CC

MJU-DP-401
Oct.,1994

On perturbative Ward identity in finite temperature Quantumelectrodynamics

su 8447

Takao Kaneko
Department of Physics, Meijo University
Nagoya 468, Japan

SCAN-9411355

CERN LIBRARIES, GENEVA

Ward identity (zero momentum transfer) is examined up to two loop order in finite-temperature quantumelectrodynamics. It is shown that, in one loop order, zero momentum transfer limits of vertex part have different limit values depending on the way of approach and Ward identity does not hold in the space like limit; in two loop order vertex part diverges (simple pole) in the limit and Ward identity does not hold.

An argument is presented which suggests that higher loop contributions would have higher order poles and total sum of loop expansion would have an essential singularity at zero momentum transfer.

§1 Introduction

In the previous work¹⁾ we calculated two loop order selfenergies of finite temperature ϕ^4 and ϕ^3 coupling theory using the imaginary time formalism, and have shown that there is a singularity (simple pole) at zero momentum limit of ϕ^3 coupling case. Such a divergence does not exist in one loop order, although there appear discrepancies between space like limit and time like limit²⁾.

In this work we study this problem on the vertex part of finite temperature Quantum-electrodynamics, especially in the case that photon momentum goes to zero.

Before going to our calculations, we review the mathematical structure of the appearance of a pole in two loop order while not in one loop. In our imaginary time calculations we decompose each propagators into partial fraction;

$$\begin{aligned}\frac{1}{q^2 - m^2} &= \frac{1}{2E_q} \left(\frac{1}{q_0 - E_q} - \frac{1}{q_0 + E_q} \right), \\ E_q &= \sqrt{\mathbf{q}^2 + m^2} \\ q_0 &= 2n\pi T \quad (n = 0, \pm 1, \pm 2, \dots)\end{aligned}$$

where T is temperature. Thus, in one loop calculations of Fig.1(a) we encounter the summation

$$\sum_{n=-\infty}^{\infty} \frac{1}{n+x_1} \frac{1}{n+x_2} = \pi \frac{\cot(\pi x_1) - \cot(\pi x_2)}{x_2 - x_1}, \quad (1)$$

where $1/(n+x_1)$ and $1/(n+x_2)$ emerge from propagators $1/(q^2 - m^2)$ and $1/[(p-q)^2 - m^2]$ respectively. After analytical continuations, if external momentum p goes to zero, x_2 goes to x_1 and Eq.(1) gives finite limit. In two loop calculation of Fig.1(b) we encounter the summation

$$\begin{aligned}& \sum_{n=-\infty}^{\infty} \left(\frac{1}{n+x_1} \right)^2 \frac{1}{n+x_2} \frac{1}{n+x_3} \\ &= \frac{1}{x_2 - x_1} \sum_n \left(\frac{1}{n+x_1} \right)^2 \frac{1}{n+x_3} - \frac{1}{x_2 - x_1} \sum_n \frac{1}{n+x_1} \frac{1}{n+x_2} \frac{1}{n+x_3} \\ &= \frac{1}{x_2 - x_1} \left\{ \frac{1}{x_3 - x_1} \frac{\pi^2}{\sin^2(\pi x_1)} + \pi \frac{\cot(\pi x_3) - \cot(\pi x_1)}{(x_3 - x_1)^2} \right. \\ & \quad \left. + \pi \frac{\cot(\pi x_1) - \cot(\pi x_2)}{(x_3 - x_2)(x_1 - x_2)} + \pi \frac{\cot(\pi x_1) - \cot(\pi x_3)}{(x_2 - x_3)(x_1 - x_3)} \right\}, \quad (2)\end{aligned}$$

where $1/(n+x_3)$ comes from inner loop. When $p \rightarrow 0$, $x_2 \rightarrow x_1$ but x_3 does not vary, and Eq.(2) leads to the simple pole.

