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Abstract

Two new superdeformed (SD) bands in ^{153}Dy , in addition to the three previously observed, have been studied with the GAMMASPHERE detector array. Assignments to single-neutron orbits in the SD potential are made based on the band properties. Evidence for a band interaction at very high spins, possibly involving the first $N=7$ proton intruder orbital, is observed in the strongest populated SD band. Furthermore, evidence for a $\Delta I = 2$ staggering is also found in this SD band.

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The spectroscopy of superdeformed (SD) bands continues to reveal new features of the physics of atomic nuclei under extreme conditions. Perhaps the most challenging aspects of superdeformation found to date are the occurrence of “identical” bands and the very recent discovery of SD bands exhibiting a $\Delta I = 2$ staggering. The possibilities of symmetries in the SD Hamiltonian revealed by these features are intriguing and may have important implications for the general understanding of finite many-body systems. The existence of “identical” SD bands has been known for some time now and among the first examples to be observed was the excited signature-partner pair of bands in ^{153}Dy [1], whose average transition energies are nearly identical¹ to the transition energies of the yrast ^{152}Dy SD core [2]. A large number of theoretical investigations have focussed on this and similar experimental findings. For instance, pseudo-SU(3) symmetry [3] and quantized pseudo-spin alignment [4] as well as supersymmetry [5,6] have been proposed for explaining the appearance of “identical” bands, and most recently, results applicable to ^{151}Tb have been presented, based on a relativistic mean field theory [7]. However, identical bands have also been calculated in standard Hartree-Fock calculations [8] to arise due to readjustments in the mean field.

The $\Delta I = 2$ staggering effect, which can be interpreted as a bifurcation of the SD states into $\Delta I = 4$ families, is also a new phenomenon that may be related to a four-fold rotational (C_4) symmetry in the nucleus. It was first observed in a SD band in ^{149}Gd [9]. Evidence for a similar staggering effect has also been observed for the SD bands in ^{194}Hg [10]. The origin of these effects in either mass region is not clear, nor is it clear that they are related. One interpretation involves the existence of a relatively large Y_{44} deformation. Using different versions of a phenomenological Hamiltonian, such a scenario has been analyzed in terms of a semiclassical treatment of tunneling between minima in the rotational energy surface [11],

¹The “identical” relationship between these bands and the yrast SD band in ^{152}Dy was first discussed by Nazarewicz et al. [3]

in the framework of a C_4 bifurcation [12] and within a K-mixing scheme [13]. Clearly more experimental information as well as further theoretical work is needed in order to obtain a better understanding of these intriguing features in SD nuclei. We present here further experimental evidence for $\Delta I = 2$ staggering of SD bands near $A \approx 150$ obtained in an experiment to study SD nuclear states in ^{153}Dy with the Gammasphere [14] detector array.

The observation of new orbitals has the potential of setting new constraints on nuclear models. An irregularity in the $\mathfrak{S}^{(2)}$ -moment of inertia at the top of band 1 is suggestive of a bandcrossing, possibly involving the first $N=7$ proton orbital. Furthermore, we observe two new excited SD bands that appear to form a signature-partner pair with small but increasing signature splitting. However, contrary to the previously observed excited bands 2 and 3, the two new bands 4 and 5 show significant deviation from the ^{152}Dy SD core transition energies and are assigned to the $[521]3/2$ Nilsson configuration.

Excited states in ^{153}Dy were populated using the reaction $^{110}\text{Pd}(^{48}\text{Ca}, 5n)^{153}\text{Dy}$, for which a target stack of two isotopically enriched, self-supporting Pd foils (each $\approx 0.5 \text{ mg/cm}^2$) was bombarded at a projectile energy of 220 MeV. The γ rays emitted in the compound-nucleus reactions were detected by the Gammasphere detector array, then consisting of 32 large ($\sim 75 - 80\%$ efficiency) escape-suppressed germanium detectors. Thirty of the detectors were situated symmetrically at backward and forward angles ($17 \leq \Theta \leq 37^\circ$) relative to the beam axis and the two remaining detectors were placed perpendicular to the beam. Of the order of 10^9 three- and higher-fold events were collected. In addition to the five SD bands assigned to ^{153}Dy , SD bands belonging to ^{152}Dy [2,15] were also observed in the present experiment, in particular the yrast SD band in ^{152}Dy was populated with a strength comparable to band 1 in ^{153}Dy .

