FUTS AND THE HIGGS-BOSON

SVEN HEINEMEYER

*Instituto de Física de Cantabria (CSIC-UC), Edificio Juan Jorda,
Avda. de Los Castros s/n 39005 Santander, Spain
sven.heinemeyer@cern.ch*

MYRIAM MONDRAGON

*Inst. de Física, Universidad Nacional Autónoma de México,
Apdo. Postal 20-364, México 01000 D.F., México
myriam@fisica.unam.mx*

GEORGE ZOUPANOS

*Physics Department, National Technical University of Athens,
Zografou Campus: Heroon Polytechniou 9
15780 Zografou, Athens, Greece
george.zoupanos@cern.ch*

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1. Introduction

The success of the Standard Model (SM) of Elementary Particle Physics is seriously limited by the presence of a plethora of free parameters. An even more disturbing fact is that the best bet for Physics beyond the SM namely the minimal supersymmetric extension of the SM (MSSM), which is expected to bring us one step further towards a more fundamental understanding of Nature, introduces around a hundred additional free parameters. To reduce the number of free parameters of a theory, and thus render it more predictive, one is usually led to introduce a symmetry. Grand Unified Theories (GUTs) are very good examples of such a procedure.^{1,2,3,4,5} For instance, in the case of minimal $SU(5)$, because of (approximate) gauge coupling unification, it was possible to reduce the gauge couplings by one and give a prediction for one of them. In fact, LEP data⁶ seem to suggest that a further symmetry, namely $N = 1$ global supersymmetry (SUSY)^{7,8} should also be required to make the prediction viable. GUTs can also relate the Yukawa couplings among themselves,

again $SU(5)$ provided an example of this by predicting the ratio M_τ/M_b ⁹ in the SM. Unfortunately, requiring more gauge symmetry does not seem to help, since additional complications are introduced due to new degrees of freedom, in the ways and channels of breaking the symmetry, and so on.

A natural extension of the GUT idea is to find a way to relate the gauge and Yukawa sectors of a theory, that is to achieve Gauge-Yukawa Unification (GYU).^{10,11,12} A symmetry which naturally relates the two sectors is supersymmetry, in particular $N = 2$ SUSY.¹³ It turns out, however, that $N = 2$ supersymmetric theories have serious phenomenological problems due to light mirror fermions. Also in superstring theories and in composite models there exist relations among the gauge and Yukawa couplings, but both kind of theories have phenomenological problems, which we are not going to address here.

Finite Unified Theories (FUTs) are $N = 1$ supersymmetric Grand Unified Theories (GUTs) which can be made finite to all-loop orders, including the soft supersymmetry breaking sector. The constructed *finite unified* $N = 1$ supersymmetric $SU(5)$ GUTs predicted correctly from the dimensionless sector (Gauge-Yukawa unification), among others, the top quark mass.^{14,15} Eventually, the full theories can be made all-loop finite and their predictive power is extended to the Higgs sector and the s-spectrum.¹⁶ For a detailed discussion see Refs. 11, 17, 18. Here we limit ourselves to a brief review.

Consider a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g . The superpotential of the theory is given by

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k, \quad (1)$$

where m^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge invariant tensors, and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G . All the one-loop β -functions of the theory vanish if the β -function of the gauge coupling $\beta_g^{(1)}$, and the anomalous dimensions of the Yukawa couplings $\gamma_i^{j(1)}$, vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G), \quad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i), \quad (2)$$

where $\ell(R_i)$ is the Dynkin index of R_i , and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of G . A theorem given in Ref. 19, 20, 21, 22 then guarantees the vanishing of the β -functions to all-orders in perturbation theory. This requires that, in addition to the one-loop finiteness conditions (2), the Yukawa couplings are reduced in favour of the gauge coupling.

In the soft breaking sector, the one- and two-loop finiteness for the trilinear terms h^{ijk} can be achieved by²³

$$h^{ijk} = -MC^{ijk} + \dots = -M\rho_{(0)}^{ijk} g + O(g^5). \quad (3)$$

It was also found that the soft supersymmetry breaking (SSB) scalar masses in Gauge-Yukawa and finite unified models satisfy a sum rule^{24,25}

$$\frac{(m_i^2 + m_j^2 + m_k^2)}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)} + O(g^4) \quad (4)$$

for i, j, k , where $\Delta^{(2)}$ is the two-loop correction, which vanishes when all the soft scalar masses are the same at the unification point.

2. $SU(5)$ Finite Unified Theories

We will examine here all-loop Finite Unified theories with $SU(5)$ gauge group, where the reduction of couplings has been applied to the third generation of quarks and leptons. An extension to three families, and the generation of quark mixing angles and masses in Finite Unified Theories has been addressed in Ref. 26, where several examples are given. These extensions are not considered here. The particle content of the models we will study consists of the following supermultiplets: three $(\bar{\mathbf{5}} + \mathbf{10})$, needed for each of the three generations of quarks and leptons, four $(\bar{\mathbf{5}} + \mathbf{5})$ and one $\mathbf{24}$ considered as Higgs supermultiplets. When the gauge group of the finite GUT is broken the theory is no longer finite, and we will assume that we are left with the MSSM.

