma_5$ matrices. We arrive at the following concise identity

$$g < (\bar{\psi}(y) \gamma_5 \psi(y) + (y \leftrightarrow z)) (\bar{\psi}(y) \gamma_5 \psi(z) + (y \leftrightarrow z))$$

$$- (\bar{\psi}(y) \psi(y) + (y \leftrightarrow z)) (\bar{\psi}(y) \psi(z) + (y \leftrightarrow z)) >$$

$$= Tr \int \phi^+ \lambda(y) \phi_\lambda(z) d\lambda + (y \leftrightarrow z)$$

(10)

Here (in (5) also) we emphasize that the rule for the chronological order is applied not only for $\psi$ and $\bar{\psi}$ but also for $\phi_\lambda$ and $\bar{\phi}_\lambda$ as if $\phi_\lambda$ and $\bar{\phi}_\lambda$ were fermionic field variables too. For example, in rhs, $< \phi^+ \lambda(y) \phi_\lambda(z) >$ are $\phi^+ \lambda(y) \phi_\lambda(z)$ or $- \phi_\lambda(z) \phi^+ \lambda(y)$ according to if $y_0 > z_0$ or $z_0 > y_0$.

3. Terms from the path integral measure (the anomaly)

Now we have to calculate rhs of (10) explicitly. The local one $\int \phi^+ \lambda(x) \phi_\lambda(x) d\lambda$ appears in the dilatation anomaly. It diverges and has been regularized by Fujikawa [8]. The $\int \phi^+ \lambda(y) \phi_\lambda(z) d\lambda$ instead is regular, as will be seen below.

The simplest way to estimate the rhs of (10) seems to be to introduce a fifth time $s$, called the proper time by Dirac, and to reexpress eq. (6) as

$$H \varphi_\lambda(x, s) = \lambda \varphi_\lambda(x, s) = i \frac{d}{ds} \varphi_\lambda(x, s)$$

(11)

with the Hamiltonian $H = \gamma^0 \partial$ which is the conjugate variable to $s$. Then due to the relation $\partial_x \theta(x_0 - y_0) = \delta(x_0 - y_0)$ the matrix with chronologically ordered elements

$$B^0_s(y, y^0 : x, s) = \int \phi^+ \lambda(y, s') \phi_\lambda(x, s) d\lambda$$

(12)

obeys the equation

$$(\gamma^0 \partial - i \partial_s) \phi^+ \lambda(y, s^0 : x, s) = -i \delta \epsilon_0 \delta^0(x - y) \delta(s - s')$$

(13)

Its solution is

$$B(y, s^0 : x, s) = \int d\alpha \exp \{i(H - i \partial_s) \alpha \} \delta^0(x - y) \delta(s - s')$$

(14)

which, after some manipulation, can be brought into the following form,

$$\frac{1}{(2\pi)^3} \int d^3 k e^{-ik \cdot y} e^{-iH(s^0 - s)} e^{ik \cdot x}$$

(15)

(see Appendix A).

In the trace of the matrix $B$, only the terms with an even number of $\gamma$ survive. Therefore one can save the labor by starting from the second order equation

$$H_2 \varphi_\eta(x, \tau) = \eta \varphi_\eta(x, \tau) = i \partial_\tau \varphi_\eta(x, \tau)$$

$$= i (\partial_s)^2 \varphi_\lambda(x, s)$$

(16)
where in euclidean space

\[ H_2 = H^2 = p^+ p = \{ p - 2g(S + \gamma_5 P) \} \{ p + 2g(S - \gamma_5 P) \} \]  

(17)

and \( \eta = \lambda^2 \). In this space the \( S \) and \( P \) stand for \( \psi^+ \gamma^\alpha \psi \) and \( \psi^+ \gamma^\alpha \gamma_5 \psi \) respectively. For the Wick’s rotated \( \gamma \) matrices we take the convention of Fujikawa [6]. The \( \tau \), the conjugate variable to \( H_2 \), is also called the proper time, and has been used by Schwinger for a similar purpose [17]. With this \( H_2 \), the trace of the matrix \( B \) is

\[ \text{Tr } B(x, y) = -\frac{i}{(2\pi)^2} \text{Tr} \int d^4k \, e^{-iky} e^{-iH_2(\tau - \tau')} \delta^{4x} \]  

(18)

which, at \( x = y \), is identical to Fujikawa’s formula for the regularization of the anomaly, if one replaces the Euclidean time \( i(\tau - \tau') \) by Fujikawa’s \( 1/M^2 \) [18].