Incontrast, in calculations of Fig.1(c) we encounter the summation

$$\begin{aligned} & \sum_{n=-\infty}^{+\infty} \frac{1}{n+x_1} \frac{1}{n+x_2} \frac{1}{n+x_3} \frac{1}{n+x_4} \\ &= \frac{\pi}{(x_2-x_1)(x_4-x_3)} \times \left\{ \frac{\cot(\pi x_1) - \cot(\pi x_3)}{x_3-x_1} - \frac{\cot(\pi x_1) - \cot(\pi x_4)}{x_4-x_1} \right. \\ & \quad \left. - \frac{\cot(\pi x_2) - \cot(\pi x_3)}{x_3-x_2} + \frac{\cot(\pi x_2) - \cot(\pi x_4)}{x_4-x_2} \right\} \end{aligned} \quad (3)$$

In this case, when $p \rightarrow 0$, x_2 goes to x_1 and x_4 goes to x_3 and the limit is finite.

Thus, we may infer that, when external momentum goes to zero, there appears a pole if three propagators coincide but only two equal propagators do not lead to a pole.

§2 Ward identity in one loop order

At first we calculate one loop contributions to the electron selfenergy and the vertex part in finite temperature Quantumelectrodynamics, as shown in Fig.2. As usually we decompose each propagators into partial fractions;

$$S_F(p) = \frac{1}{2E_p} \left(\frac{\gamma_0 E_p - \gamma_i p_i + m}{p_0 - E_p} + \frac{\gamma_0 E_p + \gamma_i p_i - m}{p_0 + E_p} \right),$$

$$D_F(k) = \frac{1}{2E_k} \left(\frac{1}{k_0 - E_k} - \frac{1}{k_0 + E_k} \right),$$

$$E_p = \sqrt{\mathbf{p}^2 + m^2}, \quad E_k = \sqrt{\mathbf{k}^2}$$

for electron and photon, respectively. We take Feynmann gauge and zero chemical potential. After rather lengthy calculations we obtain the results; electron selfenergy of Fig.2(a),

$$\begin{aligned} \Sigma(p) &= e^2 \int d^3 \mathbf{q} \frac{1}{4E_q E_{p-q}} \\ & \times \left\{ \gamma_\alpha X(p-q) \gamma_\alpha \left\{ \frac{\tanh(E_{p-q}/2T) - \coth(E_q/2T)}{p_0 + E_q - E_{p-q}} - \frac{\tanh(E_{p-q}/2T) + \coth(E_q/2T)}{p_0 - E_q - E_{p-q}} \right\} \right. \\ & \left. + \gamma_\alpha \bar{X}(p-q) \gamma_\alpha \left\{ \frac{\tanh(E_{p-q}/2T) - \coth(E_q/2T)}{p_0 - E_q + E_{p-q}} - \frac{\tanh(E_{p-q}/2T) + \coth(E_q/2T)}{p_0 + E_q + E_{p-q}} \right\} \right\} \end{aligned} \quad (4)$$

where

$$X(p) = \gamma_0 E_p - \gamma_i p_i + m$$

$$\bar{X}(p) = \gamma_o E_p + \gamma_i p_i - m,$$

and vertex part of Fig.2(b)

