The basic quantity in general relativity is the metric, the Christoffel symbols $\Gamma^a_{bc}$ being derived from it. On the other hand in gauge theories the vector potential $A^a$ plays a role, forming a Lagrangian $\mathcal{L}$, but still the linear in $F^{ab}$ is not discussed in this paper, it needs special consideration.

We will see that IVP is a notion which makes all these distinctions, the first condition to be addressed is the other hand in gauge theories it has been pointed out, $\mathcal{L}$ is quadratic with respect to $A^a$, $F^{ab}$, and $\Pi^a$.

The Lagrangian

$$\mathcal{L} = \frac{1}{2} \Pi^a \Pi^a$$

while the Lagrangian for the gauge field is quadratic with respect to $A^a$.

$$\mathcal{L} = \frac{1}{4} F^{ab} F_{ab}$$

in accordance with $[g]$ Eq. (1), in general in gauge theories $A^a$ is linear with respect to the Riemann tensor $\mathcal{R}^{abcc}_d$ in contrast to that the successful gauge theories are based upon a compact group of gauge transformations. In contrast to this, general relativity is invariant under the local Lorentz transformations. The Lorentz group is non-compact. In general relativity no other distinctions are important. The most important for our consideration are the following:

1. General relativity is invariant under the local Lorentz transformations. The Lorentz group is non-compact. Therefore, the classical equations of motion in the two theories are quite different. In general relativity the definition makes the classical equation of motion in the two theories be quite different, in contrast with higher covariant derivatives, see $[g]$, the Lagrangian is more sophisticated then in the level of the Lagrangian. This is a special phenomenon in which instatons play a crucial role. Generally speaking an instanton may be treated as a pseudo-particle, which is not detected in the vacuum state.

2. There are no gravitons on the basic level. Space-time is fundamental. There are no dimensional coupling constants in the theory. Obvious here is a non-Abelian gauge theory in which there is IVP in the vacuum state. Then this theory is more appropriate to describe the effects of gravity. Gravity arises due to a smooth variation of the orientation of the condensates through the variables of Riemann geometry satisfying the Einstein equations of general relativity.

3. There is a non-Abelian gauge theory in which there is IAP in the vacuum state. Then this theory is a remedy which sweeps away all these distinctions. The first question to be addressed is the possibility of constructing a gauge theory from general relativity. These statements are valid on the classical level only. On the quantum level the gauge potential contains some other symbols, Riemann tensor etc. are functions of the gauge field. The Einstein equations of general relativity are shown to be equivalent to the quantum equations of the Yang-Mills field. The same is a pseudo-particle, which is not detected in the vacuum state.

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Suppose that there is a non-Abelian gauge theory in which there is IVP in the vacuum state. Then this theory is a pseudo-particle, which is not detected in the vacuum state. Gravity arises due to a smooth variation of the orientation of the condensates through the variables of Riemann geometry satisfying the Einstein equations of general relativity.

There are no gravitational waves in general relativity. In this paper we want to demonstrate that conventional gauge theory can provide the basis for gravity. The theory contains no dimensional coupling constants in the theory. Obviously there is no apparent something pseudo-particle in the usual gauge theory which yields the gravitational field.

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the effects of gravity then the number of parameters of the gauge group is to be six (or more). The simplest compact
group with six parameters is SO(4). Therefore let us consider the usual SO(4) gauge theory with the conventional
Lagrangian which includes the term (2) describing the non-Abelian gauge field. Gauge theories may differ in the
number of generations of the scalars and fermions, their masses and coupling constants which results in a variety of
properties of the theories. In this work we assume that among these theories there exists a gauge theory with the
necessary property - IAP.

The considered gauge algebra so(4) is the direct sum of two su(2), so(4) = su(2) + su(2). We can choose the
generators for one su(2) to be (−1/2)η, and the generators for the other to be (−1/2)η, and refer to these
algebras as su(2)η and su(2)η. Here η, η, the ’t Hooft symbols, a = 1, 2, 3, ij = 1, ..., 4. The strength of the
gauge field in this notation is

\[ F_{\mu \nu}^{ij} = -(1/2)(F_{\mu \nu}^a \eta_{aij} + \tilde{F}_{\mu \nu}^a \bar{\eta}_{aij}) , \]

where \( F_{\mu \nu}^a \) belongs to su(2)η and \( \tilde{F}_{\mu \nu}^a \) belongs to su(2)η.

