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Starting from general assumptions about creation and annihilation operators we propose a new generalised approach to deformed statistics, which permits to distinguish between two different situations; the first, in which we may define an intrinsically defined statistics, and the second one, in which this is not possible, because the symmetry properties of the system are determined by the vacuum state. The dynamical evolution of the generalised particles (guons) is derived by exploiting the Lie - admissible structure of the g - algebra.

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1) INTRODUCTION

In the last few years there has been increasing interest in particles obeying statistics different from Bose or Fermi. The observation by Green [1] that different types of statistics are allowed in the context of field theory has produced an enormous amount of the so called para - statistical fields [2]. The commutation relations for these fields are trilinear in the creation and annihilation operators and characterised by an integer p , which is the order of para - statistics. The case when p is not an integer has been recently studied [3] in order to provide theories in which the Pauli exclusion principle and / or Bose statistics can be slightly violated.

More recently another possibility has been considered, which correspond to the case where no assumption is made about the parameter p [4 - 6].

The idea is to consider "quon" creation and annihilation operators satisfying the relations

$$a(f)a^+(g) - qa^+(f)a(g) = \langle f, g \rangle 1 \quad (1)$$

where f, g are test functions, i.e. elements of the one - particle space with inner product $\langle f, g \rangle$, and $a(f)$ and $a^+(f)$ depend linearly on f . As proved by Fivel the deformation parameter q must be real in the interval $-1 < q < 1$ [7]. Relations (1) with $q=0$ play a role in the theory of freely independent random variables [8 - 11], and in the study of commutation relation of collective degrees of freedom in a many - component system with anomalous statistics [12]. If we consider a system with a single degree of freedom, for which the test function is one dimensional, we obtain the relation $aa^+ - qa^+a = 1$, i.e. the q oscillator. It was first introduced by Arik

and Coon [13] and Kuryshkin [14], and later rediscovered [15,16] in the context of quantum $SU(2)$ [17].

Very recently an interesting generalisation of statistics was proposed which possesses some operatorial characteristics. In this type of approach the c-number q which appear in the q -deformed algebras is replaced by an operator which permits to give the generalised statistics interesting properties [18 - 20]. This new proposal was recently applied to the black body radiation with interesting results [21,22].

In this paper, starting from general assumptions, we introduce a generalised approach to deformed statistics, which permits to distinguish between two different situations; the first one in which we have an intrinsically defined statistics, and the second one in which this is not possible.

2] THE GENERALISED STATISTICS

In order to introduce such a generalisation we suppose to have operators

N_s, a_s, a_s^+ which satisfy the relations

$$\begin{aligned} [N_r, a_s] &= -\delta_{rs} a_s \\ [N_r, a_s^+] &= \delta_{rs} a_s^+ \end{aligned} \quad (2)$$

with

$$N_r = a_r^+ a_r \quad . \quad (3)$$

From (2) and (3) we have the general relations

$$\begin{aligned}
a_r^+ [a_r, a_s] + [a_r^+, a_s] a_r &= -\delta_{rs} a_s \\
a_r^+ [a_r, a_s^+] + [a_r^+, a_s^+] a_r &= \delta_{rs} a_s^+
\end{aligned} \tag{4}$$

which are our starting point. We search for a solution of conditions (4) which generalise the Bose and Fermi case. It is easy to show that the simplest possibility is

$$\begin{aligned}
[a_r, a_s] &= \hat{O} a_s a_r ; & \hat{O} a_r^2 &= 0 \\
[a_r^+, a_s] &= -\hat{O} a_r^+ a_s - \delta_{rs} \\
[\hat{O}, a_r^+] &= 0
\end{aligned} \tag{5}$$

in which $\hat{O}$ is a linear operator; in fact if we define the $\hat{g}$ operator as