$$\begin{aligned} \Lambda_\mu(p, k) = e^2 \int d^3 \mathbf{q} \frac{1}{8E_q E_{p-q} E_{p-q+k}} \\ \times \left\{ \frac{Y_1^\mu}{E_{p-q+k} - E_{p-q} - k_o} \left\{ \frac{\coth(E_q/2T) + \tanh(E_{p-q}/2T)}{p_0 - E_q - E_{p-q}} - \frac{\coth(E_q/2T) + \tanh(E_{p-q+k}/2T)}{p_0 + k_o - E_q - E_{p-q+k}} \right. \right. \\ \left. \left. + \frac{\coth(E_q/2T) - \tanh(E_{p-q}/2T)}{p_0 + E_q - E_{p-q}} - \frac{\coth(E_q/2T) - \tanh(E_{p-q+k}/2T)}{p_0 + k_o + E_q - E_{p-q+k}} \right\} \right. \\ + \frac{Y_2^\mu}{E_{p-q+k} + E_{p-q} + k_o} \left\{ -\frac{\coth(E_q/2T) + \tanh(E_{p-q}/2T)}{p_0 - E_q - E_{p-q}} + \frac{\coth(E_q/2T) - \tanh(E_{p-q+k}/2T)}{p_0 + k_o + E_{p-q+k}} \right. \\ \left. - \frac{\coth(E_q/2T) - \tanh(E_{p-q}/2T)}{p_0 + E_q - E_{p-q}} + \frac{\coth(E_q/2T) + \tanh(E_{p-q+k}/2T)}{p_0 + k_o + E_q + E_{p-q+k}} \right\} \\ + \frac{Y_3^\mu}{E_{p-q+k} + E_{p-q} - k_o} \left\{ \frac{\coth(E_q/2T) - \tanh(E_{p-q}/2T)}{p_0 - E_q + E_{p-q}} - \frac{\coth(E_q/2T) + \tanh(E_{p-q+k}/2T)}{p_0 + k_o - E_q - E_{p-q+k}} \right. \\ \left. + \frac{\coth(E_q/2T) + \tanh(E_{p-q}/2T)}{p_0 + E_q + E_{p-q}} - \frac{\coth(E_q/2T) - \tanh(E_{p-q+k}/2T)}{p_0 + k_o + E_q - E_{p-q+k}} \right\} \\ + \frac{Y_4^\mu}{E_{p-q+k} - E_{p-q} + k_o} \left\{ -\frac{\coth(E_q/2T) - \tanh(E_{p-q}/2T)}{p_0 - E_q + E_{p-q}} + \frac{\coth(E_q/2T) - \tanh(E_{p-q+k}/2T)}{p_0 + k_o - E_q + E_{p-q+k}} \right. \\ \left. - \frac{\coth(E_q/2T) + \tanh(E_{p-q}/2T)}{p_0 + E_q + E_{p-q}} + \frac{\coth(E_q/2T) + \tanh(E_{p-q+k}/2T)}{p_0 + k_o + E_q + E_{p-q+k}} \right\} \Bigg\} \quad (5) \end{aligned}$$

where

$$\begin{aligned} Y_1^\mu &= \gamma_\alpha X(p-q) \gamma_\mu X(p-q+k) \gamma_\alpha \\ Y_2^\mu &= \gamma_\alpha X(p-q) \gamma_\mu \bar{X}(p-q+k) \gamma_\alpha \\ Y_3^\mu &= \gamma_\alpha \bar{X}(p-q) \gamma_\mu X(p-q+k) \gamma_\alpha \\ Y_4^\mu &= \gamma_\alpha \bar{X}(p-q) \gamma_\mu \bar{X}(p-q+k) \gamma_\alpha. \end{aligned}$$

Using these expressions we can examine Ward identity in the limit $k \rightarrow 0$. Eq.(4) and Eq.(5) can be easily separated to temperature dependent and independent parts

$$\Sigma(p) = \Sigma^o(p) + \Sigma^T(p)$$

$$\Lambda_\mu(p, k) = \Lambda_\mu^o(p, k) + \Lambda^T(p, k)$$

In the limit $k \rightarrow 0$, temperature dependent part of vertex function approaches to different value in the spacelike limit (first $k_o \rightarrow 0$ and then $\mathbf{k} \rightarrow 0$) and time like limit ($\mathbf{k} \rightarrow 0$ then $k_o \rightarrow 0$).

In the timelike limit we obtain the relation

$$\Lambda_0^T(p, o) = -\frac{\partial}{\partial p_o} \Sigma^T(p), \quad (6)$$

however, in the spacelike limit we get

$$\Lambda_0^T(p, o) = -\frac{\partial}{\partial p_o} \Sigma^T(p) + e^2 \int d^3 \mathbf{q} \frac{n'_F(E_{p-q})}{E_{p-q}} \left\{ \frac{\gamma_o E_{p-q} + 3\gamma_i(p-q)_i + 2m}{(p_o + E_{p-q})^2 - E_q^2} + \frac{\gamma_o E_{p-q} - 3\gamma_i(p-q)_i - 2m}{(p_o - E_{p-q})^2 - E_q^2} \right\} \quad (7)$$

where n'_F is the differentiation of Fermi-Dirac distribution function with respect to its argument. As for the temperature independent part we can confirm usual zero temperature Ward identity.

