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Electromagnetic corrections to the threshold pion-pion charge exchange amplitude and the lifetime of ponium

U. Moor, G. Rasche

Institut für Theoretische Physik der Universität, Winterthurerstrasse 190,
CH-8057 Zürich, Switzerland

W.S. Woolcock

Department of Theoretical Physics, IAS, The Australian National University,
Canberra ACT 0200, Australia

Abstract

We give the electromagnetic corrections (due to the mass difference between $\pi^\pm$ and π^0 and the Coulomb potential) to the s -wave threshold parameters in the coupled two-channel ($\pi^+\pi^-$, $\pi^0\pi^0$) system. We estimate that the uncertainty in this correction to the threshold charge exchange amplitude (which determines the lifetime of ponium, i.e. the $\pi^+\pi^-$ bound state) is considerably less than the uncertainty in the prediction of chiral perturbation theory for this quantity. We conclude that a measurement of the lifetime for the decay of ponium to $\pi^0\pi^0$ with an accuracy of about 10% would provide a significant test of the chiral perturbation theory prediction.

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1. Introduction

The experimental observation of ponium has recently been reported (Afanasyev *et al.* [1]). The $\pi^+\pi^-$ atoms are produced in s -wave states, with the principal quantum number n distributed according to n^{-3} . A proposal (Czapek *et al.* [2]) has been made for an experiment to be performed at CERN by a Bern-CERN-Dubna-Moscow collaboration, to measure the lifetime $\tau(1,0)$ for the decay of ponium with $n=1$, $l=0$ into $\pi^0\pi^0$. The aim is to achieve an accuracy of 10%. In analysing the proposed experiment it is completely satisfactory to use the (not quite exact) relation given in ref. [1],

$$\tau(n,0) = \tau(1,0)n^3,$$

between the lifetimes for decay from a state with principal quantum number n and from the ground state.

The expression for $\tau(1,0)$ in the rest frame of ponium is given in ref. [1] in the form

$$\tau(1,0)^{-1} = \Gamma(1,0) = \frac{8\pi}{9} \left[\frac{2(2\mu_c + W_C(1) - 2\mu_0)}{\frac{1}{2}\mu_c} \right]^{\frac{1}{2}} (a_0^0 - a_0^2)^2 |\psi_C(0)|^2. \quad (1.1)$$

Here, μ_c, μ_0 are the physical masses of $\pi^\pm, \pi^0$ respectively, while a_0^T ($T=0,2$) are the s -wave $\pi\pi$ scattering lengths for total isospin T . Further, $W_C(1)$ is the pure nonrelativistic Coulomb energy of the ground state, while $\psi_C(0)$ is the pure Coulomb ground state wavefunction at the origin. In sect. 2 we shall give a detailed treatment of the decay of ponium based on our study of pionic hydrogen (Rasche and Woolcock [3]) and shall discuss a number of small corrections to Eq. (1.1), whose physical origin is clear and whose size can be reliably estimated.

For the present, we draw from Eq. (1.1) the conclusion given in ref. [1] that, if the effect of the electromagnetic interaction on the isospin invariant scattering length combination $(a_0^0 - a_0^2)$ can be accurately estimated, an experimental determination of $\tau(1,0)$ with an accuracy of 10% would yield a value of $(a_0^0 - a_0^2)$ with an accuracy of 5%. Now chiral perturbation theory (χ PT) makes predictions for $a_0^0 - a_0^2$ which are described at length by Gasser

and Leutwyler [4]. In sect. 3 we shall discuss these predictions, estimate the uncertainties involved, and show that $(a_0^0 - a_0^2)$ is believed known with an accuracy of about 4%. The proposed experiment should then provide a significant test of the reliability of χ P.T.

The main aim of this paper is to investigate the modification of these results by the inevitable presence of the electromagnetic interaction. We estimate the size and uncertainty of the resulting correction to $(a_0^0 - a_0^2)$. The effect of the electromagnetic interaction is to shift the pion mass from its "strong" value μ to the physical masses μ_c and μ_o and to introduce the $\pi^+\pi^-$ Coulomb potential V_C . Both of these effects will be studied using a model of the two-channel $(\pi^+\pi^-, \pi^0\pi^0)$ system which is formally identical to the model of the coupled (π^-p, π^0n) system given by Rasche and Woolcock [5]. This model will be summarised in sect. 4.