In order to integrate \( \text{Tr } B(x, y) \), it is convenient to use a new variable \( k' = k + i(x - y)M^2/2 \). Then \( \text{Tr } B(x, y) \) becomes proportional to \( \exp\{-(x - y)^2M^2/4\} \) in Minkowski space, and is small except at \( x - y \leq 1/M \). Furthermore it merges smoothly into the local value \( \text{Tr } B(x, y) \) as \( y \) approaches \( x \) (Appendix B). For conciseness in the following, we limit ourselves to the local value. Then the process of calculation is the same as for dilatation anomaly, except that \( M \) is large but is kept finite for the reason given later.

4. The WT identity

Developing \( \text{Tr } B \) in terms of \( M \) and \( 1/M \), and retaining terms independent of \( 1/M \), we obtain the following result for the identity (10) (see again Appendix B),

\[ g < S^2 - P^2 > = \frac{1}{2\pi^2} (M^2g^2(S^2 - P^2)) \]

\[ + \frac{1}{2}g^2(\partial_\mu S)(\partial^\mu S) - 2g^4S^4 \]

\[ - \frac{1}{2}g^2(\partial_\mu P)(\partial^\mu P) - 2g^4P^4 \]

\[ + 4g^4S^2P^2 \]  

(19)

where \( S = \bar{\psi}(x)\psi(x) \) and \( P = \bar{\psi}(x)\gamma_5\psi(x) \) as is mentioned previously, but in the Minkowsky space.

As will be seen below, one can easily confirm that the above eq.(19) is consistent with the formula for \( S \) and \( P \) derived in the past in refs.[11-13]. We first mention that \( M \) is equal to the Euclidean cutoff momentum, denoted as \( \Lambda \), as is shown on page 3678 of [18]. Then, with the use of the gap equation, \( 2\pi^2g(S^2 - P^2) - M^2g^2(S^2 - P^2) \) is equal to \(-m^2g^2(S^2 - P^2)\), and the identity (19) is reduced to

\[ 0 = < m^2g^2(S^2 - P^2) + \frac{1}{2}g^2(\partial_\mu S)(\partial^\mu S) \]

\[ - 2g^4S^4 - \frac{1}{2}g^2(\partial_\mu P)(\partial^\mu P) - 2g^4P^4 + 4g^4S^2P^2 > \]  

(20)

This is indeed identical to the sum of the two equations (12) and (13) of ref.[11], except for the total derivative terms which disappear when integrated. Our idea is now found to be on the right road.
5. Relation to the DS equation and the BS amplitude

Our remaining task is to clarify how the new equations 19 and 20 are related to the DS equations and BS amplitude. Our $S = (\bar{\psi} \gamma_5 \psi)$ and $P = (\bar{\psi} \gamma_5 \psi)$ contain not only quark propagators, but also bound states (Higgs and $\pi$ in particular). We separate these two kinds of components. We first introduce the Legendre transform of $G = \log Z$, denoted conventionally by $\Gamma$. Replacing $\psi$ and $\bar{\psi}$ in the lhs of eq.(10) by $-i\delta/\delta \tilde{\eta}$ and $+i\delta/\delta \eta$ acting on the Legendre transformed $Z$ respectively, we obtain the following result,

$$< \tilde{\psi}^a(y) (\tilde{\psi}(y) \gamma_5 \psi(y)) \tilde{\psi}^b(z) > = S_{cc}(yy) S_{bc}(zy) - S_{cc}(yy) S_{ba}(zy) + 4G_{ab}^1$$

(21)

where

$$iS_{ab}(xx') = \delta^2 G/\delta \tilde{\eta}^a(x) \delta \eta^b(x').$$

(22)

The $4G_{ab}^1$ is

$$4G_{ab}^1 = \delta^4 G/\delta \tilde{\eta}^a(t) \delta \eta^b(z) \delta \eta^b(y) \delta \tilde{\eta}^a(x)$$

(23)

and is related to the vertex $4\Gamma$ through the formula

$$\delta^4 G/\delta \tilde{\eta}^a(t') \delta \eta^b(z) \delta \eta^b(y) \delta \tilde{\eta}^a(x) = i \int S_{f} \Gamma_{f'} e^{i \theta(t')} S_{f'} e^{i \theta(t')} S_{f'} e^{i \theta(t')} (\tilde{q} y z) e^{i \theta(t')} \tilde{q} e^{i \theta(t')} \tilde{q}$$

with

$$4\Gamma_{f} e^{i \theta(t')} (\tilde{t} y z) e^{i \theta(t')} \tilde{q} e^{i \theta(t')} \tilde{q}$$

(24)

which is (C5) of the appendix C in the lowest order approximation. The $4G^1$ represents the four leg processes and still contains quark contributions in addition to those of the Higgs and $\pi$.