§3 Two loop calculations

In two loop order, there exist five diagrams for vertex part, which are shown in Fig. 3. There may arise divergence in the zero limit of external photon momentum. According to the inference stated in the Introduction, only Fig. 3(a) would have singularity in $k \rightarrow 0$ limit. In other graphs only two propagators coincide in $k \rightarrow 0$ limit. In fact, we calculated all graph contributions in Fig.3 and, after rather lengthy calculations, confirmed that contributions from Fig.3(b) $\sim$ (e) do not lead to divergence. In the intermediate stage of calculations, there appear several singular terms, however, in the final results they all cancel out. We show the explicit expression of the contribution from graph Fig.3(a) in Appendix. The singular part is given as follows;

in the spacelike limit (see Appendix)

$$\begin{aligned} \Lambda_\mu(p, o)_{\text{singular}} &\sim \frac{e^4}{2^8} \int d^3 \mathbf{q} \int d^3 \mathbf{r} \frac{1}{E_q^3 E_{q-r} E_r} \\ &\times \left\{ \left[\lim_{k \rightarrow 0} \frac{1}{E_{q+k} - E_q} \right] \frac{\text{sech}^2(E_q/2T)}{2T} \right. \\ &\times \sum_{i=1}^4 \left\{ \frac{Z_{1i}^\mu(k=0)}{(E_q + E_i)[(p_o - E_q)^2 - E_{p-q}^2]} + \frac{Z_{8i}^\mu(k=0)}{(E_q - E_i)[(p_o + E_q)^2 - E_{p-q}^2]} \right\} \end{aligned} \quad (8)$$

and in the time like limit

$$\begin{aligned} \Lambda_\mu(p, k=0)_{\text{singular}} &\sim \frac{-e^4}{28} \int d^3\mathbf{q} \int d^3\mathbf{r} \frac{1}{E_q^3 E_{q-r} E_r} \left\{ \lim_{k_0 \rightarrow 0} \frac{1}{k_0} \right\} \\ &\times \left\{ \frac{\text{sech}^2(E_q/2T)}{T} \sum_{i=1}^4 \left\{ \frac{Z_{1i}^\mu(k=0)}{(E_q + E_i)[(p_0 - E_q)^2 - E_{p-q}^2]} - \frac{Z_{8i}^\mu(k=0)}{(E_q - E_i)[(p_0 + E_q)^2 - E_{p-q}^2]} \right\} \right. \\ &\left. + \frac{\tanh(E_q/2T)}{E_q} \sum_i \left\{ \frac{Z_{4i}^\mu(k=0)}{(E_i - E_q)[(p_0 + E_q)^2 - E_{p-q}^2]} + \frac{Z_{5i}^\mu(k=0)}{(E_i + E_q)[(p_0 - E_q)^2 - E_{p-q}^2]} \right\} \right\}. \quad (9) \end{aligned}$$

Thus, in this order the Ward identity can not hold because electron self energy $\Sigma(p)$ does not show any singular behaviour for finite p .

In higher loop orders, we can infer that higher order poles would appear, as will be discussed in next chapter. If this be true, the Ward identity Eq.(6) does not hold in every order in perturbations at finite temperature.

§4 Discussions

Finally we wish to present arguments which suggest that the singularity would be an essential singularity in the sense of the theory of functions³⁾. To be definite we consider the selfenergy of φ^3 coupling scalar theory and examine N loop contribution as shown in Fig.4. This contribution contains following summation

$$\sum_{q_0} \left(\frac{1}{q_0 + E_q} \right)^N \frac{1}{q_0 + k_0 + E_{q+k}} \prod_{i=1}^{N-1} \frac{1}{q_0 + F_i}$$

where $1/(q_0 + F_i)$ is contribution from its inner loop and F_i is function of $\mathbf{q}$ and $\mathbf{r}_i$. This leads to the series summation

$$S_N = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{n + x_1} \right)^N \frac{1}{n + x_2} \prod_{i=1}^{N-1} \frac{1}{n + y_i}$$

which can be deformed as

$$S_N = \frac{1}{x_2 - x_1} \sum_n \left(\frac{1}{n + x_1} \right)^N \prod_{i=1}^{N-1} \frac{1}{n + y_i} - \frac{2}{(x_2 - x_1)(y_{N-1} - y_{N-2})} S_{N-1} \quad (10)$$

where S_{N-1} is analogous contribution from $N - 1$ loop diagram of the type Fig.4. When we take the limit $k \rightarrow 0$, x_2 goes to x_1 and then the first term of Eq.(10) gives simple pole and the second term gives one order higher pole than the $N - 1$ loop contribution. Thus, after the zero temperature renormalization, the sum of all contributions from the

loop expansion of this type is the sum of infinite series in which the next term has a pole of one order higher than the former one. Such a sum gives an essential singularity. So we may infer the exact selfenergy has an essential singularity at $k = 0$.