The detailed numerical results will be given and discussed in sect. 5. We shall show that the electromagnetic correction does not spoil the test of χ P.T. which would result from a measurement of $\tau(1,0)$ with an accuracy of 10%.

2. The lifetime of ponium

We take Eq. (1.1) as our starting point, and explain the various corrections which need to be made to it. It is clear immediately that the kinematical factor is a nonrelativistic approximation. The correct relativistic expression is given by the replacement

$$\left[\frac{2(2\mu_c + W_C(1) - 2\mu_o)}{\frac{1}{2}\mu_c} \right]^{\frac{1}{2}} \rightarrow \frac{2q_o}{\mu_c}, \quad (2.1)$$

where q_o is given by

$$2\mu_c + W_C(1) = 2\sqrt{\mu_o^2 + q_o^2}, \quad (2.2)$$

so that q_o is the magnitude of the momentum of each π^0 in the rest frame of the atom before the decay.

Next there is a correction due to the influence of the strong interaction on the pure Coulomb wavefunction, which results in the replacement

$$\psi_C(0) \rightarrow \psi(0) = \psi_C(0)(1 + p_1(1,0)\epsilon_0(1,0) + \dots), \quad (2.3)$$

where

$$p_1(1,0) = \frac{1}{2}(1 + \gamma) = 0.7886\dots, \quad (2.4)$$

$$\epsilon_0(1,0) = \frac{4A_{cc}(W_C(1))}{B}, \quad (2.5)$$

and

$$B = \left(\frac{1}{2}\alpha\mu_c\right)^{-1} \quad (2.6)$$

is the lowest Bohr radius of ponium, α being the fine structure constant. We defer a discussion of the quantity A_{cc} for a moment.

A further correction is most simply introduced by noting that

$$W_C(1) = -\frac{1}{4}\alpha^2\mu_c, \quad |\psi_C(0)| = (\pi B^3)^{-1}, \quad (2.7)$$

so that

$$\pi|\psi_C(0)|^2 = -\mu_c W_C(1)/B, \quad (2.8)$$

using Eq. (2.6). Just as in our treatment of pionic hydrogen in ref. [3], the pure nonrelativistic Coulomb energy needs to be modified by a relativistic correction and by a vacuum polarisation correction.

A number of corrections come together in treating the factor $(a_0^0 - a_0^2)^2$ in Eq. (1.1). The correct replacement is

$$\frac{2}{9}(a_0^0 - a_0^2)^2 \rightarrow |A_{oc}(W_C(1))|^2. \quad (2.9)$$

We are using c to denote the (closed) $\pi^+\pi^-$ channel and o to denote the (open) $\pi^0\pi^0$ channel and we are neglecting the presence of the $\gamma\gamma$ channel. With the replacements (2.1), (2.3) and (2.9) we come to the lifetime formula

$$\tau(1,0)^{-1} = \Gamma(1,0) = \frac{8}{B} |1 + p_1(1,0)\varepsilon_0(1,0) + \dots|^2 (-W_C(1))q_o |A_{oc}|^2, \quad (2.10)$$

which can be obtained directly from the results given on page 411 of ref. [3].

The quantities A_{cc} and A_{oc} are elements of a 2×2 matrix A which is given in terms of the *real* symmetric matrix K of s -wave amplitudes by

$$A^{-1} = K^{-1} - i \begin{pmatrix} 0 & 0 \\ 0 & q_o \end{pmatrix}. \quad (2.11)$$

Since A_{cc} already appears in a small correction, it is completely justified to take the isospin invariant limit

$$A_{cc} = K_{cc}(0) = \frac{1}{3}(2a_0^0 + a_0^2), \quad (2.12)$$

neglecting the electromagnetic correction to K_{cc} . Moreover, from Eq. (2.11),

$$|A_{oc}|^2 = K_{oc}^2 / (1 + q_o^2 K_{oo}^2). \quad (2.13)$$

The denominator in Eq. (2.13) gives a unitarity correction. There is also a subthreshold correction which comes from the fact that A_{oc} in Eq. (2.10) and therefore K_{oc}, K_{oo} and q_o in Eq. (2.13) should be evaluated at the pionium ground state energy $W_C(1)$.

