The relation between pole and running masses for gluino and squarks

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Abstract

We calculate the relation between the pole and running masses for gluino and squarks in supersymmetric QCD with soft supersymmetry breaking in one loop approximation. We find that the difference between running and pole masses could be up to 15 %.

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[†]In Honour of Prof.Dr. Rudolf Rodenberg 65th Birthday

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1 Introduction

At present the supersymmetric extensions of the standard Weinberg-Salam model are the single realistic explanations of the gauge hierarchy problem [1]. The soft supersymmetry breaking terms in such theories come from an account of the hidden sector in corresponding supergravity extensions of the supersymmetric models. According to standard lore [2] (see, however [3]) such soft supersymmetry breaking terms are given at the Planck scale and to relate the high energy soft supersymmetry breaking terms with soft supersymmetry breaking terms at the scale $M_{SUSY} \sim 1TeV$ renormalization group equations for running masses are used [4]. So the knowledge of the concrete model for the hidden sector allows us to find the running masses of the SUSY particles at the scale of the supersymmetry breaking. However in the experiment the pole masses are measured. Therefore it is necessary to calculate the relation between pole and running masses.

In QCD the relation between pole and running quark masses in one loop approximation in the $\overline{MS}$ -scheme has the form [5]

$$m_p = (1 + \frac{4\alpha_s}{3\pi})m(m) \quad (1)$$

Recently the relation between pole and running masses has been found in two-loop approximation [6].

In this paper we calculate the relation between pole and running masses for gluino and squarks in supersymmetric QCD with soft supersymmetry breaking in one loop approximation. We find that the difference between running and pole masses could be up to 15 %.

The running mass for fermions usually is determined in the $\overline{MS}$ -scheme, where the relation between bare and renormalized masses is [7]

$$m_B = Z_m(n, g)m \quad (2)$$

,where

$$Z_m(n, g) = \sum_{k=0}^{k=\infty} \frac{c_n(g)}{(n-4)^k} \quad (3)$$

Analogous relation takes place for boson masses. Within the context of dimensional regularization such scheme corresponds to the subtraction of the pole singularities $(n-4)^{-k}$ for the propagators and vertexes. The following formulae [8] are used in our calculations:

$$\int d^n p \frac{1}{m^2 - p^2 - i\epsilon} = -\Delta m^2 - i\pi^2 m^2 + i\pi^2 \ln\left(\frac{m^2 - i\epsilon}{\mu^2}\right) + O(n-4) , \quad (4)$$

$$\begin{aligned} & \int d^n p \frac{1}{(m_1^2 - p^2 - i\epsilon)(m_2^2 - (p+k)^2 - i\epsilon)} = \\ & \Delta - i\pi^2 \int_0^1 dx \ln\left(\frac{k^2 x^2 + x(m_2^2 - m_1^2 - k^2) + m_1^2}{\mu^2}\right) + O(n-4) \end{aligned} \quad (5)$$

with $\Delta = -\frac{2i\pi^2}{n-4} + i\delta$, $\delta = \pi^2(\gamma - \ln\pi)$, $\gamma = \text{Euler's constant}$. The $\overline{MS}$ -scheme corresponds to the subtraction of the δ from the expressions (4) and (5). Consider for simplicity the scalar field with the cubic selfinteraction

$$L = \frac{A\phi^3}{6} \quad (6)$$

One loop correction to the scalar propagator

$$D(p^2) = \frac{i}{(p^2 - m^2 - \Sigma(p^2))} \quad (7)$$

in the $\overline{MS}$ -scheme is

$$\Sigma(p^2) = \frac{A^2}{32\pi^2} \left(\int_0^1 dx \ln\left(\frac{p^2 x^2 - p^2 x + m^2}{\mu^2}\right) \right) , \quad (8)$$

where μ is the renormalization point. The pole mass in one loop approximation is

$$m_p^2 = m^2(m) + \Sigma(m^2) = m^2 - 0.186 \frac{A^2}{16\pi^2} , \quad (9)$$

where $m(m) = m$ is the running mass at the point $\mu = m$. The fermion propagator is represented in the form

$$D_F(p) = \frac{i}{p_\mu \gamma^\mu (1 - L\Delta Z_L(p^2) - R\Delta Z_R(p^2)) - m - \Sigma_F(p^2)} , \quad (10)$$

where $L(R) = \frac{1 \pm \gamma_5}{2}$. The fermion pole mass in one loop approximation is determined by the formula

$$m_p = m + \Sigma_F(m^2) + \frac{m(\Delta Z_L(m^2) + \Delta Z_R(m^2))}{2} \quad (11)$$

2 Gluino mass

Corrections to the gluino mass are determined by the diagrams depicted on figs.(1a, 1b). In the $\overline{MS}$ -scheme neglecting the masses of all quarks inside of the loop we find that

$$m_{g,p} = m_g(m_g) \left[1 + \frac{3\alpha_s}{\pi} + \left(\frac{\alpha_s}{4\pi} \right) \sum_{sq} \int_0^1 dx x \ln[x^2 + x(\frac{m_{sq}^2}{m_g^2} - 1)] \right] \quad (12)$$