We assume that there is a condensate of polarized instantons belonging to su(2)η and a condensate of polarized
antiinstantons belonging to su(2)η. Now let us consider the interaction of the sufficiently weak and slowly varying
gauge field with this polarized state. From now on the Euclidean formulation is used. The interaction of one instanton
with a smoothly varying weak field was considered in Ref. [7], see also [8]. For the case of the SU(2) gauge group the
effective action describing this interaction is

\[ S_{m,s} = (2\pi^2 \rho^3 / g^3) \eta_{\mu \nu} \tilde{F}_{\mu \nu}^a \bar{C}_{ab} , \]

where \( \rho \) is the radius of instanton, and \( \bar{C}_{ab} \in SO(3) \) is a matrix describing the orientation of the instanton. The
bar over \( \tilde{F}_{\mu \nu} \) reminds us that the instanton under consideration belongs to su(2)η. Eq.(4) remains valid for the
antiinstanton as well if we substitute \( \eta \eta \rightarrow \eta \eta , \tilde{F}_{\mu \nu} \rightarrow F_{\mu \nu} \), and \( \bar{C}_{ab} \rightarrow C_{ab} \) where \( C_{ab} \in SO(3) \) is a matrix
describing the orientation of the antiinstanton.

Now let us apply this result to the vacuum with IAP. If we suppose the dilute gas approximation for the polarized
instantons and antiinstantons to be valid then from Eq.(4) with the help of Eq.(3) we deduce that the interaction of
the slowly varying weak field with this vacuum is described by the Lagrangian

\[ \mathcal{L} = - (\eta_{\mu \nu} \eta_{bij} M_{ab} + \bar{\eta}_{\mu \nu} \bar{\eta}_{bij} \bar{M}_{ab}) F_{\mu \nu}^{ij} . \]

Here \( M_{ab} , \bar{M}_{ab} \) are defined as

\[ M_{ab} = \pi^2 \langle (1/g^2) \rho^3 n(\rho, C) C_{ab} \rangle , \]
\[ \bar{M}_{ab} = \pi^2 \langle (1/g^2) \rho^3 \bar{n}(\rho, \bar{C}) \bar{C}_{ab} \rangle , \]

where \( n(\rho, C) \) is the concentration of the su(2)η instantons with radius \( \rho \) and orientation given by the matrix \( C_{ab} \),
and \( \bar{n}(\rho, \bar{C}) \) is the concentration of the su(2)η antiinstantons with orientation given by the matrix \( \bar{C}_{ab} \). The brackets \( \langle \rangle \) in (6),(7) describe the average over microscopic fluctuations of the gauge field. The existence of IAP means that
there are condensates of polarized su(2)η instantons and su(2)η antiinstantons which give the nonzero contribution
to the right-hand sides of (6), (7)

\[ M_{ab} = (f/4) C_{ab}^{cond} , \quad \bar{M}_{ab} = (\bar{f}/4) \bar{C}_{ab}^{cond} , \]

where \( C_{ab}^{cond} \in SO(3) \) is the matrix describing the orientation of the condensate of polarized instantons and \( \bar{C}_{ab}^{cond} \in SO(3) \) is the matrix describing the orientation of the condensate of polarized antiinstantons. The constants \( f, \bar{f} \)
characterize the intensity of these condensates. We will suppose them to be equal, and hence \( f = \bar{f} \).

It is useful to present the matrixes \( C_{ab}^{cond} , \bar{C}_{ab}^{cond} \) with the help of a matrix \( h^{ij} \in SO(4) \) which satisfies the equations

\[ h^{ik} h^{lj} \eta_{kl} = C_{ab}^{cond} \eta_{ij} , \quad h^{ik} h^{lj} \bar{\eta}_{kl} = \bar{C}_{ab}^{cond} \bar{\eta}_{ij} , \]

and describes the orientation of the condensate of instantons and antiinstantons.