It turns out that the relativistic and vacuum polarisation corrections to $W_C(1)$, the unitarity correction and the subthreshold correction are all negligible for calculating the lifetime. The relativistic kinematic correction (2.1), with q_o evaluated at the pionium ground state energy via Eq. (2.2), and the correction (2.3) due to the influence of the strong interaction on the Coulomb wavefunction are just large enough to need to be taken into account. They can be reliably calculated, so that the single significant uncertainty in applying the lifetime formula (2.10) concerns the electromagnetic correction to K_{oc} : we need an estimate of the difference

$$K_{oc} - \frac{\sqrt{2}}{3}(a_0^2 - a_0^0)$$

which arises from the presence of the electromagnetic interaction. We describe the model for doing this in sect. 4., but first we turn in sect. 3 to the predictions of χ P.T for a_0^0, a_0^2 which are the starting point for our calculations.

3. The s -wave pion-pion scattering lengths

In the absence of the electromagnetic interaction the strong interaction of the pions is assumed to be governed by QCD. QCD gives almost isospin invariant results for the $\pi\pi$ system: a mass difference of even a few MeV between the up and down quarks gives practically no difference between the strong masses of $\pi^\pm$ and π^0 and almost no isospin breaking of the scattering amplitudes. When the electromagnetic interaction is turned on, the mass of the π^0 hardly changes, while that of the $\pi^\pm$ changes to μ_c . A discussion of the physical mechanisms involved is given by Stephenson, Maltman and Goldman [6]. Thus the strong pion mass μ is very close to μ_o . The statements given in this paragraph can all be found in the papers of Gasser and Leutwyler [4, 7].

The predictions of QCD for hadronic processes like $\pi\pi$ scattering can be obtained by means of χ P.T. Gasser and Leutwyler [4] have shown how to systematically determine the low-energy structure of the Green functions in QCD by means of a simultaneous expansion in powers of the external momenta and of the quark masses. They obtain in particular the low energy representation of the $\pi\pi$ scattering amplitudes. Taking account of their

definition of the scattering lengths in Eqs. (18.3) and (18.4), the results which we need may be taken from Eqs. (20.6) and (20.4):

$$a_0^0 = \frac{7}{32\pi} \frac{\mu}{F_\pi^2} \left\{ 1 + \frac{1}{3} \mu^2 \langle r^2 \rangle_S^\pi - \frac{\mu^2}{672\pi^2 F_\pi^2} (15\bar{l}_3 - 353) \right\} + \frac{25}{4} \mu^4 (a_2^0 + 2a_2^2), \quad (3.1)$$

$$2a_0^0 - 5a_0^2 = \frac{3}{4\pi} \frac{\mu}{F_\pi^2} \left\{ 1 + \frac{1}{3} \mu^2 \langle r^2 \rangle_S^\pi + \frac{41\mu^2}{192\pi^2 F_\pi^2} \right\}. \quad (3.2)$$

In Eqs. (3.1) and (3.2) the s -wave and d -wave scattering lengths a_0^T and a_2^T ($T=0,2$) have the usual dimensions (mass) $^{-1}$ and (mass) $^{-5}$ respectively.

The strong pion mass μ is taken as μ_o , where μ_o is given in the 1994 review of particle properties (Montanet *et al.* [8]) as

$$\mu_o = 134.9764(8) \text{ MeV}.$$

The pion decay constant F_π is also given in ref. [8]:

$$F_\pi = 92.4(3) \text{ MeV}.$$

Gasser and Leutwyler give

$$1 + \frac{1}{3} \mu^2 \langle r^2 \rangle_S^\pi = 1.12(4),$$

$$\bar{l}_3 = 2.9 \pm 2.4,$$

$$a_2^0 + 2a_2^2 = (19.6 \pm 9) 10^{-4} \mu^{-5}.$$

In our numerical work we shall always give the s -wave scattering lengths in fm. With the numbers just given, we find from Eqs. (3.1) and (3.2) that

$$a_0^0 = 0.283 \text{ fm}, a_2^0 = -0.061 \text{ fm}, \frac{\sqrt{2}}{3} (a_0^2 - a_0^0) = -0.162(7) \text{ fm}. \quad (3.3)$$

The values of a_0^0, a_2^0 in Eq. (3.3) are used as starting values for the calculations to be described in sect. 4. The error is given only for the quantity $\frac{\sqrt{2}}{3} (a_0^2 - a_0^0)$ and arises from the combined effect of the errors on the input data.