A predictive Gauge-Yukawa unified $SU(5)$ model which is finite to all orders, should have the following properties:

- (1) One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)j} \propto \delta_i^j$.
- (2) Three fermion generations, in the irreducible representations $\bar{\mathbf{5}}_i, \mathbf{10}_i$ ($i = 1, 2, 3$), which obviously should not couple to the adjoint $\mathbf{24}$.
- (3) The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss two versions of the all-order finite model. The model of Ref. 14, 15, which will be labeled **A**, and a slight variation of this model (labeled **B**), which can also be obtained from the class of the models suggested in Ref. 27 with a modification to suppress non-diagonal anomalous dimensions.

The superpotential which describes the two models takes the form^{14,15,28}

$$\begin{aligned} W = & \sum_{i=1}^3 \left[\frac{1}{2} g_i^u \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \mathbf{10}_i \bar{\mathbf{5}}_i \bar{H}_i \right] \\ & + g_{23}^u \mathbf{10}_2 \mathbf{10}_3 H_4 + g_{23}^d \mathbf{10}_2 \bar{\mathbf{5}}_3 \bar{H}_4 + g_{32}^d \mathbf{10}_3 \bar{\mathbf{5}}_2 \bar{H}_4 \\ & + \sum_{a=1}^4 g_a^f H_a \mathbf{24} \bar{H}_a + \frac{g^\lambda}{3} (\mathbf{24})^3, \end{aligned} \quad (5)$$

where H_a and $\bar{H}_a$ ($a = 1, \dots, 4$) stand for the Higgs quintets and anti-quintets.

The main difference between model **A** and model **B** is that two pairs of Higgs quintets and anti-quintets couple to the $\mathbf{24}$ in **B**, so that it is not necessary to mix them with H_4 and $\bar{H}_4$ in order to achieve the triplet-doublet splitting after the

symmetry breaking of $SU(5)$.²⁸ Thus, although the particle content is the same, the solutions to the finiteness equations and the sum rules are different, which will reflect in the phenomenology.

The non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ for model **FUTA**, which are the boundary conditions for the Yukawa couplings at the GUT scale, are:

$$\begin{aligned} (g_1^u)^2 &= \frac{8}{5} g^2, (g_1^d)^2 = \frac{6}{5} g^2, (g_2^u)^2 = (g_3^u)^2 = \frac{8}{5} g^2, \\ (g_2^d)^2 &= (g_3^d)^2 = \frac{6}{5} g^2, (g_{23}^u)^2 = 0, (g_{23}^d)^2 = (g_{32}^d)^2 = 0, \\ (g^\lambda)^2 &= \frac{15}{7} g^2, (g_2^f)^2 = (g_3^f)^2 = 0, (g_1^f)^2 = 0, (g_4^f)^2 = g^2. \end{aligned} \quad (6)$$

In the dimensionful sector, the sum rule gives us the following boundary conditions at the GUT scale for this model:^{28,29,30}

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = m_{H_d}^2 + m_{\mathbf{5}}^2 + m_{\mathbf{10}}^2 = M^2, \quad (7)$$

and thus we are left with only three free parameters, namely $m_{\mathbf{5}} \equiv m_{\mathbf{5}_3}$, $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ and M .

For the model **FUTB** the non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ give us:

$$\begin{aligned} (g_1^u)^2 &= \frac{8}{5} g^2, (g_1^d)^2 = \frac{6}{5} g^2, (g_2^u)^2 = (g_3^u)^2 = \frac{4}{5} g^2, \\ (g_2^d)^2 &= (g_3^d)^2 = \frac{3}{5} g^2, (g_{23}^u)^2 = \frac{4}{5} g^2, (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5} g^2, \\ (g^\lambda)^2 &= \frac{15}{7} g^2, (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2} g^2, (g_1^f)^2 = 0, (g_4^f)^2 = 0, \end{aligned} \quad (8)$$

and from the sum rule we obtain:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2, m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}, m_{\mathbf{5}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}, \quad (9)$$

i.e., in this case we have only two free parameters $m_{\mathbf{10}} \equiv m_{\mathbf{10}_3}$ and M .

3. Predictions of the $SU(5)$ Models

We confront now the predictions of the four models with the experimental data, starting with the heavy quark masses (see Refs. 16, 18 for more details). Since the gauge symmetry is spontaneously broken below M_{GUT} , the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (6), the $h = -MC$ relation eq.(3), and the soft scalar-mass sum rule (4) at M_{GUT} , as applied in each of the models. Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below M_{GUT} their evolution is assumed to be governed by the MSSM. We further assume a unique supersymmetry breaking scale M_s and therefore below that scale the effective theory is just the SM.