Above arguments would be valid also for vertex part of ϕ^3 theory and also for Quantumelectrodynamics. In general, at an essential singular point, the function does not have unique limit and can take any value possibly except one value (Picard's theorem³)) so, in our case, the Ward identity Eq.(6) would not hold uniquely. In order to guarantee the Ward identity we would need to specify the procedure of taking the limit $k \rightarrow 0$.

Appendix

$$\begin{aligned}
\Lambda_{\mu}^{(a)}(p, k) &= \frac{e^4}{2^8} \int d^3 \mathbf{q} \int d^3 \mathbf{r} \frac{1}{E_q^2 E_{q-r} E_r E_{q+k} E_{p-q}} \\
&\times \sum_{i=1}^4 \left\{ Z_{1i}^{\mu} \left\{ \frac{1}{2T \cosh^2(E_q/2T)(p_0 + E_{p-q} - E_q)(k_0 - E_{q+k} + E_q)(E_i + E_q)} \right. \right. \\
&\quad - \frac{\tanh(E_q/2T) + \tanh(E_i/2T)}{(p_0 + E_{p-q} + E_i)(k_0 - E_{q+k} - E_i)(E_i + E_q)^2} \\
&\quad + \frac{\tanh(E_q/2T) - \tanh(E_{q+k}/2T)}{(p_0 + k_0 + E_{p-q} - E_{q+k})(k_0 - E_i - E_{q+k})(k_0 + E_q - E_{q+k})^2} \\
&\quad \left. + \frac{\tanh(E_q/2T) - \tanh(E_{p-q}/2T)}{(p_0 + k_0 + E_{p-q} - E_{q+k})(p_0 + E_i + E_{p-q})(p_0 - E_q + E_{p-q})^2} \right\} \\
&+ Z_{2i}^{\mu} \left\{ \frac{1}{2T \cosh^2(E_q/2T)(p_0 + E_{p-q} - E_q)(k_0 + E_{q+k} + E_q)(E_i + E_q)} \right. \\
&\quad - \frac{\tanh(E_q/2T) + \tanh(E_i/2T)}{(p_0 + E_{p-q} + E_i)(k_0 + E_{q+k} - E_i)(E_i + E_q)^2} \\
&\quad + \frac{\tanh(E_q/2T) + \tanh(E_{q+k}/2T)}{(p_0 + k_0 + E_{p-q} + E_{q+k})(k_0 - E_i + E_{q+k})(k_0 + E_q + E_{q+k})^2} \\
&\quad \left. + \frac{\tanh(E_q/2T) - \coth(E_{p-q}/2T)}{(p_0 + k_0 + E_{p-q} + E_{q+k})(p_0 + E_i + E_{p-q})(p_0 - E_q + E_{p-q})^2} \right\} \\
&+ Z_{3i}^{\mu} \left\{ \frac{\tanh(E_q/2T) + \tanh(E_i/2T)}{(p_0 + E_{p-q} + E_i)(k_0 - E_{q+k} - E_i)(E_i + E_q)(E_q - E_i)} \right. \\
&\quad + \frac{\tanh(E_q/2T)}{E_q(p_0 + E_{p-q} + E_q)(k_0 - E_{q+k} - E_q)(E_i - E_q)} \\
&\quad + \frac{\tanh(E_q/2T) - \tanh(E_{q+k}/2T)}{(p_0 + k_0 + E_{p-q} - E_{q+k})(k_0 - E_i - E_{q+k})(k_0 - E_q - E_{q+k})(k_0 + E_q - E_{q+k})} \\
&\quad \left. + \frac{\tanh(E_q/2T) - \coth(E_{p-q}/2T)}{(p_0 + k_0 + E_{p-q} - E_{q+k})(p_0 + E_i + E_{p-q})(p_0 + E_q + E_{p-q})(p_0 - E_q + E_{p-q})} \right\} \\
&+ Z_{4i}^{\mu} \left\{ \frac{\tanh(E_q/2T) + \tanh(E_i/2T)}{(p_0 + E_{p-q} + E_i)(k_0 - E_{q+k} - E_i)(E_i + E_q)(E_q - E_i)} \right. \\
&\quad + \frac{\tanh(E_q/2T)}{E_q(p_0 + E_{p-q} + E_q)(k_0 + E_{q+k} - E_q)(E_i - E_q)}
\end{aligned}$$