We also make calculations with a value of μ slightly different from μ_o in order to test the effect of a small uncertainty in μ . For $\mu = 136 \text{ MeV}$ we have

$$a_0^0 = 0.285 \text{ fm}, a_2^0 = -0.061 \text{ fm}, \frac{\sqrt{2}}{3} (a_0^2 - a_0^0) = -0.163(7) \text{ fm}. \quad (3.4)$$

In this section we have presented the results of standard χ PT given in ref. [4]. We remark however that Meissner [9] in his review discusses a generalisation (Stern, Sazdjian and Fuchs [10]) of the usual low energy expansion, in which in each order additional terms are included which in the standard scheme are relegated to higher orders. The hope is that the convergence of the expansion will be improved. For each order, the modified scheme contains more parameters than does standard χ PT; already at leading order there is one additional free parameter. The predictions of the theory are then rather open; the low energy parameters in the effective lagrangian may be fixed by fitting to experimental data on $\pi\pi$ scattering. From the viewpoint of ref. [10] then, the proposed experiment on the lifetime of pionium will provide an important additional piece of information which will help in determining these parameters.

We emphasise that in the present work we are calculating the electromagnetic correction to the threshold s -wave charge exchange amplitude and that the value of this correction is hardly affected at all by the uncertainty in the starting values of a_0^0 and a_2^0 .

4. The electromagnetic corrections to the s -wave scattering lengths

The coupled Schrödinger equations for the two-channel ($\pi^+\pi^-, \pi^0\pi^0$) system close to the $\pi^+\pi^-$ threshold involve the two physical masses μ_o and μ_c and the corresponding channel momenta (in the c.m. frame) q_c and q_o .

When the identity of the two particles in the second channel is taken into account, the equations turn out to have the same form as those for the two-channel ($\pi^- p, \pi^0 n$) system treated by Rasche and Woolcock [5]. The strong potentials $U_3(r), U_1(r)$ (for $T = \frac{3}{2}, \frac{1}{2}$) are now replaced by $U_2(r), U_0(r)$ (for $T = 2, 0$).

As in ref. [5] we approximate these potentials by square wells with the same radius r_N so that, for $T=0,2$,

$$\begin{aligned} U_T(r) &= U_T, \quad r < r_N, \\ U_T(r) &= 0, \quad r > r_N. \end{aligned} \quad (4.1)$$

We prefer r_N to the notation r_0 of ref. [5]. The quantities U_0, U_2 may have to be taken to be energy dependent. Once r_N is chosen, the scattering lengths a_0^0, a_0^2 determine the values of U_0, U_2 respectively at threshold ($q_c = 0$). In the following U_0, U_2 will denote these threshold values; they are all that is of interest to us for the present purpose. If we were concerned with electromagnetic corrections to the slope parameters b_0^T of a low energy expansion of the s -wave scattering amplitudes, as we were in ref. [5], we would also need to fix the derivatives of U_T with respect to q_c^2 at $q_c^2 = 0$.

The first decision is to choose a suitable range of values of the radius r_N . Since the lowest mass for t -channel exchange in $\pi\pi$ scattering is $2\mu_o$, r_N is certainly less than $0.5\mu_o^{-1}$. There are indications of a considerably shorter range. For example, the virton-quark model of Efimov, Ivanov and Lyubovitskii [11] generates a potential with a range which we estimate to be about $0.39 \text{ fm}(0.27\mu_o^{-1})$. This is consistent with the potential being dominated by the exchange of a scalar-isoscalar ϵ meson with a mass of around 500 MeV. We have made calculations with r_N equal to $0.2\mu_o^{-1}, 0.3\mu_o^{-1}$ and $0.5\mu_o^{-1}$ in order to cover a wide range of possibilities.