As a first step, we compare the predictions of the two models, **FUTA** and **FUTB** (both with $\mu > 0$ and $\mu < 0$), with the experimental values of the top and bottom quark masses. We use for the top quark the value for the pole mass³¹

$$M_t^{exp} = (173.1 \pm 1.3) \quad (10)$$

where we notice that the theoretical values for M_t may suffer from a correction of $\sim 4\%$.¹¹ For the bottom quark mass we use the value at M_Z ³²

$$m_b(M_Z) = 2.83 \pm 0.10 \text{ GeV} \quad (11)$$

to avoid errors that come from the further running from the M_Z to the m_b mass, and where we have taken the Δb effects into account.

From fig. 1 it is clear that the model **FUTB** with $\mu < 0$ is the only one where both top and bottom quark masses lie within experimental limits. In this case the value of $\tan\beta$ is found to be ~ 48 . Thus, we will concentrate now on the results

Fig. 1. $m_b(M_Z)$ as function of M (top) and M_{top} as function of M (bottom) for models **FUTA** and **FUTB**, for $\mu < 0$ and $\mu > 0$.

for **FUTB**, $\mu < 0$. In the case where all the soft scalar masses are universal at the unification scale, there is no region of M below $\mathcal{O}(\text{few TeV})$ in which $m_{\tilde{\tau}} > m_{\chi^0}$ is satisfied (where $m_{\tilde{\tau}}$ is the lightest $\tilde{\tau}$ mass, and m_{χ^0} the lightest neutralino mass). This problem can be solved naturally, thanks to the sum rule (9).

Furthermore, we impose the conditions of successful radiative electroweak symmetry breaking, $m_{\tilde{\tau}}^2 > 0$ and $m_{\tilde{\tau}} > m_{\chi^0}$, plus the constraints coming from B physics, namely the experimental bounds on $\text{BR}(b \rightarrow s\gamma)$ and $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ (which we have evaluated with Micromegas³³). This way, we find a prediction for the lightest Higgs mass and the s -spectra. From the analysis we find that the lightest observable particle (LOSP) is either the stau or the second lightest neutralino, with mass starting around ~ 500 GeV.

The prediction of the lightest Higgs boson mass as a function of M is shown in Fig. 2. The light points shown are in agreement with the two B -physics observables listed above. The lightest Higgs mass ranges in

$$M_h \sim 121 - 126 \text{ GeV}, \tag{12}$$

where the uncertainty comes from variations of the soft scalar masses, and from finite (i.e. not logarithmically divergent) corrections in changing renormalization scheme. To this value one has to add ± 3 GeV coming from unknown higher order corrections.³⁴ We have also included a small variation, due to threshold corrections at the GUT scale, of up to 5% of the FUT boundary conditions. Thus, taking into account the B physics constraints results naturally in a light Higgs boson that fulfills the LEP bounds.^{35,36} See Refs. 18, 16 for a more detailed analysis.

The prediction for the lightest Higgs mass, M_h as function of M for the model **FUTB** with $\mu < 0$ is shown in Fig. 2. The full particle spectrum of model **FUTB** with $\mu < 0$, again compliant with quark mass constraints and the B physics observables is shown in Fig. 3. The masses of the particles increase with increasing values of the unified gaugino mass M .

One can see that large parts of the spectrum are in the kinematic reach of the LHC. A numerical example of such a light spectrum is shown in Table 1. The colored

Table 1. A representative spectrum of a light **FUTB**, $\mu < 0$ spectrum.

Mbot(M_Z)	2.71 GeV	Mtop	172.2 GeV
Mh	123.1 GeV	MA	680 GeV
MH	679 GeV	MH $^\pm$	685 GeV
Stop1	1876 GeV	Stop2	2146 GeV
Sbot1	1849 GeV	Sbot2	2117 GeV
Mstau1	635 GeV	Mstau2	867 GeV
Char1	1072 GeV	Char2	1597 GeV
Neu1	579 GeV	Neu2	1072 GeV
Neu3	1591 GeV	Neu4	1596 GeV
M1	580 GeV	M2	1077 GeV
Mgluino	2754 GeV

Fig. 2. The highest Higgs mass M_h as function of the unified gaugino mass M .

Fig. 3. The particle spectrum of model **FUTB** with $\mu < 0$, where the points shown are in agreement with the quark mass constraints and the B -physics observables. The light points on the left are the various Higgs boson masses. The dark points following are the two scalar top and bottom masses, followed by the gluino and the lighter scalar tau masses. The darker points to the right are the two chargino masses followed by the lighter shaded points indicating the neutralino masses.

part of this spectrum as well as the lightest Higgs boson should be (relatively easily) accessible at the LHC.

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