TRANSVERSE MODE-COUPLING INSTABILITY
IN THE PRESENCE OF DETUNING IMPEDANCE

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In non-axisymmetric beam pipes, a detuning impedance is present in addition to the usual driving impedance. The impact of this additional impedance contribution on the Transverse Mode-Coupling Instability (TMCI) has been re-examined in detail, comparing four different approaches: (i) the two-particle model solved analytically; (ii) the circulant-matrix formalism with two (or many) azimuthal modes but only one radial mode solved semi-analytically; (iii) the circulant-matrix formalism with many azimuthal and radial modes solved numerically and (iv) past HEADTAIL macroparticle tracking simulations. The results of the several benchmarks performed are discussed and the possible next steps are outlined.

INTRODUCTION

In axially asymmetric structures, in addition to the driving (or dipolar) impedance (coming from the linear force proportional to the transverse displacement of the source particle), a detuning (or quadrupolar) impedance (coming from the linear force proportional to the transverse displacement of the tail/test particle) also exists [1]. This effect is purely geometric and therefore also exists when the relativistic velocity factor \( \beta \approx 1 \). It is worth noting that another detuning impedance is coming from the non-relativistic velocity factor \( \beta < 1 \) even for axially symmetric structures [2], but it will not be discussed in this paper. In Ref. [1], the effect of the detuning impedance on the TMCI has been discussed revealing an increase of the intensity threshold. However, most of the asymmetric cases are close to the flat-chamber case (with the vertical half-gap (much) smaller than the horizontal one) and in this case the vertical instability threshold is very close to the round case, meaning that only a marginal gain is expected. Nevertheless, as the horizontal intensity threshold is predicted to be (much) higher (see for instance Fig. 1 where it can be seen that the horizontal intensity threshold is a factor \( \sim 2 \) higher compared to the round chamber case, which is very close to the vertical threshold), the proposal of using linear coupling between the two transverse planes was made. As discussed in Ref. [3], a gain of \( \sim 30\% \) was initially predicted with a simple analytical approach, which was then confirmed by HEADTAIL macro-particle tracking simulations (see also Ref. [4] where these results have been checked).

In Ref. [1], it was mentioned that the betatron tune spread introduced by the detuning impedance (which varies along the longitudinal axis) should be beneficial. However, recent pyHEADTAIL simulations [5] revealed that the detuning impedance can have a destabilising effect as the intensity threshold was found to be in some cases lower in the horizontal plane than in the vertical one. A destabilising effect has been also observed with some other pyHEADTAIL simulations [6]. This is why we decided to review in detail all the instability mechanisms with detuning impedance, starting from the coasting-beam theory, where a new instability mechanism, involving the coupling between the fast and slow waves, has been described [7]. The purpose of this paper is to review in detail the different aspects related to the TMCI (without chromaticity), before studying in more detail in the future the impact of chromaticity.

To try and fully understand the past HEADTAIL macroparticle tracking simulations of Fig. 1, the paper is structured as follows. In Section 1, the simplest two-particle model [8] (with only the 2 azimuthal modes 0 and -1) is extended to include the effect of the detuning impedance (corresponding to a constant wake). Then, the more involved circulant-matrix formalism [1] with different azimuthal modes but only one radial mode is discussed in Section 2, comparing theory to tracking simulation using the BimBim code, which is an implementation of the circulant-matrix model in the Python language [9]. Finally, the results from the full BimBim tracking simulation with the circulant-matrix formalism is presented in Section 3 to analyse in detail the effect of the radial modes, before concluding on this study.

Figure 1: HEADTAIL macroparticle tracking simulations of the horizontal and vertical mode-frequency shifts and instability growth rates vs. bunch intensity for the case of
a CERN SPS bunch interacting with the broad-band impedance of a flat chamber [4]. Courtesy of B. Salvant.

TWO-PARTICLE MODEL

All the details of the computation can be found in Ref. [10], where the famous two-particle model discussed in Ref. [8] is extended to include the effect of a (constant) detuning wake. The final results are shown in Fig. 2, where it can be seen that the quadrupolar wake is always beneficial whatever its value or sign with respect to the dipolar wake. The reason for the stabilisation is that the two modes are pushed in opposite directions by the quadrupolar wake, with the net effect of coupling for higher intensities and eventually decoupling. These results are consistent with those of Ref. [1]. The possible next steps would consist in using a more realistic shape of the wake (linear, oscillating, resistive-wall type) and including nonzero chromaticity.

CIRCULANT-MATRIX FORMALISM WITH ONLY ONE RADIAL MODE

Extending Ref. [1], using the “air-bag” model with a constant wake (given below as the constant term of a resonator wake, which will be used after) and considering first only 2 modes (0 and -1), the system is fully described by the following matrix to be diagonalised [11]

\[
\begin{pmatrix}
-1 + \frac{\kappa}{2} & \frac{2 I_{\text{norm}}}{\pi^2} (1 - \kappa) \\
\frac{2 I_{\text{norm}}}{\pi^2} (-1 - \kappa) & \frac{2 I_{\text{norm}}}{\pi^2} (-1 + \kappa)
\end{pmatrix}
\]