On-line gain adjustments were performed using strong, low-lying yrast lines in ^{153}Dy . In this typical thin-target, heavy-ion experiment, the final nuclei recoil out of the target and the majority of the yrast γ decay takes place in vacuum from nuclei with a well-defined velocity distribution. However, due to the very large collective E2 transition strengths ($\sim 2000 \text{ W.u.}$), the lifetimes of the SD states are much shorter than those for the low-lying

yrast states. Thus, the emission of γ rays in the SD bands is so fast that the highest-lying states decay during the slowing down of the nuclei in the target material and these γ rays are emitted at a different (higher) velocity distribution than the low-lying, low-spin yrast lines. Using the measured values of Q_t for band 1 in ^{152}Dy [16] and assuming feeding times similar to the SD lifetimes, an appropriate velocity curve $f(\tau)$ was calculated as a function of γ -ray energy. This provided an off-line gain matching optimized for SD transitions. A detailed description of this procedure will be published elsewhere [17]. The peaks at the top of the bands that otherwise were split corresponding to the main forward and backward detector angles reverted to their expected resolution and the less intense peaks are now more visible in the spectra. For instance, for the 1409 keV peak in band 1 the FWHM was reduced from 11.1 keV to 6.2 keV and the improvement was even better for the higher-lying 1455 keV and 1500 keV transitions. By varying the Q_t values applied in the correction we find that the average measured value for ^{152}Dy minimizes the linewidths for the SD bands in both ^{152}Dy and ^{153}Dy ; therefore, not only do our data agree (within errors) with the previous lifetime measurements in ^{152}Dy but also verify the assumption that the lifetimes of the SD states in ^{153}Dy are very similar. It is worth noting (for later reference) that due to the forward-backward symmetry of the detector system, in the limit of high statistics, these corrections only affect the widths and not the centroids of the peaks.

Spectra for the SD bands in ^{153}Dy observed in the present experiment are shown in Fig.1. The transition energies are given in Table I. Bands 1-3 were found in the earlier work by Johansson et al. [1]. We have extended the previously observed bands by several transitions both to higher and lower spins and in addition observe two new, considerably weaker, excited bands, 4 and 5. Furthermore, the accuracy in the determination of the transition energies is higher than that obtained in the previous work due to the greater resolving power achieved with high-statistics, high-fold data. The isotopic assignment of the new SD bands is based partly on coincidences with low-lying ^{153}Dy lines, even though relatively severe contamination in the spectra of the two new SD bands makes this somewhat inconclusive. However, SD structures are well studied in the neighboring nuclides populated

in the experiment (including ^{152}Dy) and the present γ -ray sequences have not been previously assigned as SD bands in this mass region.

The Fermi levels in the SD nucleus ^{153}Dy are situated at the major proton shell gap at $Z = 66$ and one particle above a similarly large neutron shell gap at $N = 86$. Therefore it is reasonable to assume that static pair correlations are of limited strength in this system. In order to compare the experimental and theoretical spectra of states in the SD minimum we show the experimental Routhians for bands 1-5 in Fig.2 and the theoretical (unpaired) single-particle Routhians for a ‘Universal’ Woods-Saxon potential (see Ref. [18] and references therein) are shown in Fig.3. Spin assignments for bands 1-3 in Fig. 2 were adopted from the work of Ragnarsson [19], and we chose spin assignments for the new SD bands 4 and 5 which are consistent with these, i.e., the lowest level of band 4 was assigned to $59/2\hbar$ and the lowest level of band 5 was assigned to $65/2\hbar$. Using results from Total-Routhian-Surface calculations [18] the deformation parameters for the neutron diagram (4a) were chosen appropriately for the lowest-lying excited configurations in ^{153}Dy ($\beta_2 = 0.59, \beta_4 = 0.12, \gamma = 0$). This is a smaller deformation than predicted for the yrast SD 7_3 configuration ($\beta_2 = 0.62, \beta_4 = 0.12, \gamma = 0$), which we use for the proton Routhian plot (4b) in order to interpret the behavior of band 1 (see below). The neutron orbits closest to the Fermi Surface are the $[752]5/2$ (7_3), $[521]3/2$, $[514]9/2$ and $[402]5/2$ Nilsson levels. In the calculation, the three last configurations produce signature-pair bands out of which only the $[521]3/2$ configuration is predicted to have any significant signature splitting.

Comparing Fig.2 and Fig.3, there appears to be a good qualitative agreement between the experimentally observed SD bands and the theoretical neutron single-particle levels. The assignment of band 1 to the 7_3 intruder configuration [1] seems unambiguous. For bands 2 and 3 the apparent lack of signature splitting suggests either the $[402]5/2$ or the $[514]9/2$ orbits which are well described by the strong coupling scheme. In the calculation the $[402]5/2$ and the $[514]9/2$ levels are almost perfectly parallel as a function of rotational frequency, only separated by around 150 keV (which is well inside the uncertainty of the calculation). It is in principle possible that both these configurations are present, giving rise to two pairs