$$\begin{aligned}
& + \frac{\tanh(E_q/2T) + \tanh(E_{q+k}/2T)}{(p_0 + k_0 + E_{p-q} + E_{q+k})(k_0 - E_i + E_{q+k})(k_0 - E_q + E_{q+k})(k_0 + E_q + E_{q+k})} \\
& + \frac{\tanh(E_q/2T) - \coth(E_{p-q}/2T)}{(p_0 + k_0 + E_{p-q} + E_{q+k})(p_0 + E_i + E_{p-q})(p_0 + E_q + E_{p-q})(p_0 - E_q + E_{p-q})} \Big\} \\
& + Z_{5i}^\mu \Big\{ \frac{\tanh(E_q/2T) + \tanh(E_i/2T)}{(p_0 + E_{p-q} + E_i)(k_0 - E_{q+k} - E_i)(E_i + E_q)(E_q - E_i)} \\
& + \frac{\tanh(E_q/2T)}{E_q(p_0 + E_{p-q} - E_q)(k_0 - E_{q+k} + E_q)(E_i + E_q)} \\
& - \frac{\tanh(E_q/2T) + \tanh(E_{q+k}/2T)}{(p_0 + k_0 + E_{p-q} - E_{q+k})(k_0 - E_i - E_{q+k})(k_0 + E_q - E_{q+k})(k_0 - E_q - E_{q+k})} \\
& - \frac{\tanh(E_q/2T) + \coth(E_{p-q}/2T)}{(p_0 + k_0 + E_{p-q} - E_{q+k})(p_0 + E_i + E_{p-q})(p_0 - E_q + E_{p-q})(p_0 + E_q + E_{p-q})} \Big\} \\
& + Z_{6i}^\mu \Big\{ - \frac{\tanh(E_q/2T) - \tanh(E_i/2T)}{(p_0 + E_{p-q} + E_i)(k_0 + E_{q+k} - E_i)(E_i + E_q)(E_q - E_i)} \\
& + \frac{\tanh(E_q/2T)}{E_q(p_0 + E_{p-q} - E_q)(k_0 + E_{q+k} + E_q)(E_i + E_q)} \\
& - \frac{\tanh(E_q/2T) - \tanh(E_{q+k}/2T)}{(p_0 + k_0 + E_{p-q} + E_{q+k})(k_0 - E_i + E_{q+k})(k_0 + E_q + E_{q+k})(k_0 - E_q + E_{q+k})} \\
& - \frac{\tanh(E_q/2T) + \coth(E_{p-q}/2T)}{(p_0 + k_0 + E_{p-q} + E_{q+k})(p_0 + E_i + E_{p-q})(p_0 - E_q + E_{p-q})(p_0 + E_q + E_{p-q})} \Big\} \\
& + Z_{7i}^\mu \Big\{ - \frac{1}{2T \cosh^2(E_q/2T)(p_0 + E_{p-q} + E_q)(k_0 - E_{q+k} - E_q)(E_i - E_q)} \\
& + \frac{\tanh(E_q/2T) - \tanh(E_i/2T)}{(p_0 + E_{p-q} + E_i)(k_0 - E_{q+k} - E_i)(E_q - E_i)^2} \\
& - \frac{\tanh(E_q/2T) + \tanh(E_{q+k}/2T)}{(p_0 + k_0 + E_{p-q} - E_{q+k})(k_0 - E_i - E_{q+k})(k_0 - E_q - E_{q+k})^2} \\
& - \frac{\tanh(E_q/2T) + \coth(E_{p-q}/2T)}{(p_0 + k_0 + E_{p-q} - E_{q+k})(p_0 + E_i + E_{p-q})(p_0 + E_q + E_{p-q})^2} \Big\} \\
& + Z_{8i}^\mu \Big\{ - \frac{1}{2T \cosh^2(E_q/2T)(p_0 + E_{p-q} + E_q)(k_0 + E_{q+k} - E_q)(E_i - E_q)} \\
& + \frac{\tanh(E_q/2T) - \tanh(E_i/2T)}{(p_0 + E_{p-q} + E_i)(k_0 + E_{q+k} - E_i)(E_q - E_i)^2}
\end{aligned}$$

