A PROPERTY OF RECOMBINATION IN POLARIZED HADRONIC TARGETS

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Abstract

The triple gluon-ladder vertex is shown to project the outgoing gluon in either polarization state with equal probability up to the leading double-$\ln(x)\ln(Q^2)$ approximation. This implies that the $Q^2$-evolution of $\Delta G(x,Q^2)$ is free from recombination effects to this level of approximation.

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1 Introduction

The $A$-dependence in deep inelastic scattering experiments off nuclei has been a long studied problem. The many levels of description of this behavior make it a textbook example of quantum mechanical reasoning. The simple intuitive explanation is that the nucleons at the “front face” of the nuclei shadow those behind them. Quantum mechanically the effect at very low energy is explained at a phenomenological level by the vector-dominance-model (VDM)[1]. Here the spacelike photon is supposed to have coupled to a vector meson which then interacts with the hadrons in the nuclei. Due to the strong interaction between the meson and nucleons the interaction is likened once again to occur at the front face of the nuclei.

At the microscopic level it was Zakharov and Nikolaev[2] who in a classic paper explained how the parton model accounts for the $A$-dependence at large $Q^2$ through recombination of the constituents. This description, in recent times, has been quantified by perturbative QCD in the small-$x$ high $Q^2$ regime [3, 4, 5]. The QCD picture is that the standard $Q^2$-evolved ladders recombine via the triple gluon-ladder vertex, the basic element of the theory.

The QCD argument has reasonable physical underpinnings and in the large-$Q^2$ regime shows consistency to experiment. The question thus arises on how to put it to further tests that reveal qualitatively new features. In this paper we will show how scattering polarized protons or nuclei off polarized nuclei could provide a simple test of recombination effects. It will be shown that the triple gluon-ladder vertex, the motive of recombination, at leading-$\ln x$ order decorrelates gluon polarization. This means, given two gluon densities of polarization $\alpha, \alpha'$, if they recombine through the triple gluon-ladder vertex, the outgoing “recombined” legs will be in either polarization state with equal probability; a property which we will refer to as disordering of polarization.

To understand the possible observable outcomes of this mechanism, let us recall that nuclear shadowing arises from several sources, of which recombination is only of relevance at large $Q^2$ and small-$x$. At low $Q^2$, shadowing is a nonperturbative effect with recombination playing an insignificant role. However, the implication of polarization disordering is that as one scales up in $Q^2$, recombination will not modify whatever the low energy shadowing effects are in $\Delta G$. This statement is valid up to the leading double-$\ln(x)\ln(Q^2)$ approximation only. Our purpose in this paper is only to identify this hard mechanism. We are not claiming that the effect is measurable. Here, the complications arise both from present day experimental limitations and unknown nonperturbative inputs. Due to the simplicity of this mechanism and its distinct qualitative effects, we feel there may be use at some stage for the theoretical considerations given below.

2 Polarization Disordering

For our calculations we will work in the light-cone frame. For a momentum vector $p = (p_0, p_1, p_2, p_3)$ we have $p_\pm = p_0 \pm p_3$ with the plus component $p_+$ expressed through its momentum fraction $z_\nu$ as $p_+ = z_\nu P$. We will use the light-cone axial gauge so that the gluon propagator $D^{ab}_\mu(k)$ is,

$$D^{ab}_\mu(k) = \frac{k^{ab}}{k^2 - i\epsilon} \sum_{\alpha = 1}^2 e^{[\alpha]}(k) e^{(\alpha)}(k)$$

(1)

where

$$e^{[\alpha]} = \left[ g_\mu - \frac{p_\mu \nu + k_\mu \nu}{n \cdot k} \right] \tilde{e}^{[\alpha]}$$

(2)

with

$$(0, 1, 0, 0)$$

2
\[ \varepsilon^{(2)} = (0, 0, 1, 0) \]

and \( \eta = (-1, 0, 0, 1) \) so that \( \eta \cdot k = k_\perp \).