(1)

with

\[
I_{\text{norm}} = \frac{N e^2}{2\gamma m_0 \omega_B \omega_s C} \times \frac{\omega_B R_t}{\omega_s} \quad \text{and} \quad \kappa = \frac{D(z)}{W(z)},
\]

(2)

where \(N\) is the number of protons in the bunch, \(e\) the elementary charge, \(\gamma\) the relativistic mass factor, \(m_0\) the proton rest mass, \(\omega_B\) the transverse betatron angular frequency, \(\omega_s\) the synchrotron angular frequency, \(C\) the machine circumference, \(\omega_t\) the resonance angular frequency

(of the broad-band resonator impedance), \(R_t\) the transverse shunt impedance (of the broad-band resonator impedance), \(Q\) the quality factor (of the broad-band resonator impedance) and \(\omega_r = \omega_t \sqrt{1 - 1/(4Q^2)}\). The strength of the detuning (quadrupolar) impedance \(D(z)\) with respect to the driving (dipolar) horizontal one \(W(z)\) is characterised by the factor \(\kappa\), with \(z\) the longitudinal coordinate along the accelerator. Similar results as with the two-particle model are obtained (see Fig. 3). Note the different convention compared to the one used in Section 1, with for instance \(\kappa = +1\) for the case of the classical thick-wall resistive wall impedance in horizontal and \(\kappa = -1/2\) in vertical.

Figure 2: Summary of the results obtained by extending the two-particle model, discussed in the past with dipolar (D) wake only, to include also the quadrupolar (Q) wake.

Figure 3: Similar result as the one obtained with the two-particle model (see Fig. 2) but using the simplified circulant-matrix formalism discussed above.

Considering the case of a broad-band resonator impedance \((Q = 1)\), still with only one radial mode but with many azimuthal modes, an excellent agreement has also been obtained between theory and the tracking code BimBim, as can be seen in Figs. 4-7 [12].

Figure 4: Comparison between theory (left) and the BimBim code (right) for the case of a broad-band resonator impedance vs. the product between the resonance frequency and the full (4-sigma) bunch length.
One should be careful when comparing the different $\kappa$-cases, as for each case $I_{\text{norm}}$ is normalised by the dipolar impedance (which includes a Yokoya dipolar factor [13]): 1 for round ($\kappa = 0$), $\pi^2/24$ for flat $x$ ($\kappa = 1$) and $\pi^2/12$ for flat $y$ ($\kappa = -1/2$). Applying this to the case $f_r\tau_b = 2.8$ (see Fig. 7), the intensity threshold is higher in $x$ by the factor $\sim 2.1$ (compared to $\sim 2$ in Fig. 1 from the HEADTAIL code) and higher in $y$ by the factor $\sim 1.3$ (compared to $\sim 1$ in Fig. 1). The effect of the asymmetry (flat chamber) on the TMCI intensity threshold (for this SPS case) can be therefore explained mainly by the Yokoya dipolar factor, as already discussed in Ref. [4].

Figure 5: Comparison between theory (left) and the BimBim code (right) for the case of a broad-band resonator impedance with $f_r\tau_b = 1.6$: (a) $\kappa = 0$; (b) $\kappa = -1$; (c) $\kappa = +1$.

Figure 6: Comparison between theory (left) and the BimBim code (right) for the case of a broad-band resonator impedance with $f_r\tau_b = 2.8$: (a) $\kappa = 0$; (b) $\kappa = -1$; (c) $\kappa = +1$.

Figure 7: Comparison between the horizontal ($\kappa = +1$) and vertical ($\kappa = -1/2$) planes for the case of a broad-band resonator impedance with $f_r\tau_b = 2.8$ (similar to Fig. 1).

CIRCULANT-MATRIX FORMALISM WITH MANY RADIAL MODES

A detailed analysis of the effect of the radial modes has been performed in Ref. [14]. Depending on the configuration, radial modes may affect the TMCI in either direction. The modes involved in the dominant instability mechanism are usually different and a solid benchmark would require simulations in various configurations. A TMCI between radial modes with the same azimuthal mode number is observed with short bunches and $\kappa = 1$, but the convergence couldn't be reached and this should be further studied in the future.

Figure 8: Results from the BimBim code for the case of Fig. 1: (left) horizontal and (right) vertical plane vs. (normalised) intensity. The results with many radial modes (a) are compared to the case with only one radial mode (b).
Comparing the results from the BimBim code to the ones obtained in the past with HEADTAIL (see Fig. 1), very similar results have been obtained, as can be seen in Fig. 8 (a). In Fig. 8, the results with many radial modes are also compared to the case with only one radial mode and it is interesting to observe that, in both cases, similar intensity thresholds are obtained but the modes which couple change.

CONCLUSION

The effect of the asymmetry (flat chamber) on the TMCI intensity threshold (for the CERN SPS case of Fig. 1) can be explained mainly by the Yokoya driving factor, as already discussed in Ref. [4]. The detuning impedance is found to be always beneficial for the TMCI with all the different approaches discussed in this paper. The next step should consist in including the chromaticity in the picture (the BimBim code is already all set to perform this analysis) to see if (and how) in some cases the detuning impedance can become detrimental in the horizontal plane as observed in some tracking simulations with pyHEADTAIL [5]. For the moment, a destabilising effect of the detuning impedance could only be explained for coasting beams through the coupling between the slow and the fast waves [7].

REFERENCES