The coupled Schrödinger equations are given in matrix form in Eq. (53) of ref. [5]:

$$(\mathbb{1}_2 d^2/dr^2 + Q^2 - 2MU(r) - 2MV(r))R(r) = 0, \quad (4.2)$$

where

$$Q = \begin{pmatrix} q_c & 0 \\ 0 & q_o \end{pmatrix}, \quad 2M = \begin{pmatrix} \mu_c & 0 \\ 0 & \mu_o \end{pmatrix}, \quad (4.3)$$

$$U(r) = \begin{pmatrix} \frac{2}{3}U_0(r) + \frac{1}{3}U_2(r) & \frac{\sqrt{2}}{3}(U_2(r) - U_0(r)) \\ \frac{\sqrt{2}}{3}(U_2(r) - U_0(r)) & \frac{1}{3}U_0(r) + \frac{2}{3}U_2(r) \end{pmatrix}, \quad V(r) = \begin{pmatrix} V_C(r) & 0 \\ 0 & 0 \end{pmatrix}. \quad (4.4)$$

The assumption of charge independence is thus built into the matrix $U(r)$. The dependence of $U_T(r)$ on r is given by Eq. (4.1). As we have explained, we are interested only in the $\pi^+\pi^-$ threshold, so that, in Eq. (4.2), $q_c = 0$, q_o is evaluated at $q_c = 0$ and U_0, U_2 are the strictly nuclear potentials at $q_c = 0$.

The calculation of the electromagnetic corrections is carried out in two steps. In the first we approximate $V_C(r)$ in the rather crude way used in ref. [5]:

$$\begin{aligned} V_C(r) &= -1.2\alpha/r_N, \quad r < r_N, \\ V_C(r) &= -\alpha/r, \quad r > r_N. \end{aligned} \quad (4.5)$$

Then all the formal developments of ref. [5] can be used. If W is the constant 2×2 matrix

$$W = \begin{pmatrix} \frac{2}{3}U_0 + \frac{1}{3}U_2 - 1.2\alpha/r_N & \frac{\sqrt{2}}{3}(U_2 - U_0) \\ \frac{\sqrt{2}}{3}(U_2 - U_0) & \frac{1}{3}U_0 + \frac{2}{3}U_2 \end{pmatrix},$$

then Eqs. (77)-(82) of Ref. [5] give the 2×2 matrix d and its determinant D . Note that $2m = \mu_c$, $2m_o = \mu_o$. Eqs. (67) - (71), with $\rho = 0$, $\rho_o = q_o r_N$, then yield the scattering length matrix $A(0)$ (denoted by a in Eq. (83)). The Coulomb functions in Eqs. (68) - (71) are defined earlier in ref. [5]. We draw attention to differences between the notation of ref. [3] and that of ref. [5]. The matrix K of ref. [3] is identical with the matrix A of ref. [5]. The matrices K, A of ref. [5] are therefore different from the matrices K, A of ref. [3]. We consistently use the notation of ref. [3] in this paper.

The second step in the calculation is to introduce a more realistic model for V_C , in which we treat $\pi^\pm$ as uniformly charged spheres of radius r_c . From the experimental value of Amendolia *et al.* [12],

$$\left[\langle r^2 \rangle_{\text{ch}}^\pi \right]^{\frac{1}{2}} = 0.663(6) \text{ fm},$$

we deduce the value

$$r_c = 0.856 \text{ fm}.$$

The potential between two uniformly charged spheres is given in Appendix B of Oades and Rasche [13]. The effect on K_{oc} of changing from the potential V_C of Eq. (4.5) to the potential between two uniformly charged spheres is calculated with the necessary accuracy using the first order results in Eq. (33) of ref. [14]; only the terms involving the radial integrals are required. Full details are given in ref. [15].

5. Results and discussion

We begin by considering all the small corrections which were discussed in sect. 2. With the values

$$\mu_c = 139.56995(35) \text{ MeV}, \quad \alpha^{-1} = 137.0359895(61)$$

given in ref. [8] it follows from Eq. (2.7) that

$$W_C(1)(\text{nonrelativistic}) = -1858.07 \text{ eV}.$$

Austen and de Swart [16] give the relativistic energy levels of a spin 0–spin 0 system bound by the Coulomb potential, while Shafer [17] gives the vacuum polarisation correction to the energy levels. The relativistic correction to $W_C(1)$ is -0.08 eV and the vacuum polarisation correction is -0.95 eV , so that

$$W_C(1)(\text{corrected}) = -1859.10 \text{ eV}. \quad (5.1)$$

These corrections thus give a relative change of $5.5 \cdot 10^{-4}$.