We now adopt the spectral representation for the quark propagator,

$$S_{ab}(x, y) = \int \frac{1}{\alpha(p)^2 - \beta(p)^2} e^{i \theta(p - y)} dp$$

(26)

and assume that $\alpha(p)$ and $\beta(p)$ are independent of $p$, implying that the bubble approximation is introduced for the quark line. Choosing the sum of momenta of two incoming quarks, $p_3 + p_4 = p_1 + p_2 = q$ as an independent variable, we obtain (see Appendix D)

$$S_{cc} S_{ba} + 4G_{ab}^1 = \frac{i(2\pi)^{-8}}{(4\pi)^2} g o^4 \log \frac{\Lambda^2}{\mu^2}$$

$$4^2 \int q \, dq \left\{ 1 - \frac{r^2}{q^2 - 4m^2} + O \left( \frac{1}{q^2 - 4m^2}, r^2 \right) \right\}$$

(27)

where $r = p_1 + p_4$. The $O\left(\frac{1}{q^2 - 4m^2}, r^2\right)$ indicates higher order terms in $1/(q^2 - 4m^2)$, includes further factors such as $1/q^2$ etc., and has to be disregarded in the bubble approximation.

In the same way the first term of lhs of eq.(10) is

$$< \tilde{\psi}^a(y) (\tilde{\psi}(y) \gamma_5 \psi(y) \gamma_5 \psi(z)) > = S_{cc}^2(yy) S_{ba}^2(zy)$$

$$- S_{cc}^2(yy) S_{ba}^2(zy) + 4G_{ab}^5$$

(28)
with
\[ 4G_{ab}^5 = \delta^4 G/\delta \eta^4(y)(\delta \eta^4(y)\delta \bar{\eta}^4(y))(\gamma_5 \delta \bar{\eta}^4(z))^b \]
(29)

Corresponding to (27) we obtain
\[ -S_{ac}^5S_{ca}^5 - 4G^5 = -\frac{i(2\pi)^{-8}}{(4\pi)^7} g^2 a^4 \log \frac{\Lambda^2}{\mu^2} \]
\[ 4^2 \int dr dq \left\{ 1 - 3 \frac{r^2}{q^2} + O\left( \frac{1}{q^4}, r^2 \right) \right\} \]  
(30)

The first terms in (27) and (30), which are too highly divergent to be renormalized off, cancel each other in eqs. (10) and (20) (see also ref. [13]).

The second terms in (27) and (30), which we denote as \( D_\phi \) and \( D_\pi \) respectively, are evidently the Higgs (mass of Higgs is twice of \( m \)) and \( \pi \) propagators, and are exactly equal to the BS amplitudes (2,5) and (2,7) of [14], when one integrates over \( \ell \) of \( \Gamma ([A,4] of [14]) \) from \( \mu \) to \( \Lambda \) (see also [15]) (note that there appears a difference by a factor \( \log \frac{\Lambda^2}{\mu^2} \) from [14] as the result of the way of integration adopted here, as seen in (D1)).

The sum (27) plus (30) multiplied by \( g^2 \), is now simply \( D_\phi - D_\pi \),
\[ g^2(S_{ac}S_{ca} - S_{ac}^5S_{ca}^5 + 4G^1 - 4G^5) = -i(D_\phi - D_\pi) \]  
(31)

The eqs. (21) and (22) are now expressed in terms of propagators as
\[ g^2 < S^2 - P^2 >= g^2(S_{ac}S_{ca} - S_{ac}^5S_{ca}^5) - i(D_\phi - D_\pi) \]  
(32)

which is consistent with the conventional expressions for \( S \) and \( P \) (for example see p. 7 of [15])
\[ gS = gv + \phi \]
\[ gP = \pi \]  
(33)

with \( m = 2g \nu \).

