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Three problems related to the heavy quarks*

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Abstract

It is pointed out that the interpretation of leptonic and semileptonic decays of mesons containing one heavy (anti)quark requires a better understanding of boosts for two-body systems, of two-body wave functions at small separation and, perhaps, of the role of the light quark condensate.

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We briefly present three results. These results have their intrinsic value, but we will stress that they expose open problems, whose solutions would be of great interest for heavy quark physics.

1 BOOST

The first result and problem can be explained in connection with the semileptonic decay $\bar{B} \rightarrow D^* l \nu$. This is a well-known showpiece of the heavy quark effective theory (HQET). All the six formfactors can be expressed in terms of a single Isgur-Wise function $\xi(\omega)$, where the argument ω is the product of the four-velocities of the two mesons. Geometrically, ω is the Lorentz factor of the recoiling D^* in the centre of mass frame of the decay. Thus, it equals one for zero recoil and increases, when the recoil increases. In HQET the heavy mesons are analogous to the hydrogen atom. The heavy quark sits at the centre and just supplies a static field, while the light antiquark (more generally the light stuff) is described by some state vector. Up to a simple kinematic factor, the Isgur-Wise function is the overlap of these state vectors for the two mesons:

$$\xi(\omega) = \sqrt{\frac{2}{\omega + 1}} \langle l, v | l, v' \rangle, \quad (1)$$

where $|l, v\rangle$ denotes the state vector of the light stuff, when the velocity of the heavy quark, taken as equal to that of the meson, is v . According to HQET, for the zero recoil case $v = v'$ the state vectors of the light stuff for the $\bar{B}$ and D^* mesons coincide. Then the overlap, and consequently the Isgur-Wise function, equals one. With increasing recoil both the kinematic factor and the overlap decrease. Thus one can write

$$\xi(\omega) = 1 - \rho^2(\omega - 1) + O[(\omega - 1)^2]. \quad (2)$$

The parameter ρ^2 is very difficult to measure. Typical experimental values are $1 \leq \rho^2 \leq 2$. A recent theoretical analysis based on sum rules finds $\rho^2 \approx 1.0$ [1]. A lattice calculation [2] yields $\rho^2 \approx 1.2$.

Any model supplying wave functions for the light stuff can be used to calculate the Isgur-Wise function. It came as a surprise that the otherwise very successful nonrelativistic quark model gives, if not corrected, $\rho^2 = 0.6$ i.e. a much too flat Isgur-Wise function [3]. The first result we report on,

is that in order to get a good result it is necessary to make a relativistic calculation. This depends little on the details of the model, having been derived both from the MIT bag model [4] modified by fixing the heavy quark in the centre of the bag [5], [6] and from the nonrelativistic quark model modified by using some relativistic kinematics [7]. Nonrelativistically

$$\rho^2 = \frac{1}{4} + \frac{1}{3} m_q^2 \langle r^2 \rangle, \quad (3)$$

where m_q is the mass of the light antiquark. The corresponding relativistic formula [4] is

$$\rho^2 = \frac{1}{2} + \frac{1}{3} E_q^2 \langle r^2 \rangle \approx 1.2. \quad (4)$$

The additional $\frac{1}{4}$ results from Lorentz contraction, and the replacement of m_q by E_q is typical for relativistic calculations.

In simple quark models, in the heavy quark limit the mass of the meson M is the sum of the mass m_Q of the heavy quark sitting at the centre of the meson and of the energy E_q of the light quark. Thus the energy $E_q = M - m_Q$ coincides with the much discussed parameter $\bar{\Lambda}$ of HQET. The relation

$$\rho^2 = \frac{1}{2} + \frac{1}{3} \bar{\Lambda}^2 \langle r^2 \rangle \quad (5)$$

yields, for $\rho^2 \approx 1$, the estimate $\bar{\Lambda} \approx 0.4$ GeV, which is reasonable. Typical estimates for $\bar{\Lambda}$ are between 0.3 GeV and 0.7 GeV.

This gives importance to the question: how reliable is our formula? The wave functions, which correctly reproduce the spectroscopy of the mesons, are probably roughly correct, but the description of the boost of the meson is open to doubt. Incidentally, the failure of the uncorrected nonrelativistic quark model calculation of the parameter ρ^2 can be traced to the use of the Galilean boost. The boosts in the relativistic models are Lorentzian, but much simplified. E.g. in the bag model all the binding energy is ascribed to the light antiquark. It would be of great interest to know how to boost a system of two interacting quarks. In full generality this problem is very hard. In the quantum field theory of positronium it has been discussed for years with no generally accepted conclusion. In the case of heavy mesons, however, where one of the two quarks is heavy, the problem might be more tractable.