Substituting Eqs.(8),(9) into Eq.(5) and using the identity [9]

\[ \eta_{\mu \nu} \eta_{aij} + \bar{\eta}_{\mu \nu} \bar{\eta}_{aij} = 2(\delta_{\mu} \delta_{\nu} - \delta_{\mu} \delta_{\nu}) , \]

one finds that the Lagrangian (5) may be written as

2
Remember that the Latin letters \( i, j \) label the indexes of variables in the isotopic space while the Greek letters \( \mu, \nu \) label the indexes in the coordinate space. The symbols \( h_{\mu \nu} \), \( A_{\mu} \) play a role of the generators of the gauge transformations, see Eq.(3), while the symbols \( g_{\mu \nu} \) describe the orientation of instantons in the coordinate space, see Eq.(4). Eq.(10) gives a match between the indexes of isotopic and coordinate spaces. It makes it useful to consider the matrix \( h^{i \mu} \) in Eq.(11) with one Latin index and one Greek one. This matrix plays a role of the order parameter of the problem.

From Eq.(11) one deduces that there appears the corresponding term in the action:

\[
\Delta S = - f \int h^{i \mu} h^{r \nu} F_{\mu \nu}^{ij} \det h \, d^4 x .
\]

This action is invariant under two types of transformations. First, it, preserves gauge symmetry. Gauge transformations have a form

\[
F_{\mu \nu}^{ij} = U^{ij}(x)U^{k \ell}(x)F_{\mu \nu}^{k \ell}, \quad H^{\mu \nu} = U^{ki}(x)h^{k \mu} .
\]

Second, it is invariant under the transformations of the coordinates \( x^\mu \to x'^\mu \)

\[
F_{\mu \nu}^{ij} = \frac{\partial x^\sigma}{\partial x'^\mu} \frac{\partial x'^\tau}{\partial x^\nu} F_{\sigma \tau}^{ij}, \quad H^{\mu \nu} = \frac{\partial x^\sigma}{\partial x'^\mu} h^{\sigma \nu} .
\]

The factor \( \det h(\equiv \det h_{ij}) \), where \( h_{ij}^{-1} \) denotes the matrix inverse to \( h^{ij} : h^{i \mu} h^{j \nu} = \delta_{ij} \) in Eq.(12) compensates for the variation of the volume \( d^4 x \) due to the identity \( \det h = \det [\partial x^\mu / \partial x'^\mu] \), which follows from Eq.(14).

Up to now we assumed the condensate to be homogeneous and the matrix \( h^{ij}(x) \) was considered as a global orthogonal matrix, \( h^{ij} \in SO(4) \). This is true for some particular coordinate frame and particular gauge condition, see (13),(14). Similar consideration may be fulfilled for the case of the non-homogeneous condensate as well. In this case the order parameter may be shown to be described by the matrix \( h^{ij}(x) \) as well, but there are two important distinctions.

First, the matrix \( h^{ij}(x) \) varies in space. For non-homogeneous condensate this variation is nontrivial, i.e. it can not be eliminated with the help of the transformations (13),(14). Second, \( h^{ij}(x) \) is an arbitrary matrix having 16 independent parameters.

Consider the interaction of a weak and smooth gauge field with the non-homogeneous condensate which also varies smoothly in space. Then it may be shown that this interaction is described by the same action (12) assuming that \( h^{ij} = h^{ij}(x) \) is a function of \( x \). A simple argument in favour of this result is a fact that locally, at any point \( x_0 \), the order parameter \( h^{ij}(x_0) \) may be transformed with the help of the coordinate transformation (14) to be an orthogonal matrix \( h^{ij}(x_0) \in SO(4) \). Then the expression (11) for the Lagrangian is valid in the vicinity of \( x_0 \). The point \( x_0 \) is arbitrary, therefore we can integrate the Lagrangian evaluating the action (12).

The action (12) depends on the vector potential \( A_{\mu}^{ij}(x) \) and the matrix \( h^{ij}(x) \). \( A_{\mu}^{ij}(x) \) is a slowly varying vector potential having the trivial topological structure (at least on the microscopic level). In contrast to that \( h^{ij}(x) \) describes the orientation of polarized instantons and antiinstantons which are the degrees of freedom of the field with nontrivial topological structure. This allows us to consider \( A_{\mu}^{ij}(x) \) and \( h^{ij}(x) \) as separate variables. Note that for the considered weak field the term quadratic in \( F_{\mu \nu}^{ij} \) given by the Lagrangian (2) is much smaller compared to the linear term (12). Therefore we can neglect the action coming from Lagrangian (2).

