SOLA INVERSIONS FOR THE RADIAL STRUCTURE OF THE SUN

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A SELF-CONSISTENT APPROACH TO FILTERING OUT NEAR-SURFACE UNCERTAINTIES FROM HELIOSEISMIC INVERSIONS

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INTRODUCTION

The inverse problem of finding the structure of the solar interior can be written as

\[ \frac{\delta \omega_i}{\omega_i} = \int \left[ K_i^{(1)}(r) \delta f_1(r) + K_i^{(2)}(r) \delta f_2(r) \right] \, dr + F_{\text{surf}}(\omega_i) S_i, \]

where \( \delta \omega_i \) is the difference in the frequency \( \omega_i \) of the \( i \)th mode between the solar data and a reference model, \( f_1 \) and \( f_2 \) are an appropriate pair of model parameters, and \( S_i \) (cf. Christensen-Dalsgaard 1991) is proportional to the inertia of the mode. The \( F_{\text{surf}} \) is an unknown function of frequency resulting from the near-surface errors in the physics of the reference model and frequency calculation. Our aim is to filter out this term, which is assumed to be a slowly varying function of frequency. In the process, however, a contribution to \( S_i \delta \omega_i / \omega_i \) from \( \delta f_1 \) and \( \delta f_2 \) near the surface is also filtered out. Thus the corresponding contributions from the kernels \( K_i^{(r)} \) should be suppressed, for consistency.

FORMULATION

Guided by asymptotics, we make a two-spline fit to the scaled frequency differences:

\[ S_i \frac{\delta \omega_i}{\omega_i} \approx \mathcal{H}_1 \left( \frac{\omega_i}{L} \right) + \mathcal{H}_2(\omega_i) \]

(cf. Christensen-Dalsgaard et al. 1989). Here \( L = l + 1/2 \), where \( l \) is the degree of mode \( i \). The functions \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \) are splines; in particular,

\[ \mathcal{H}_2(\omega) = \sum_k d_k \phi_k(\omega), \]

where the \( \phi_k \) are the expansion functions for the spline, and the coefficients \( d_k \) are linearly related to the data through the coefficients \( D_k \):

\[ d_k = \sum_i D_{ki} S_i \frac{\delta \omega_i}{\omega_i}. \]

Note that \( \mathcal{H}_2(\omega) \) corresponds asymptotically to the function that we wish to filter out.

Our aim is to remove from the data a slowly-varying function of frequency. We achieve this by first using a low-pass filter that will isolate in the scaled data the slowly-varying contribution from \( F_{\text{surf}} \) and also near-surface contributions from the functions \( \delta f_1 \) and \( \delta f_2 \). This can then be subtracted from the original scaled data. To this end we introduce a suitably chosen mesh \( \{ \omega_\alpha \} \) and then construct a matrix \( F_{\alpha \beta} \) that has the effect of a low-pass filter (cf. Pérez Hernández & Christensen-Dalsgaard 1994). The result of filtering \( \mathcal{H}_2 \) can be represented at the points in the mesh as

\[ \mathcal{H}_2^\prime(\omega_\alpha) = \sum_\beta F_{\alpha\beta} \mathcal{H}_2(\omega_\beta). \]

Since we need to evaluate the effect of the filtering at all the frequency points \( \omega_i \), we introduce a new spline fit, to \( \mathcal{H}_2^\prime \), with spline basis \( \tilde{\phi}_i \):

\[ \mathcal{H}_2^\prime(\omega) = \sum_i \tilde{d}_i \tilde{\phi}_i(\omega), \quad \text{where} \quad \tilde{d}_i = \sum_\alpha \tilde{D}_{\alpha i} \mathcal{H}_2^\prime(\omega_\alpha). \]

Thus, finally, the effect of applying the fit and filtering can be written as

\[ \mathcal{H}_2^\prime(\omega_i) = \sum_j F_{ij} S_j \frac{\delta \omega_j}{\omega_j}, \]

where

\[ F_{ij} = \sum_{\alpha, \beta, k} \tilde{\phi}_i(\omega_\alpha) \tilde{D}_{\alpha i} F_{\alpha \beta} D_{kj} \phi_k(\omega_\beta). \]