When expressed in terms of \( v \), \( \phi \) and \( \pi \) by eq. (33), our identity (20) represents the vacuum condition for the effective potential to be at a minimum, that is, zero. The result is consistent with the chiral scalar model from the effective field approach [11-15]. Here, \( < S^2 - P^2 > \) in eq. (20) is replaced by \( \frac{1}{3} (S^2 - P^2) \) corresponding to the bubble approximation.

Compared to this effective field approach, in Schwinger’s path integral formalism [3,4 and 5] the result is always given in terms of propagators only. Through the relation (32), the identity (20) presents an equation for propagators of quark, \( \phi \) and \( \pi \). In our particular case the identity becomes very simple as will be explained below.

While the identity (10) is a function of \( y \) and \( z \), the identity (20) depends upon \( x \) only, and consequently all the propagators including \( D_\phi \) and \( D_\pi \) are constants. Owing to the gap equation the quark propagator \( S \) is \( m/2g \) and \( S^5 \) vanishes. Then eq. (20) becomes simply
\[ D_\phi = \frac{m^2}{8} = \frac{(gS)^2}{2} \] with \( D_\pi = 0 \),
(34)

stating again that the Higgs mass is twice of the quark mass in the bubble approximation.

Thus our WT identity (10) is fully consistent with the result from the effective potential approach in the low energy domain [12-14].
Discussion

Throughout the above calculation we have assumed the cutoff momentum $M$ to be large such that $O(1/M^2)$ is small. The higher dimensional interaction terms which have been discussed vigorously in [19] and [20] in particular, appear in the $O(1/M^2)$ in our treatment. Evidently these terms have to be examined also, if a relatively small cutoff momentum is chosen.
We have seen that our identity leads to the same result as the effective potential approach [11-16] in the low energy region. This situation is quite different for events with large momentum transfer. In our treatment, the Higgs is not inserted as an effective field $\phi$. Instead the Higgs's propagator appears in the vertex function in a natural manner, and is expected to behave quite differently from the result obtained from the effective field $\phi$ for very high energy events.

It has been known that the effective potential approach predicts results precisely and rapidly for the low energy phenomena. On the contrary, we have known for some time that it is very dangerous to apply the effective theorems for high energy processes [11-21]. In future experiments beyond the LEP energy the effective potential approach will not always be reliable.

Our approach can handle such high energy problems since our identity is an exact relation. In this connection, it is interesting to compare, for example the improved gap equation (2,74) of [22] obtained from the refined effective potential, to our identity. The improvement made in their eq.(2,74) corresponds to taking into account further higher terms $O(x, x, 1/M^2)$ in our (B4). However our $O(x, x, 1/M^2)$ contains terms not appearing in the effective potential. It will be still more interesting to examine the auxiliary field propagator of sect. 4 of ref.[22] by inspection of our non-perturbative formula and try to see how our identity predicts differently from their refined effective potential at very high energy.

Also, in comparison with the orthodox Feynman-Dyson perturbation calculations, our treatment can produce the results more quickly by saving an immense amount of labor. Still, further ingenuity will be needed to solve the identity as the energy goes up higher.
Appendix A

Expressing $\delta(s - s')$ as

$$\delta(s - s') = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk e^{i(s - s')},$$

one can rewrite the solution (14) as

$$B(y, s' : x, s) = \frac{1}{2\pi} \int d\alpha e^{i(H - \alpha)} \delta(x - y) e^{i(s - s')}$$

$$= \int d\alpha \delta(\alpha + s - s') e^{iH} \delta(x - y)$$

$$= e^{-iH(s - s')} \delta(x - y)$$

$$= \frac{1}{(2\pi)^3} \int d^3k e^{-iky} e^{-iH(s - s')} e^{ikx}$$

which is the expression (15).
Appendix B

First we rewrite $Tr B(x,y)$ (18)

$$
\int d^4k e^{-i2kx}e^{-\Phi^+/M^2}e^{ik(x-y)} = \int d^4k e^{i(k(x-y))}e^{-\Phi^+(x,k)\Phi(x,k)/M^2}
$$

(B1)

where $\Phi(x,k)$ is defined as $\Phi(x,k)f(\bar{\psi},\psi) = \delta^{ikx}f(x,k)M^2$, $f(\bar{\psi},\psi)$ being a test functional. There are two choices for the expression of the derivative operator $\partial$ in $\Phi(x,k)$ and $\Phi^+(x,k)$. If one adopts $\partial = \partial/\partial x$, we will get involved with a complicated calculation. Alternatively we adopt the expression $(\partial\psi/\partial x)\delta/\delta\psi + (\partial\bar{\psi}/\partial x)\delta/\delta\bar{\psi}$ for $\partial$ in the present paper. The $\Phi^+(x,k)\Phi(x,k)$ commutes with $ik(x-y)$, and we get the result rapidly. As a matter of fact, both choices should lead to the same result.