2 WAVE FUNCTIONS

The second result and problem can be explained by comparing the leptonic decays of B and D mesons. The usual picture is that in order to annihilate into a lepton pair, the quark and the antiquark constituting the decaying meson must meet. The probability for that to happen is proportional to the square of the absolute value of the wave function at the origin. The subsequent annihilation process is for two partons sitting on top of each other and, therefore, its probability should not depend on the geometry of the meson. Thus, the decay width for the decay into leptons should be proportional to $|\psi(0)|^2$ with a proportionality coefficient, which may depend on the quark masses etc., but not e.g. on the size of the meson. The decay constant for the pseudoscalar meson P , which is proportional to the square root of the corresponding leptonic width, is given by the formula [8]

$$f_P = \sqrt{12}|\psi(0)|/\sqrt{M_P}. \quad (6)$$

The factor $1/\sqrt{M_P}$ is necessary for dimensional reasons. The replacement of the meson mass M_P by the heavy quark mass would change little, since $M_P \approx m_Q$, and the light quark mass cannot replace M_P , because the decay constant should not tend to infinity for $m_q \rightarrow 0$. The factor $\sqrt{12}$ results from a specific calculation, but the inverse proportionality of the decay constant to the square root of the heavy meson mass seems difficult to avoid. The problem is that this implies $f_B \approx 0.6f_D$, while recent estimates favour (cf. [9] for references) $f_B \approx f_D$. We have pointed out in ref. [9] that the desired result can be obtained, if one forgets about wave functions and uses semilocal duality instead. This is in fact being done in the recent estimates. The problem, however, arises: what is wrong with the wave functions? Wave functions are of great value for depicting the structure of quantum objects and it would be very interesting to know, how the wave function argument should be modified to get the estimates considered plausible.

3 PICTURE

The semileptonic decays of heavy mesons into light mesons are very different from the rather well understood semileptonic decays of heavy mesons into heavy mesons. For instance (data are from [10]) the fact that $\Gamma(\bar{B} \rightarrow$

$D^*l\nu) \approx 3\Gamma(\bar{B} \rightarrow Dl\nu)$ simply follows from the remark that, neglecting hyperfine effects, the D^* is like three copies of the D . On the other hand, the explanation that $\Gamma(\bar{D} \rightarrow K^*l\nu) \approx 0.5\Gamma(\bar{D} \rightarrow Kl\nu)$ requires a calculation, which only in some models gives the desired result. Here we would like to stress the fact that even the successful models correspond to very different physical pictures.

The new result, which we present, is the model from ref. [11]. The model is for the decays at zero recoil, while for the q^2 dependence the usual pole formulae are assumed. This model, which is built so as to stay as close as possible to the picture for the heavy to heavy transitions, reproduces correctly all the measured semileptonic decay widths for the heavy to light decays. The decay amplitude contains the overlap of the spectator quark wave functions in the initial and that in the final state. This is known from HQET theory of the heavy to heavy transitions. The evolution of the active quark is, however, more complicated. First the heavy quark goes over into a light quark localized at the centre of the meson, where the heavy quark had been. Then one calculates the overlap of the wave function of this localized state with the final wave function of the corresponding quark, which is taken from a bag model for the final meson. The probability of the first step is guessed using arguments of consistency and simplicity.

This physical picture can be contrasted with the picture following from [12]. There it is argued from sum rules that the crucial factor in the heavy to light semileptonic decays is the interaction of the valence quarks with the condensate. Perhaps the quark from a $q\bar{q}$ pair in the condensate annihilates with the quark from the b decay, while the antiquark enters the final meson as a new valence parton. This would eliminate the small overlap between the localized state and the state of the antiquark in the meson. The question is, of course, at what cost. On the other hand pole models, where e.g. a B meson emits a pion and goes over into a B^* meson, which then decays into a lepton pair, have also been much studied. Such processes would be obviously important, if the mass of the lepton pair were close to the mass of the B^* . Because of kinematics it cannot be very close, but it can be argued that most of the physical region is close enough.

This short comparison shows that in the theory of the heavy to light semileptonic decays even the qualitative picture is controversial. The situation is, as usual, further complicated by the fact that different pictures may

be applicable in different kinematical regions and that because of some duality two pictures which look completely different may in fact be equivalent.

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