It follows from Eqs. (2.2) and (5.1) that

$$q_o(W_C(1)) = 35.509(2) \text{ MeV}. \quad (5.2)$$

This is the correct value using relativistic kinematics. The nonrelativistic expression for q_o given in the original lifetime formula (1.1) is

$$[\mu_c(2\mu_c + W_C(1) - 2\mu_o)]^{\frac{1}{2}} = 35.805 \text{ MeV}.$$

The relativistic kinematic correction is therefore nearly 1%.

For the correction to the Coulomb wavefunction at the origin due to the influence of the strong interaction, we have to evaluate

$$|1 + p_1(1,0)\varepsilon_0(1,0) + \dots|^2 = 1 + 2p_1(1,0)\text{Re}\varepsilon_0(1,0),$$

where, from Eq.(2.5),

$$\text{Re}\varepsilon_0(1,0) = 4K_{cc}/B.$$

With K_{cc} taken from χ PT, neglecting the electromagnetic correction,

$$K_{cc} = \frac{2}{3}a_0^0 + \frac{1}{3}a_0^2 = 0.1683 \text{ fm}.$$

Then, with $B = 387.489(1) \text{ fm}$ and $p_1(1,0)$ given by Eq. (2.4),

$$2p_1(1,0)\text{Re}\varepsilon_0(1,0) = 2.7(2) \cdot 10^{-3}. \quad (5.3)$$

The uncertainty in this number comes almost entirely from the uncertainty in K_{cc} , which in turn comes from the combined uncertainties in the prediction of χ PT and in the electromagnetic correction to $(\frac{2}{3}a_0^0 + \frac{1}{3}a_0^2)$.

The unitarity correction is $(1 + q_o^2 K_{oo}^2)$, and it is sufficient to take

$$q_o^2 K_{oo}^2 = q_o^2 (\frac{1}{3}a_0^0 + \frac{2}{3}a_0^2)^2 = 9.4 \cdot 10^{-5},$$

with an uncertainty of about 10%. The subthreshold correction is even smaller. We do not go into details, but simply give the result that the change in K_{oc}^2 in going from the $\pi^+\pi^-$ threshold to $W_C(1)$ is given by

$$\Delta(K_{oc}^2)/K_{oc}^2 = -2.4 \cdot 10^{-5}.$$

To obtain this number one uses the slope parameters b_0^0, b_0^2 given in ref. [4] using χ P.T. The uncertainty is again about 10%.

We now have all the numbers for the lifetime formula (2.10). Using Eq. (2.13) as well, we find that

$$K_{oc}^{\text{exp}}(\text{fm}) = 0.30830(4) [\tau(1,0)(\text{fs})]^{-\frac{1}{2}}. \quad (5.4)$$

The units are clearly indicated, and K_{oc} is the value at the $\pi^+\pi^-$ threshold. About 70% of the uncertainty in the conversion factor in Eq. (5.4) comes from the uncertainty given in Eq. (5.3). The only other significant uncertainty comes from q_0 . Even so, the full uncertainty is completely negligible when compared with the expected experimental error in the measurement of the pionium lifetime.

The proposed experiment is expected to yield via Eq. (5.4) a value of K_{oc} with an accuracy of 5%. To compare this number with the prediction of QCD evaluated using standard χ P.T, the value of $\frac{\sqrt{2}}{3}(a_0^2 - a_0^0)$ given in Eq. (3.3) needs to be corrected for electromagnetic effects, using the model described in sect. 4. We give in Table 1 the final results for K_{oc} (in fm), for the choices $0.2\mu_o^{-1}, 0.3\mu_o^{-1}$ and $0.5\mu_o^{-1}$ for r_N , and for $\mu = \mu_o$ and $\mu = 136 \text{ MeV}$. We do not give the details for the two steps of the calculation described in sect. 4. In fact the model used for V_C is of little importance: only about 6% of the total correction arises from the presence of V_C , so that the electromagnetic mass difference between the charged and neutral pions accounts for nearly all the correction. Nor do we separate here the inner and outer corrections, as was done in ref. [5]. Further details of the results, including results for a wider range of choices of the strong pion mass μ , are given in ref. [15].