By construction, the low-pass filter has essentially no effect on \( F_{\text{surf}} \), provided this is indeed slowly varying as we assume. Hence, applying the filtering to Eq. (1) gives

\[ \sum_j F_{ij} \delta \omega_j = \int \sum_j F_{ij} S_j \left[ K_j^{(1)}(r) \delta f_1(r) + K_j^{(2)}(r) \delta f_2(r) \right] \, dr + F_{\text{surf}}(\omega_i). \]

By subtracting Eq. (9) from Eq. (1) we finally obtain

\[ \sum_j (\delta f_j - F_{ij}) S_j \delta \omega_j = \int \left[ \hat{K}_j^{(1)}(r) \delta f_1(r) + \hat{K}_j^{(2)}(r) \delta f_2(r) \right] \, dr, \]

where

\[ \hat{K}_j^{(\alpha)} = \sum_j (\delta f_j - F_{ij}) S_j K_j^{(\alpha)}. \]

Hence the new filtered data on the left of Eq. (10) are related to the unknown functions \( \delta f_1, \delta f_2 \) by new kernels \( \hat{K}_j^{(1)}, \hat{K}_j^{(2)} \). Finally we note that the filtering defined by Eqs. (2) - (5) can be based on just a subset of modes. In that case the index \( j \) in \( F_{ij} \) runs over just those modes in the subset.
RESULTS

To illustrate the effectiveness of the filtering technique, we have calculated mode kernels for different pairs of functions, using a "standard" solar model. The details of the construction of the filter $F_{\alpha\beta}$ were given by Pérez Hernández & Christensen-Dalsgaard (1994). The filter was constructed to suppress all signals arising from the near-surface region with a radius of $r \geq 0.995 R_\odot$.

In order to calculate the coefficients $F_{ij}$, we use all modes in the modeshet in Libbrecht et al. (1990) that have degrees $\ell \leq 100$ and lower turning points located between the base of the convection zone and $r = 0.995 R_\odot$. For these modes the effects of near-surface errors are essentially independent of $l$ (assuming that the effect of departures from spherical symmetry are negligible) so that Eq. (2) is a reasonable approximation. We have taken $\phi_1$ and $\phi_2$ to be identical, namely a B-spline basis on 25 equally-spaced knots. To illustrate the effectiveness of filtering, we have used kernels for $f_1 = c^2$ and $f_2 = \rho$, where $c$ is the adiabatic sound speed and $\rho$ is the density.

Fig. 1 shows the amplitude of kernels for $c^2$ at constant $\rho$ at a fixed radial location $r$, plotted as a function of frequency. For the original mode kernels this shows an oscillatory pattern, which becomes more slowly-varying with frequency as $r$ is chosen closer to the surface. The filtered kernels have smaller amplitude at these radii, showing that filtering has suppressed the sensitivity to near-surface structural changes. Suppression is almost total for $r > 0.995 R_\odot$, as was intended. Fig. 2 shows kernels for sound speed at constant density, and for density at constant sound speed. In this case we show individual kernels before and after filtering. They show once again that the effect of filtering is to suppress the kernels in the near-surface region.

CONCLUSION

We have shown here that by applying suitable filters to the frequency differences and the mode kernels it is possible to pre-process the frequency differences prior to applying any of the standard inversion techniques in order to eliminate the frequency-dependent component which arises from the near surface uncertainties. In the process we also suppress the corresponding contributions from the kernels relating frequency and structural differences. We are currently applying this technique to the full inversion problem, and will report those results elsewhere. Preliminary results from applying a least squares inversion to artificial data indicate that filtering gives more accurate results, particularly in $r \geq 0.9 R_\odot$, than including $F_{\text{surf}}$ in the fit (as in, for example, Antia & Basu 1994).

REFERENCES