Then the (B1), that is,

$$
e^{-\Phi^+(x,k)\Phi(x,k)/M^2}e^{i3k(x-y)}
$$

(B2)

can be rewritten in terms of the translated momentum $k' = k + i(x-y)M^2/2$ as

$$
\exp -\{(x-y)^2M^2/4\} \exp \{ -k^2 + M^2(x-y)\beta + 2ik\beta \}
+ \beta^2 + \beta(2g)(S + \gamma S) - (2g)(S + \gamma S)\beta - (2g)^2(S^2 - P^2)/M^2
$$

(B3)

The presence of the terms $(x-y)\beta$ in the last expression implies the translational invariance of the result.

Because of the factor $\exp -\{(x-y)^2M^2/4\}$ the (B3) is small except when $x \approx y$ and can be expressed as

$$
e^{-\{x-y\}^2M^2/4}e^{-\{x-y\}\beta} \left( M^4 + A(x,x) + O(x,x,1/M^2) + O(x-y) \right),
$$

(B4)

after the integration over $k'$ from 0 to $M$.

Among the terms of the power series in $1/M^2$, $A(x,x)$ represents terms independent of $M^2$, and the $O(x,x,1/M^2)$ indicate the rest.

The $O(x-y)$ is a power series in $(x-y)$ and arises as a result of the factorization of $\exp -\{(x-y)\beta\}$. The final result for $A(x,x)$ is in the Minkowski space

$$
A(x,x) = -\frac{1}{\pi^2} \{ M^2g^2(S^2 - P^2) 
+ \frac{1}{2}g^2(\partial_{\mu}S)(\partial_{\mu}S) - 2g^4S^4 
- \frac{1}{2}g^2(\partial_{\mu}P)(\partial_{\mu}P) - 2g^4P^4 
+ 4g^4S^2P^2 \}
$$

(B5)

with $S = \bar{\psi}(x)\psi(x)$ and $P = \bar{\psi}(x)\gamma_5\psi(x)$, up to total derivatives.

The first term proportional to $M^2$ in (B4) is independent of the interaction, and has to be normalized off. The rest of (B4) approaches smoothly $A(x,x)$ as $y$ approaches $x$. 

11
Appendix C

There are several ways to obtain $^4G^1$ in terms of the Legendre transform $\Gamma$. We have started from the relation

$$\frac{\delta \tilde{G}^H(y')}{\delta \tilde{G}^E(x')} = \delta \Omega, \delta(x' - y') = \int \frac{\delta \tilde{G}^E(x'')}{\delta \tilde{G}^E(x')} \delta \tilde{G}^H(y') dx''$$

Applying $\delta / \delta \eta^E(z')$ and $\delta / \delta \tilde{G}^J(t')$, we arrive at the following relation

$$- \int S_{\psi \psi}(z'' z') S_{\tilde{G} \tilde{G}}(t'' t') \frac{\delta^3 \tilde{G}}{\delta \tilde{G} J(t') \delta \eta^E(z') \delta \psi E(x'')} dx'' dz'' dt''$$

$$= \int \Gamma_{\psi \psi}^2(x' x'') \frac{\delta^3 \tilde{G}}{\delta \tilde{G}^J(t') \delta \eta^E(z') \delta \psi E(x'')} \delta \psi E(x'') \delta \psi E(x'') dx''$$

from which we obtain $^4G^1$ as

$$\frac{\delta^3 \tilde{G}}{\delta \tilde{G}^J(t') \delta \eta^E(z') \delta \psi E(x')} = i \int S_{\tilde{G} \psi}(t'') S_{\tilde{G} \psi}(y)$$

$$\Gamma_{\psi \psi}^2(x' x') = \delta \psi^E(x') \psi^E(x)$$

is the inverse of the propagator. In the lowest order, the vertex function is simply

$$^4\Gamma_{\psi \psi \psi}(t' y' y'') = \delta \psi^E(x') \delta \psi^E(x)$$

$$= 2g(\delta_{\psi \phi} - \delta_{\psi \psi} \delta_{\phi \phi}) \delta(x' - t') \delta(x' - y')$$