$$\hat{g} = \hat{O} + 1 \tag{6}$$

the relations (4) are satisfied provided we have

$$\begin{aligned}
[a_r, a_s]_g &= 0 & [\hat{g}, a_r^+] &= [\hat{g}^+, a_r] = 0 \\
[a_r, a_s^+]_g &= \delta_{rs}
\end{aligned} \tag{7}$$

where $[A, B]_g = AB - \hat{g}BA$ is the $\hat{g}$ -mutator [14]. From the first of (7) we have immediately that

$$\hat{g}^2 = 1 \tag{8}$$

The condition $\hat{O}a_r^2 = 0$ which appears in (5) is a sort of generalisation of the Pauli principle; it may be written as

$$(\hat{g} - 1)a_r^2 = 0 \quad . \quad (9)$$

It is worth nothing that it does imply neither $\hat{g} = 1$ (bosons) nor $a_r^2 = 0$ (fermions), but it contains also a more general possibility referred to as the guon case [18 - 22].

The second assumption which we assume to be hold is the validity of microscopic causality. Consider two observables $A(x)$ and $B(x)$: if microcausality is respected, the commutator

$$[A(x), B(y)] \quad (10)$$

must be zero for space - like separation $(x-y)^2 < 0$. We assume as is customary that observables are bilinear in the fields, which for simplicity we choose real scalars. From quantum field theory we know that the same commutation relations of a_i, a_i^+ are satisfied by the fields, for example

$$[\phi(x), \phi(y)]_g = 0 \quad . \quad (11)$$

From the definition of the $\hat{g}$ -mutator it is easy to get for three generic operators A, B, C

$$[AB, C] = A[B, C]_g - [C, A]_g B + [A, \hat{g}]CB \quad (12)$$

so if $[A, \hat{g}] = 0$ the expression simplifies in the sum of two $\hat{g}$ -mutators. From the assumed bilinearity of fields we obtain immediately that the condition of microscopic causality is satisfied if and only if

$$[\hat{g}, \phi(x)] = 0 \quad . \quad (13)$$

If we assume a linear expansion of $\phi(x)$ as a function of a_i and a_i^+ we get

$$[\hat{g}, a_i] = [\hat{g}, a_i^+] = 0 \quad . \quad (14)$$

Together with condition (7) this means that

$$\hat{g} = \hat{g}^+ \quad (15)$$

so the operator $\hat{g}$ is both hermitian and unitary.

We may summarise this result as follows :

Suppose to have a system of identical particles which fulfill the relations (2) and satisfy the microscopic causality; then a general algebra that connects the annihilation and creation operators is given by

$$\begin{aligned} [a_i, a_k]_g &= 0 ; & [\hat{g}, a_i] &= [\hat{g}, a_i^+] = 0 \\ [a_i, a_k^+]_g &= \delta_{ik} & [A, B]_g &= AB - \hat{g}BA \end{aligned} \quad (16)$$

with $\hat{g} = \hat{g}^+ ; \hat{g}^2 = 1$.

3] PHYSICAL ANALYSIS OF THE G - ALGEBRA

An important feature of g - statistics is that it permits a defined symmetry property of multiparticle states under particle interchange. This is completely different from the quon case; in fact suppose to consider the vector

$$|s\rangle = (a_1^+ a_2^+ - \beta a_2^+ a_1^+) |0\rangle = 0 \quad (17)$$

for some β . The application of relation $[a_j, a_k^+]_q = \delta_{jk}$ to $a_1 |s\rangle = 0$ and $a_2 |s\rangle = 0$ yields

$$(a_2^+ + qa_1^+ - \beta a_1^+ - \beta qa_2^+) |0\rangle = 0 \quad (18)$$

wherefrom we get $q^2 = 1$, or $q = \pm 1$. In the case of g - statistics we should have obtained $\hat{g}^2 = 1$, which does not imply $\hat{g} = \pm 1$.