Inspection of Table 1 shows that the uncertainty in K_{oc} due to the electromagnetic correction is in fact quite small. If we assume that μ lies in a 1 MeV range centred on μ_o and that r_N lies between $0.2\mu_o^{-1}$ and $0.4\mu_o^{-1}$, with a most likely value a little below $0.3\mu_o^{-1}$, then we can say that

$$K_{oc}^{\text{exp}} = -0.168(9) \text{ fm}. \quad (5.5)$$

About 20% of the uncertainty given for K_{oc} in Eq. (5.5) comes from the electromagnetic correction and the remainder from the prediction of χ P.T (see Eqs. (3.3) and (3.4)). The theoretical prediction of K_{oc} is therefore accurate to about 5%, if one uses the results of Gasser and Leutwyler [4].

If on the other hand one adopts the viewpoint of ref. [10], discussed in sect. 3, then our results show that, if the proposed experiment yields a certain value K_{oc}^{exp} , the electromagnetic correction leads to

$$\frac{\sqrt{2}}{3}(a_0^2 - a_0^0) = K_{oc}^{\text{exp}} + 0.006(2),$$

all quantities being in fm. This is then another piece of information for determining the low energy parameters in the effective lagrangian of χ P.T.

We believe that we have been realistic in our choice of the ranges in which r_N and μ lie. We expect the simple model of strong square well potentials to be satisfactory, since the work of Zimmermann [18] on pion-nucleon scattering shows that the electromagnetic corrections are quite insensitive to the shape of the nuclear potential. We conclude that, taking proper account of electromagnetic effects, an experimental determination of the lifetime of the ground state of pionium with an accuracy of 10% would provide a good test of the predictions of QCD and standard χ P.T for low-energy pion-pion scattering.

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References

- [1] L. Afanasyev *et al.*, Phys. Lett. B308 (1993) 200
- [2] G. Czapek *et al.*, Lifetime measurement of $\pi^+\pi^-$ atoms to test low energy QCD predictions, CERN/SPSLC 92-44 (1992)
- [3] G. Rasche and W.S. Woolcock, Nucl. Phys. A381 (1982) 405
- [4] J. Gasser and H. Leutwyler, Ann. of Phys. 158 (1984) 142
- [5] G. Rasche and W.S. Woolcock, Helv. Phys. Acta 49 (1976) 455
- [6] G.J. Stephenson, K. Maltman and T. Goldman, Phys. Rev. D43 (1991) 860
- [7] J. Gasser and H. Leutwyler, Phys. Lett. C87 (1982) 77
- [8] L. Montanet *et al.*, Phys. Rev. D50 (1994) 1172
- [9] Ulf-G. Meissner, Rep. Prog. Phys. 56 (1993) 903
- [10] J. Stern, H. Sazdjian and N.H. Fuchs, Phys. Rev. D47 (1993) 3814
- [11] G.V. Efimov, M.A. Ivanov and V.E. Lyubovitskii, Sov. J. Nucl. Phys. 44 (1986) 296
- [12] S.R. Amendolia *et al.*, Nucl. Phys. B277 (1986) 168
- [13] G.C. Oades and G. Rasche, Nucl. Phys. B20 (1970) 333
- [14] G. Rasche and W.S. Woolcock, Helv. Phys. Acta 49 (1976) 435
- [15] U. Moor, Diploma Thesis, University of Zürich (1994)
- [16] G.J.M. Austen and J.J. de Swart, Phys. Rev. Lett. 50 (1983) 2039
- [17] R.E. Shafer, Phys. Rev. 163 (1967) 1451
- [18] H. Zimmermann, Helv. Phys. Acta 48 (1975) 191

Table 1

Results for the s -wave scattering amplitude K_{oc} (in fm) at the $\pi^+\pi^-$ threshold for various choices of the range r_N of the strong potential and the strong pion mass μ

	$r_N = 0.2\mu_o^{-1}$	$r_N = 0.3\mu_o^{-1}$	$r_N = 0.5\mu_o^{-1}$
$\mu = \mu_o$	-0.1688	-0.1674	-0.1661
$\mu = 136 \text{ MeV}$	-0.1677	-0.1668	-0.1658