The basis of our analysis will be the nonlinear evolution equations involving recombination that were obtained by Mueller and Qiu[3]. The equations for the gluon density \( G(x, Q^2) \) and quark density \( q(x, Q^2) \) are,

\[
Q^2 \frac{\partial}{\partial Q^2} G(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} P_{gg} \left( \frac{x}{y} \right) G(y, Q^2) - \frac{4\pi^3}{N^2 - 1} \left( \frac{\alpha C_2(G)}{\pi} \right)^2 \int_x^1 \frac{dy}{y} y^2 G^{(2)}(y, Q^2; y, Q^2) \tag{3a} \\
Q^2 \frac{\partial}{\partial Q^2} q(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} f P_{qg} \left( \frac{x}{y} \right) G(y, Q^2) - \frac{2\alpha f}{N(N^2 - 1)Q^2} \left( \frac{4}{15} N^2 - \frac{3}{5} \right) x G^{(2)}(x, Q^2; x, Q^2) + \frac{2\alpha f}{\tau Q^2} \int_x^1 \frac{dy}{y} \gamma_{FG}(x, y) G_{rr}(y, Q^2) \tag{3b}
\]

where,

\[
q(x, Q^2) = \sum_{i=1}^{2f} q_i(x, Q^2), \tag{4}
\]

\[
l^2 \frac{\partial}{\partial l^2} G_{rr} = -\frac{4\pi^2}{N^2 - 1} \left( \frac{\alpha C_2(G)}{\pi} \right)^2 \int_x^1 \frac{dy}{y} y^2 G^{(2)}(y, x^2; y, x^2), \tag{5}
\]

\[
\gamma_{FG}(z) = -2z + 15z^2 - 30z^3 + 18z^4. \tag{6}
\]

\( G^{(2)}(x, Q^2, x', Q^2) \) above is the two-gluon density distribution, \( C_2(G) = N \), and \( f \) is the number of fermion flavors. Often for studies of loosely bound systems such as a nucleus, \( G^{(2)}(x, Q^2, x', Q^2) \) is approximated as the product of two single gluon density distributions. Our analysis holds for the general form so we will leave it as such. In regards to equations (3a) and (3b), we will only be concerned with some of their features and will elaborate on them where needed. For a general discussion of these equations, the reader is referred to the original paper[3].

Let us now examine the triple gluon-ladder vertex. This we define as everything except the two-gluon density distribution in the second term of equation (3a). What we will show is that the polarization of the gluon emerging from the bottom ladder (labeled \( m \) in figure 1) is uncorrelated to the polarizations of the incoming gluons. To understand this let us examine the helicity flow at the vertices \( V \) and \( V' \) of the triple gluon-ladder vertex in Figure 1. The vertex contraction at \( V \), for example, is by the general formula given as,

\[
e^{(a)}_b \varepsilon^{(a')\nu}_b (b - m) \varepsilon^{(a''\lambda)}_b (b, b - m, m) = -i g c_{uvw} \left[ e^{(a)}_b e^{(a')}_b (b - m)(2b - m) \cdot e^{(a''\lambda)}_b (m) - e^{(a)}_b (b - m) \cdot e^{(a'\nu)}_b (m)[b + m] \cdot e^{(a')}_b (b - m) \right] + e^{(a')}_b (b - m) \cdot e^{(a''\lambda)}_b (m)[2b - m] \cdot e^{(a')}_b (b) \tag{7}
\]

The leading-in \( x \) approximation requires retaining the most singular term at this vertex as a function of the longitudinal momentum. By noting the momentum ordering \( z_b \gg z_m \), the relevant term from (7) is then,

\[
-i g c_{uvw} e^{(a)}_b (b - m)[2b - m] \cdot e^{(a'\nu)}_b (m) = \left. -i g c_{uvw} \delta_{a\alpha} \frac{2b_\perp}{m_+} \cdot e^{(a')}_b \right. \tag{8}
\]
In words this says that the contraction is between the two polarization vectors from the upper
ladders whereas the polarization vector of the m-line is contracted with the momentum term from
the vertex. This means the polarization state of the m-gluon is uncorrelated to the polarization
states of the two impinging gluons. Recall now that it is the state of the m-gluon that defines
the gluon density as can be seen by inspection of the evolution equation (3a). Thus, based on our
above deduction about the vertex, we can conclude that the polarization state in the gluon density
distribution at the triple gluon-ladder vertex becomes disordered.

The approximation used to go from (7) to (8) is the same as what in reference [3] is called the
leading-double-logarithmic approximation (DLA) in the light-cone axial gauge. Equivalently, in the
Altarelli-Parisi formalism [6] the contraction given in (8) is the one that leads to the 1/z term in the
gluon-gluon probability kernel. As a check one can see from that paper that \( P_{\alpha_+\alpha_+}(z) = P_{\alpha_-\alpha_+}(z) \)
at 1/z order. This is indicative of the disordering effect described above.