$^4G^5$ can be obtained in a similar manner.
Appendix D

We choose the sum of momenta of two incoming particles, say $p_1 + p_2 = q$, as an independent variable, because $q$ represents the momentum of the bound states, that is, the Higgs and $\tau$ (we ignore vector and pseudo vector bound states). Let us denote the momenta of four $S_{ab}$ in $^4G^1$ as $p_1, p_2, p_3$ and $p_4$ successively. Expressing $p_1$ as $p_1 = q - p_2$, and integrating over $p_2$, from $\mu$ to $\Lambda$ we obtain

\[
^4G^1 = i4(2\pi)^8 \int \left\{ \frac{1}{8} + \frac{\alpha^2 g \log \frac{\Lambda^2}{\mu}}{(4\pi)^2} (4m^2 - q^2) \right\} \\
\left( Tr \frac{1}{p_3 - m} \frac{1}{p_4 - m} \right) dp_3 dp_4 dq
\]

The first term of $^4G^1$ is equal to $S_{ac}S_{ca}$. Here the gap equation has already been used for this term in order to eliminate $g$. (We did not employ the integration used by BHL who take into account the scaling aspect.)

The last factor of $^4G^1$, namely the product of the last two $S_{ab}$, can be expanded in terms of $q^2 - 4m^2$. The result for the second term of $^4G^1$ is

\[
\int (4m^2 - q^2) Tr \frac{1}{p_3 - m} \frac{1}{p_4 - m} dp_3 dp_4 \\
= \int dr^2 \left\{ 1 - \frac{r^2}{q^2 - 4m^2} + \frac{8m^2}{(q^2 - 4m^2)^2} \right\} \\
\left( 1 + \frac{2(qr) + r^2}{q^2 - 4m^2} \right) \left( 1 + \frac{-2(qr) + m^2}{q^2 - 4m^2} \right) \\
\int dr^2 \left\{ 1 - \frac{3r^2}{q^2 - 4m^2} + O \left( \frac{1}{q^2 - 4m^2} \right) \right\}
\]

In the same way $^4G^5$ is

\[
^4G^5 = i4(2\pi)^8 \int \left\{ \frac{1}{8} - \frac{\alpha^2 g \log \frac{\Lambda^2}{\mu}}{(4\pi)^2} q^2 \right\} \\
\left( Tr \gamma_5 \frac{1}{(p_3 - m)} \gamma_5 \frac{1}{(p_4 - m)} \right) dp_3 dp_4 dq
\]

The first term is equal to $S_{ac}^5 S_{ca}^5$. For the second term we have

\[
\int q^2 Tr \left( \gamma_5 \frac{1}{(p_3 - m)} \gamma_5 \frac{1}{(p_4 - m)} \right) dp_3 dp_4 \\
= - \int dr^2 \left\{ 1 - \frac{3r^2}{q^2} + O \left( \frac{1}{q^2} \right) \right\}
\]

REFERENCES


[16] The point of view on the symmetry breaking resulting from this transformation has been presented by us p.359 of the Bregenz Symposium “Symmetry in Science VI” 1992, ed. B. Gruber, Plenium Press, New York 1993. Here only the mass generation of quarks is treated in the ladder approximation. All terms which do not contribute to the quark mass are ignored. The Bethe-Salpeter amplitudes for bound states $\phi$ and $\pi$ (our $D_0$ and $D_4$) are neglected too.


[21] As one of the later examples, we cite the $Z \rightarrow 2\gamma$ decay. See T. HATZUDA, M. UMEZAWA:  
*Phys. Lett.* **B254** (1991) 493. All other references are cited therein.

T. HATZUDA, M. UMEZAWA: The Decay $Z \rightarrow \sigma\gamma$, CRN Report (unpublished 1990); The $Z \rightarrow \sigma\mu^+\mu^-$ as a test of the usage of PCDC in high energy phenomena, CRN Report (unpublished 1991); The Decay of $Z$ to a Dialton and a $\mu$ pair: PCDC and the Decay Form Factor, CRN Report (unpublished 1992).