Here m_g is the gluino mass and m_{sq} is the mass of the corresponding superquark. In the $\overline{DR}$ scheme [9] instead of the term $\frac{3\alpha_s}{\pi}$ in formula (12) we have to substitute $\frac{5}{4} \frac{3\alpha_s}{\pi}$. For the case of t-quark we can't neglect its mass and we have to make the replacement in the formula (12)

$$\ln[x^2 + x(\frac{m_{st}^2}{m_g^2} - 1)] \rightarrow \ln[x^2 + x(\frac{m_{st}^2}{m_g^2} - \frac{m_t^2}{m_g^2}) + \frac{m_t^2}{m_g^2}] \quad (13)$$

To estimate the difference between running and pole masses consider the case when all squark masses are degenerate and have the mass m_{sq} . For the gluino mass the relation between pole and running masses in the $\overline{MS}$ scheme is

$$m_{g,p} = m_g(m_g) \left(1 + \frac{k\alpha_s}{\pi} \right), \quad (14)$$

where in the $\overline{MS}$ -scheme $k = 4.2; 3.71; 3; 1.5; 0$ for $m_{sq}^2 = k_1 m_g^2$ ($k_1 = 4; 3; 2; 1; 0$) respectively. For the $\overline{DR}$ -scheme [9] we find that $k = 4.95; 4.46; 3.75; 2.25; 0.75$ correspondingly. For $\alpha_s = 0.1$ and for $m_{sq} = 2m_g$ we find that in the $\overline{DR}$ -scheme $m_{g,p} = 1.16m_g(m_g)$. So the correction to the gluino pole mass is not small. In formulae (12) we did not took into account the mixing of $\tilde{t}_L$ and $\tilde{t}_L^c$ squarks due to trilinear soft supersymmetry breaking term

$$L_I = A(\tilde{t}, \tilde{b})_L H_1 \tilde{t}_L^c \quad (15)$$

In general case the eigenvalues of the mass matrix for $\tilde{t}_L, \tilde{t}_L^c$ superquarks are

$$\tilde{t}'_L = \tilde{t}_L \cos(\phi) + \tilde{t}_L^c \sin(\phi) , \quad (16)$$

$$\tilde{t}_L = \tilde{t}_L^c \cos(\phi) - \tilde{t}_L \sin(\phi) \quad (17)$$

with the masses M_1 and M_2 . The modification of the formula (12) due to the mixing of $\tilde{t}_L, \tilde{t}_L^c$ superquarks is minimal, namely we have to perform the substitution (13) into the formula (12) with $m_{st_L} = M_1$ and $m_{st_L^c} = M_2$. Besides due to the mixing of $\tilde{t}_L$ and $\tilde{t}_L^c$ squarks we also have additional ultraviolet finite correction to the gluino mass

$$\delta m_t = \frac{\alpha_s}{4\pi} \sin 2\phi m_t \left[\int_0^1 dx \ln \left(\frac{m_g^2 x^2 + x(M_1^2 - m_t^2 - m_g^2) + m_t^2}{m_g^2 x^2 + x(M_2^2 - m_t^2 - m_g^2) + m_t^2} \right) \right] \quad (18)$$

Numerically we find that the typical values are $\delta m_g \sim 0.01 \sin(2\phi) m_t$.

3 Squark mass

The correction to the squark mass is determined by the diagrams drawn on fig. 2a - 2c.

In our calculations we neglect the masses of quarks. The result of the calculation is

$$m_{sq,p}^2 = m_{sq}^2(m_{sq}) \left(1 + \frac{2\alpha_s}{\pi} \right) - \frac{\alpha_s}{6\pi} K , \quad (19)$$

where

$$K = -2m_g^2 + (m_g^2 - m_{sq}^2) \left(2 + 4 \int_0^1 dx \ln \left[x^2 - x \left(1 + \frac{m_g^2}{m_{sq}^2} \right) + 1 \right] \right) \quad (20)$$

in the $\overline{MS}$ -scheme. In the $\overline{DR}$ -scheme we have to add $-2m_g^2$ to the value K (20) calculated in the $\overline{MS}$ -scheme. For the case $m_g = m_{sq}$ in $\overline{MS}$ -scheme we find that

$$m_{sq,p}^2 = \left(1 + \frac{7\alpha_s}{3\pi} \right) m_{sq}^2(m_{sq}) \quad (21)$$

Note that $m_{sq}(m_{sq}) = m_{sq}$ and $m_g(m_{sq}) = m_g$ in the formulae (20,21). For the $\tilde{t}_L$ and $\tilde{t}_L^c$ squarks the corrections due to trilinear soft supersymmetry breaking interaction (15) are

not small in general. Neglecting the mass of the Higgs boson for the equal mass of $\tilde{t}_L$ and $\tilde{t}_L^c$ squarks we find that the correction to the pole mass for $\tilde{t}$ and $\tilde{b}$ superquarks is

$$\delta m_h^2 = -k \frac{A^2 h_t^2}{8\pi^2}, \quad (22)$$

where $k = 1$ for $\tilde{t}_L$ and $\tilde{b}_L$ squarks and $k = 2$ for $\tilde{t}_L^c$ squarks.

4 Conclusion

In this paper we have calculated the relation between pole and running masses for gluino and squarks in supersymmetric QCD with soft supersymmetry breaking. The difference between pole and running masses for gluino and squarks is not small and could be up to 15 % .

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Figure Captions

Figs.1a,1b. One loop corrections to gluino propagator.

Figs.2a - 2c. One loop corrections to the squark propagator.