To make more precise our thinking, in symbolic terms the recombination vertex is a convolution
of the general form,
\[
G^{(2)}_{aa'} \ast V_{aa''a''}
\]
(9)
where \( G^{(2)}_{aa'} \) is the density distribution for two gluons in polarization states \( \alpha \) and \( \alpha' \). \( V_{aa''a''} \) is the
triple gluon-ladder vertex connecting the incoming polarization states \( \alpha \) and \( \alpha' \) to the outgoing
polarization state \( \alpha'' \), where \( \alpha \) is defined as the outer gluon density in Figure 1. Our above analysis
of \( V_{aa''a''} \) shows that
\[
V_{+a'a'} = V_{+a'a'} = V_{-a'a'} = V_{-a'a'}
\]
(10)
Since the form of the recombination term contributing to \( \Delta G \) is,
\[
\Delta G_R = \sum_{aa'} G_{aa'} \ast (V_{aa'a'} - V_{aa''a''})
\]
(11)
by using equation (10) we see that \( \Delta G_R = 0 \). As an aside notice that the incoming gluons connected
to the inner vertices in Figure 1 play no role in controlling the polarization flow across the outer
vertices \( V \) and \( V' \).

This demonstration is not complete since the diagram in figure 1 is only one among a group of
triple gluon-ladder vertices. However Mueller and Qiu [3] showed that by taking advantage of the
residual gauge freedom available in the axial gauge, one can impose the prescription for the 1/k\text{+}
denominator as,
\[
\frac{1}{k_+} \equiv \frac{1}{k_+ - i\epsilon}
\]
(12)
With this choice they demonstrated that the diagram in Figure 1 is in fact the only one at leading-
ln x order. To simplify our discussion we will adhere to their prescription.

At this stage we remind the reader that the polarization vectors in equation (2) are not the
same as for physical gluons due to the longitudinal components. However, to leading ln \( Q^2 \) order
they are equivalent. We also clarify that we are not interpreting their relation to the physical
polarization vectors in any way differently then as done in the Altarelli-Parisi formalism. For this
recall that the recombination calculation is done in the probabilistic limit of the QCD equations
which is the same limit as for the Altarelli-Parisi equations. In this limit Gribov[7] has shown that
the results are gauge independent and can be obtained, in fact, by using only the two physical
transverse polarizations of the gluons. With some consideration one can be convinced that the
transverse components of the light-cone axial gauge polarization vectors retain the same physical
information as the physical transverse vectors of Gribov. The point of this discussion is to clarify
the interpretation of our findings in terms of the physical densities that are ultimately probed.
We are simply clarifying that the correspondence here is the obvious one that one would naturally suspect.

The reason for using the axial gauge in recombination calculations is due to the simplification in the number of graphs to be evaluated, which in particular is markedly decreased by choosing an appropriate prescription for the residual gauge freedom. In the original work of Altarelli-Parisi[6] where only ladder diagrams were discussed, it is obvious by inspection on a graph by graph basis that all residual gauge choices are equivalent. This is because no expressions of the form \((1/k_-)^2\) ever arise. It is for this reason that in their derivation the issue of residual gauge choice never came up. To the authors knowledge, it was Mueller and Qiu [3] who recognized how to take advantage of gauge prescriptions to simplify the recombination calculation.

We now obtain from equation (3a) the evolution equation for \(\Delta G\) as,

\[
Q^2 \frac{\partial}{\partial Q^2} \Delta G(x, Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dy}{y} \Delta P_{qg} \left( \frac{x}{y} \right) \Delta G(y, Q^2)
\]

(13)

where

\[
\Delta G(x, Q^2) \equiv G_+(x, Q^2) - G_-(x, Q^2)
\]

(14)

Observe that this equation has no recombination effects in the leading \(\ln(x)\ln(Q^2)\) limit. This is a direct outcome of our basic observation regarding polarization flow and is a property of perturbative QCD, free from model dependent assumptions.

3 Conclusion

Recall that perturbative QCD-recombination is operative at large \(Q^2\). To understand the implications of polarization disordering from recombination, as given in eqs (13), let us consider two possible low energy scenarios. In the first case, which is in the spirit of [3], suppose shadowing effects in polarized nuclei at low-\(Q^2\) are given by some low-energy model. Then according to eqs (13), no further shadowing effects will be generated by QCD-evolution for \(\Delta G\). The second example, which is in the spirit of [8], supposes that at the low energy scale there is no shadowing. In their model, shadowing is soley attributed to QCD-recombination. Due to polarization disordering, it follows that to leading double-\(\ln(x)\ln(Q^2)\) shadowing would not be seen in \(\Delta G(x, Q^2)\) as \(Q^2\) is increased. In both examples, nonleading gluon-gluon fusion and quark-gluon fusion would be expected to give rise to recombination induced shadowing. We will not pursue that matter at present.

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References