If we take the vector

$$|s\rangle = (a_1^+ a_2^+ - \hat{g} a_2^+ a_1^+) |0\rangle = 0 \quad (19)$$

and consider the condition

$$(\hat{g} - 1)a_r^2 = 0 \quad (20)$$

we may have several possibilities:

1] $\hat{g} = 1$; the particles satisfy the Bose - Einstein statistics ;

2] $a_r^2 = 0$; the particles obey the Fermi - Dirac statistics ;

3] neither $\hat{g} = 1$ nor $a_r^2 = 0$ but $(\hat{g} - 1)a_r^2 = 0$: in such a case the statistics is the g - statistics and the particles are called guons;

4] $\hat{g} \neq 1$ but $\hat{g}|0\rangle = |0\rangle$: in this case $\hat{g}$ is not trivial, but the properties of vacuum allow particles to obey Bose - Einstein statistics ;

5] $\hat{g} \neq 1$ but $\hat{g}|0\rangle = -|0\rangle$: in such a case the particles (still due to vacuum properties) obey Fermi Dirac statistics;

6] $\hat{g}|0\rangle \neq \pm|0\rangle$ i.e. the vacuum is not eigenstate of $\hat{g}$: the statistics is the g - statistics which generalises both the Fermi and Bose statistics.

We have immediately to observe that the cases (1,2) and (4,5), despite producing the same effects, are completely different; in fact in the first two cases the statistics is connected directly to the properties of $\hat{g}$ and a, a^+ operators in a way we may define an intrinsic statistics for particles. In the cases (4,5) the statistics depends upon the properties of the vacuum state as follows

1] If the vacuum is eigenstate of $\hat{g}$ with eigenvalue $\lambda = 1$ we have Bose Einstein statistics.

2] If the vacuum is eigenstate of $\hat{g}$ with eigenvalue $\lambda = -1$ we have Fermi Dirac statistics.

3] If the vacuum is not eigenstate of $\hat{g}$, the statistics is a generalised one and the vacuum state has to be expressed as a linear combination of eigenstates of $\hat{g}$.

4] DYNAMICS OF GUON PARTICLES

We want now briefly touch the problem of the dynamical evolution connected to the g - statistics. To this aim, let us notice that the generalised algebra (16) defined by the g - mutator

$$[A, B]_g = AB - B\hat{g}A \quad (21)$$

is no longer a Lie algebra. Indeed, the bracket (21) is nothing but the product of a Lie - admissible algebra [23]

$$(A, B) \equiv ARB - BSA \quad (22)$$

where the operators R, S read, in this case

$$R = 1 \quad ; \quad S = \hat{g} \quad (23)$$

(as first realised by Jannussis and coworkers [24]).

If H is the hamiltonian of the system of guons, the time evolution of the operators a, a^+ in the Heisenberg representation is then given by the Santilli - Heisenberg equation [25]

$$i \frac{da}{dt} = (H, a) \quad (24)$$

namely, in our case

$$i \frac{da}{dt} = [H, a]_g = Ha - \hat{g}aH \quad (25)$$

The integrated form of (25) reads

$$a(t) = \exp(it\hat{g}H)a(0)\exp(-itH) \quad (26)$$

Eqs. (25), (26) (and their hermitian coniugates) fully specify the dynamical behaviour of g - particles, on account also of the fact that the the fields operators are, in general, functions of a, a^+ .

3] CONCLUSION

In conclusion, starting from general assumptions about creation and annihilation operators, it is possible to construct a generalised approach to deformed statistics. Our approach permits to distinguish between two completely different situations; the first one in which the statistics, depending directly on the properties of $\hat{g}$ and a, a^+ , has an intrinsic nature, and the second in which the multiparticle symmetry is related to the properties of ground state. The very interesting result is not that $\hat{g} \neq 1$, but, that, if $\hat{g} \neq 1$, we may define a system for which the statistics depends on the vacuum symmetry.

The possible applications, and the physical meaning of these ideas are, in our opinion, potentially very important and require further studies which we hope to develop in the near future.

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