of identical bands. However, a different possibility is that one of the $[402]5/2$ or $[514]9/2$ bandhead energies lies higher than predicted in the calculation in which case only one of them is likely to be populated. Since the lineshapes of the γ rays in bands 2 and 3 are similar within errors to the other SD bands observed in the experiment (if the bands were doublets one would expect to see at least some broadening and irregularity in the lineshapes), we find this latter scenario to be the most likely one. In addition, this discrepancy between mean field calculations and experiment in terms of a “missing” pair of either $[514]9/2$ or $[402]5/2$ bands appears systematically in the odd SD nuclei with $N \approx 87$ [21]. Bands 4 and 5 show rotational properties (in particular an apparent signature splitting) which closely resembles those calculated for the $[521]3/2$ Nilsson configuration, which is thus a natural assignment. Moreover, the lack of other orbitals with such properties near the Fermi surface further strengthens this assignment.

Considering the high degree of observed “identity” of bands (2 and 3) relative to the ^{152}Dy SD core, this feature is not reproduced by the simple single-particle calculation. The inclusion of a realistic (weak) pairing field may to some extent provide a remedy but it appears that a more “fundamental” addition to the model is needed. Since the relative properties of the observed SD bands are so well described in the model, one may speculate that the central problem lies in describing the detailed interaction between the valence particles and the core.

We now turn to the detailed properties of band 1. The γ -ray energies for this band were determined from four-fold coincidence events using a global (fraction of the four-fold total projection) background subtraction. The dynamic moment of inertia (experimentally derived as $\mathfrak{I}^{(2)} = 4\hbar^2/\Delta E_\gamma$) as a function of rotational frequency $\hbar\omega$ is shown in Fig.4. Two principal features can be seen in the $\mathfrak{I}^{(2)}$ -curve of band 1: (i) At the top of the band, the $\mathfrak{I}^{(2)}$ -moment of inertia shows a sharp increase with rotational frequency. (ii) The $\mathfrak{I}^{(2)}$ values show a small $\Delta I = 2$ stagger for rotational frequencies $\hbar\omega$ above 0.5 MeV. Thus, it appears that the energy levels in the band are experiencing small, alternating shifts.

The sudden increase in the $\mathfrak{I}^{(2)}$ -curve at the top of band 1 is interesting since such

an effect was in fact predicted in the Woods-Saxon calculations by Nazarewicz, Wyss and Johnson [18]. The prediction involves a sharp increase in aligned angular momentum as a function of rotational frequency arising from the occupation of the first $N=7$ proton intruder orbit. This configuration approaches the Fermi level and crosses the $Z=66$ SD shell gap, favored by the very high rotational frequency ($\hbar\omega \approx 0.8$ MeV) and the relatively larger quadrupole deformation of the yrast $\nu 7_3$ configuration, as is shown in the proton single-particle Routhian diagram in Fig.3. The present observation, even though it only consists of the shift of one level energy and is therefore not conclusive, seems to agree with the predictions of the Woods-Saxon calculations and may thus constitute the first observation of the $N=7$ proton intruder orbital in the nuclear potential. We note with great interest that very recently, a similar scenario has been proposed involving a possible proton-particle-hole excited SD band in ^{152}Dy [20].

In the two previously reported cases of $\Delta I = 2$ staggering in SD bands [9,10], the effect could be observed clearly and consistently with or without background subtraction in five-fold coincidence data for ^{149}Gd and in both four-fold and five-fold coincidence data for the case of band 1 in ^{194}Hg . In the present case of ^{153}Dy , the measured transition energies are sensitive to the background subtraction when applied to the three-fold data. Unfortunately, there are not sufficient statistics in the cleaner four-fold events to unambiguously establish the staggering effect in band 1. As mentioned above, triple-gated spectra using a “global” background subtraction show a staggering in the transition energies. A more “realistic” background spectrum for high-fold events includes coincidence gates on the actual peaks of interest and is therefore rich in SD transitions. When such a background is subtracted from the triple-gated spectrum the subsequent loss in intensity increases the statistical uncertainties in the transition energies to the extent that the significance of the small staggering is washed out. With this in mind the evidence for $\Delta I = 2$ staggering should be taken as somewhat tentative. However, assuming the assigned energies and errors are correct, the observed staggering sequence is sufficiently long that it would be rather improbable that it is a phenomenon of purely accidental nature. It is noteworthy that SD band 1 in ^{153}Dy is

assigned to be built on a similar $N=7$ neutron intruder configuration as band 1 in ^{149}Gd , the only other SD band in the $A \approx 150$ mass region where $\Delta I = 2$ staggering is reported. This, and the fact that only a few cases are found, despite a host of high-quality data sets on SD structures in many nuclei, suggests that the intrinsic single-particle structure may play an important role.