$$\begin{aligned}
& - \frac{\tanh(E_q/2T) - \tanh(E_{q+k}/2T)}{(p_0 + k_0 + E_{p-q} + E_{q+k})(k_0 - E_i + E_{q+k})(k_0 - E_q + E_{q+k})^2} \\
& - \frac{\tanh(E_q/2T) + \coth(E_{p-q}/2T)}{(p_0 + k_0 + E_{p-q} + E_{q+k})(p_0 + E_i + E_{p-q})(p_0 + E_q + E_{p-q})^2} \Big\} \\
& - \text{terms with } E_{p-q} \longrightarrow -E_{p-q} \Big\}
\end{aligned}$$

where

$$\begin{aligned}
Z_{1i}^\mu &= \gamma_\alpha X(q) \gamma_\beta W_i \gamma_\beta X(q) \gamma_\mu X(q+k) \gamma_\alpha \\
Z_{2i}^\mu &= \gamma_\alpha X(q) \gamma_\beta W_i \gamma_\beta X(q) \gamma_\mu \bar{X}(q+k) \gamma_\alpha \\
Z_{3i}^\mu &= \gamma_\alpha X(q) \gamma_\beta W_i \gamma_\beta \bar{X}(q) \gamma_\mu X(q+k) \gamma_\alpha \\
Z_{4i}^\mu &= \gamma_\alpha X(q) \gamma_\beta W_i \gamma_\beta \bar{X}(q) \gamma_\mu \bar{X}(q+k) \gamma_\alpha \\
Z_{5i}^\mu &= \gamma_\alpha \bar{X}(q) \gamma_\beta W_i \gamma_\beta X(q) \gamma_\mu X(q+k) \gamma_\alpha \\
Z_{6i}^\mu &= \gamma_\alpha \bar{X}(q) \gamma_\beta W_i \gamma_\beta X(q) \gamma_\mu \bar{X}(q+k) \gamma_\alpha \\
Z_{7i}^\mu &= \gamma_\alpha \bar{X}(q) \gamma_\beta W_i \gamma_\beta \bar{X}(q) \gamma_\mu X(q+k) \gamma_\alpha \\
Z_{8i}^\mu &= \gamma_\alpha \bar{X}(q) \gamma_\beta W_i \gamma_\beta \bar{X}(q) \gamma_\mu \bar{X}(q+k) \gamma_\alpha \\
W_1 &= -X(q-r) \left\{ \tanh(E_{q-r}/2T) + \coth(E_r/2T) \right\} \\
W_2 &= X(q-r) \left\{ \tanh(E_{q-r}/2T) - \coth(E_r/2T) \right\} \\
W_3 &= \bar{X}(q-r) \left\{ \tanh(E_{q-r}/2T) - \coth(E_r/2T) \right\} \\
W_4 &= -\bar{X}(q-r) \left\{ \tanh(E_{q-r}/2T) + \coth(E_r/2T) \right\} \\
E_1 &= -E_r - E_{q-r}, E_2 = E_r - E_{q-r} \\
E_3 &= -E_r + E_{q-r}, E_4 = E_r + E_{q-r}
\end{aligned}$$

References

- 1) T.Kaneko, Phys.Rev. D 49 4209 (1994).
- 2) H.A.Welden, Phys.Rev. D 47 544 (1993).
- 3) See for example E.C.Titchmarsh "The Theory of Function" (Oxford Univ.,1939)

Figure captions

- Fig.1 One loop and two loop selfenergies of ϕ^3 coupling scalar theory.
Fig.2 One loop electron selfenergy and one loop photon-electron vertex part.
Fig.3 Two loop photon-electron vertex part.
Fig.4 One of N-loop selfenergies in ϕ^3 coupling theory.

Fig. 1

(a)

(b)

F i g . 2

F i g . 3

F i g . 4