The weak and smooth nature of the field permits one to use the classical approximation:

\[
\delta(\Delta S) / \delta A_{\mu}^{ij} = 0 ,
\]

\[
\delta(\Delta S) / \delta h^{ij} = 0 .
\]

Eq.(15) gives the relation between \( h^{ij} \) and \( A_{\mu}^{ij} \)

\[
\nabla_{\mu} \left( \left( h^{i \mu} h^{j \nu} - h^{i \nu} h^{j \mu} \right) \det h \right) = 0 .
\]

Here \( \nabla_{\mu} \) is the covariant derivative in the gauge field \( \left( \nabla_{\mu} \right)^{ij} = \delta_{ij} \partial_{\mu} + A_{\mu}^{ij} \). In order to present Eq.(17) in a more convenient form let us define three quantities, \( g_{\mu \nu}, \lambda_{\mu}, \) and \( R_{\mu \nu}^{\lambda} \):

\[
g_{\mu \nu} = h_{i}^{i \mu} h_{j}^{j \nu} , \quad \lambda_{\mu} = h^{r \lambda} h^{j \nu} A_{\mu}^{i j} + h^{i \lambda} \partial_{\mu} h^{i \nu} , \quad R_{\mu \nu}^{\lambda} = h^{i \lambda} h_{i}^{i \mu} F_{\mu \nu}^{ij} .
\]
Remember that the space-time under consideration is basically flat and Eqs. (18), (19), (20) define the left-hand sides.

From (18), (19) we find that Eq. (17) may be presented in the form: 
\[
g_{\lambda \sigma}, \frac{\lambda}{\mu \nu} = \frac{1}{2}(\partial_{\mu}g_{\nu \sigma} + \partial_{\nu}g_{\sigma \mu} - \partial_{\sigma}g_{\mu \nu}).
\]
demonstrating that we may consider \( g_{\mu \nu} \) as a metric and \( \lambda_{\mu \nu} \) as a Christoffel symbol. Moreover, one finds that the quantity \( R_{\mu \nu}^{\lambda} \) introduced in Eq. (20) turns out to be equal to the Riemann tensor: 
\[
R_{\mu \nu}^{\lambda} = \partial_{\mu}g_{\lambda \nu} - \partial_{\nu}g_{\lambda \mu} + \gamma_{\nu \sigma}^{\lambda}g_{\mu \sigma} - \gamma_{\mu \sigma}^{\lambda}g_{\nu \sigma}.
\]
Considering now the second classical equation (16) one verifies with the help of Eqs. (18), (19), (20) that it results in the Einstein equations of general relativity in the absence of matter: 
\[
R_{\mu \nu} = -\frac{1}{2}g_{\mu \nu}R = 0.
\]

We come to the important conclusion. If IAP takes place in the SO(4) gauge theory then the classical approximation for this gauge theory may be described via the variables of Riemann geometry for which the Einstein equations are valid. These equations imply in particular that there exist gravitational waves. That is a pleasant surprise since the initial gauge theory possesses no graviton on the basic level. Graviton appears due to excitation of the condensate of instantons and antiinstantons.

Consider the action (12) when the classical Eq. (15) is valid. It is clear from (18), (19), (20) that it is identical to the Lagrangian of general relativity (1). The gravitational constant turns out to be 
\[
k^{-1} = 16\pi f.
\]
This relation shows that a radius and separation of instantons which give the contribution to the constant \( f \), see Eqs. (6), (8), are comparable to the Plank radius.

We see how the distinctions (i), (ii), (iii) discussed at the beginning of the paper are eliminated by IAP. The major problem is (i), the compactness of the group of local transformations. IAP solves it in a peculiar manner. The variables of the gauge theory are expressed, see (18), (19), (20), via the variables of Riemann geometry. This geometry deals with the invariance with respect to transformations of a local reference frame. These transformations may be continued from Euclidean to Minkowsky space.

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