In summary, five SD bands have been observed in ^{153}Dy . The two bands labeled 4 and 5 are observed for the first time whereas the three previously known bands have been extended appreciably to both lower and higher spin states. The first signature-partner pair of excited SD bands 2 and 3 have transition energies very close to the “quarter points” and “three-quarter points” of the ^{152}Dy SD “reference” core. However, the new bands 4 and 5, which are also assigned to be signature-partner bands, show a significant deviation from the core, reflecting a different contribution to the band properties from the single-particle configuration. The rotational patterns of both excited pairs of SD bands may be qualitatively understood from the single-particle properties of the assigned Nilsson configurations. In addition, a sudden increase in $\mathfrak{F}^{(2)}$ as a function of rotational frequency is observed at the top of band 1. This may be interpreted as evidence for a band crossing involving the first $N=7$ proton intruder orbital, and could thus constitute the first observation of this orbital in atomic nuclei. Finally, tentative evidence for a $\Delta I = 2$ staggering in the transition energies of band 1 is found.

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FIGURES

FIG. 1. Coincidence γ -ray spectra for SD bands 1 to 5 in ^{153}Dy . The spectrum for band 1 was obtained from the sum of all possible combinations of three-fold coincidence gates. The inset shows the top part of the band as derived from clean combinations of double-gated spectra. The last assigned 1500 keV transition appears weakly in the triple-gated spectrum but its relative intensity is consistent with that in the double-gated spectrum. The spectra for bands 2 - 5 are obtained from sums of all possible combinations of two-fold gates, excluding a few which bring in contaminating lines. Assigned SD transitions are marked with a “*”. In all cases, background contributions have been subtracted. The spectra have been compressed to a dispersion of 1 keV/channel for display purposes.

FIG. 2. Experimental Routhians for SD bands 1-5 in ^{153}Dy . A rigid-rotor reference with the Harris parameter $J_0 = 85.5\hbar^2\text{MeV}^{-1}$ was subtracted. The absolute energy scale is arbitrary and the relative excitation energies of the bands in the diagram assumes that bands 2,3 and 4,5 respectively are signature partners (degenerate at zero rotational frequency)

FIG. 3. Single-particle routhians for a: $N \approx 87$ calculated at a deformation ($\beta_2 = 0.59, \beta_4 = 0.12, \gamma = 0$) relevant for the excited SD bands in ^{153}Dy ; b: $Z \approx 66$ calculated at a deformation ($\beta_2 = 0.62, \beta_4 = 0.12, \gamma = 0$) relevant for band 1 in ^{153}Dy .

FIG. 4. Dynamic moment of inertia $\mathfrak{I}^{(2)}$ as a function of $\hbar\omega$ for SD band 1 in ^{153}Dy . Errors are statistical only.

TABLES

TABLE I. Transition energies (in keV) for the SD bands assigned to ^{153}Dy in the present work.

Quoted errors are statistical.

Band 1	Band 2	Band 3	Band 4	Band 5
721.4(5)	678.6(5)	702.0(5)	723.4 (15)	743.2(15)
765.9(1)	724.5(3)	747.7(3)	767.1(5)	789.8(6)
810.6(1)	770.6(1)	793.9(3)	813.2(8)	835.6(7)
855.4(1)	816.5(1)	839.9(2)	858.4(6)	881.4(7)
900.2(1)	863.1(1)	886.8(2)	904.8(7)	927.7(8)
945.4(1)	910.4(1)	934.0(3)	953.2(7)	974.2(6)
991.1(1)	957.6(2)	981.0(4)	999.0(7)	1023.0(9)
1036..9(1)	1004.0(4)	1028.5(4)	1045.7(6)	1068.8(5)
1082.6(1)	1052.5(4)	1076.3(3)	1092.8(8)	1116.5(6)
1129.1(1)	1100.0(3)	1123.9(4)	1140.3(13)	1164.4(7)
1175.0(1)	1148.2(3)	1172.6(4)	1188.3(12)	1212.6(8)
1221.8(1)	1196.6(4)	1220.2(4)	1234.9(12)	1260.9(7)
1268.3(1)	1244.5(4)	1268.1(5)	1284.8(13)	1307.6(7)
1315.5(1)	1292.6(4)	1316.2(5)	1331.4(14)	1355.3(10)
1362.0(2)	1340.8(5)	1363.7(6)	1380.8(13)	1403.6(13)
1408.6(2)	1388.8(6)	1412.6(7)	1428.3(19)	1452.2(14)
1455.3(2)	1437.8(7)	1460.4(9)		
1499.9(5)	1485.3(8)

Figure 1

Figure 2

(π, α) : solid $=(+, +1/2)$, dotted $=(+, -1/2)$, dot-dash $=(-, +1/2)$, dashed $=(-, -1/2)$

Figure 3

Figure 4