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DSF-T-94/46
INFN-NA-IV-94/46

2D electron in an external magnetic field in the presence of dissipation. ¹

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Abstract

A coherent state approach is presented to analyze the behavior of a 2D electron subject to an external magnetic field, normal to the plane, in the presence of dissipation. The phenomenon of the shrinking of the Gaussian (localization) describing the electron ground state takes place on large time scales. Its relation with the transport properties of such a system is then analyzed.

SCAN-9411429

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¹Work supported in part by MURST and by EEC contract n. SC1 - CT92 - 0789 .

1 Introduction

In the last two decades there has been considerable effort toward a better understanding of the transport properties of a Quantum Hall fluid, that is of a system of highly correlated electrons confined in a two-dimensional film (hetero-junction) and moving in a strong external magnetic field $\vec{B}$ ($\sim Tesla$), normal to the film, at very low temperature ($T \sim mK$) [1]. Even though the striking phenomenon of the quantization of the conductance transverse to an applied constant electric field $\vec{E}$ in the plane of the film, σ_H (Hall conductance), has been reproduced for the Integer as well as for the Fractional Quantum Hall fluid in different approaches [2], all of them revealing its topological nature, still little is understood about the behavior of the longitudinal resistance ρ_L in the region between plateaus (transition region), where it appears to be different than zero. Indeed the experimental observation of zero longitudinal resistance at rational values of the filling ν (corresponding to the plateaus middle points) is very well reproduced theoretically, and simply understood in terms of strongly correlated electrons (for integer values of ν) or in terms of anyons (for rational values of ν ($\nu < 1$)) with fractional electric charge, which completely fill the highly degenerate Lowest Landau Level (LLL). Electrons or anyons appear to be stuck to the Landau level, with a resulting zero mobility on large time scales.

On the other side the fact that the longitudinal resistance ρ_L is different than zero in the transition region suggests that one should take into account the effect of the interaction of the electron with the lattice vibrational modes. One way of introducing such an interaction is à la Caldeira-Leggett [3], where the electron couples to a thermal bath of harmonic oscillators, with a distribution in frequencies reproducing, in the classical limit, the dispersive (Ohmic) term $\eta\dot{q}$ (η being the viscosity constant, and $\dot{q}$ the velocity of the electron) in the equations of motion. Along this line the motion of a Bloch electron in a magnetic field and in the presence of dissipation (the dissipative Hofstadter model) has been studied in an effective lagrangian approach with a non-local term describing the dissipation, on large time scales [4].

In this paper we propose another way, local, of studying the effect of the dissipative term $\eta\dot{q}$ on the motion of an electron moving in a plane and in the presence of an external magnetic field $\vec{B}$, normal to it. In particular it is shown how localization occurs in the presence of dissipation with a resulting change in the transport properties of the system. The paper is organized as follows:

in section 2 we present the coherent states technique to study the time evolution on large time scales ($t \gg \eta^{-1}$) of the electron LLL states.

In section 3 we analyze the shrinking of the gaussian for the electron ground-state and find its relation with the transport properties of the system.

In section 4 we compare Anderson localization with the results of sec.3 and give some comments.

2 Two-dimensional electron in a constant magnetic field B in the presence of dissipation

2a. The classical case

Let us consider an electron moving in a plane in the presence of a transverse magnetic field $\vec{B}$.

By introducing dissipation in the system, the Newton equations of motion are modified by a term proportional to the velocity $\eta\dot{q}$. Their explicit form is, defining the complex variable $z(t) = x(t) + iy(t)$ ($\bar{z}(t)$ being the complex conjugate), (in the units $m = c = e = 1$):

$$\frac{d^2 z}{dt^2} = -(\eta + i\omega)\frac{dz}{dt}; \quad \frac{d^2 \bar{z}}{dt^2} = -(\eta - i\omega)\frac{d\bar{z}}{dt}. \quad (1)$$

Dissipation modifies the relations between the canonical momenta $p_z, p_{\bar{z}}$ and the velocities $v_z, v_{\bar{z}}$. In fact the new canonical momenta are defined by:

$$p_z = v_z + \frac{\eta + i\omega}{2}z; \quad p_{\bar{z}} = v_{\bar{z}} + \frac{\eta - i\omega}{2}\bar{z}. \quad (2)$$

It can be easily shown that, from eq.(2) and the canonical equations of motion:

$$\begin{aligned} \frac{dz}{dt} &= v_z; & \frac{d\bar{z}}{dt} &= v_{\bar{z}} \\ \frac{dp_z}{dt} &= \frac{f_z}{2}; & \frac{dp_{\bar{z}}}{dt} &= \frac{f_{\bar{z}}}{2} \end{aligned} \quad (3)$$

we get the Newton equations (1) (Above f_z and $f_{\bar{z}}$ are the chiral components of the "total force" $\vec{f}$).

By integrating equations (1), we obtain the following solutions

$$\begin{aligned} z(t) &= \frac{\dot{z}_0}{\eta + i\omega}(1 - \exp(-(\eta + i\omega)t)) + z_0 \\ \bar{z}(t) &= \frac{\dot{\bar{z}}_0}{\eta - i\omega}(1 - \exp(-(\eta - i\omega)t)) + \bar{z}_0 \end{aligned} \quad (4)$$

where $\omega = B$ is the cyclotron frequency and $z_0, \dot{z}_0$ are the initial "coordinate" and "velocity", respectively.

In equation (4) we can immediately see that for large t the electron will fall on the orbit center, whose coordinates are given by

$$\begin{aligned} z_\infty &= \frac{\dot{z}_0}{\eta + i\omega} + z_0 \\ \bar{z}_\infty &= \frac{\dot{\bar{z}}_0}{\eta - i\omega} + \bar{z}_0 \end{aligned} \quad (5)$$

due to the fact that the electron will loose all its initial energy after an infinite time.

2b. The quantum case

Our approach in quantizing the system is to apply the correspondence principle to the above solutions of the classical equations of motion. More precisely we will be interested in the large time scales ($t \gg \eta^{-1}$) of the evolution, that is when the electron has thermalized with the bath of harmonic oscillators.

Applying then the correspondence principle to eq.(5) we get the operatorial relations

$$\begin{aligned}\hat{z}_\infty &= \frac{\hat{z}_0}{\eta + i\omega} + \hat{z}_0 = \frac{-2}{\omega - i\eta} \frac{\partial}{\partial \bar{z}_0} + \frac{z_0}{2} \\ \hat{z}_\infty^\dagger &= \frac{\hat{z}_0^\dagger}{\eta - i\omega} + \hat{z}_0^\dagger = \frac{2}{\omega + i\eta} \frac{\partial}{\partial z_0} + \frac{\bar{z}_0}{2}\end{aligned}\quad (6)$$

where $\hat{z}_0$ is the position operator and use has been done of the following expansion for the velocity operators $\hat{z}_0, \hat{z}_0^\dagger$ (see eq.(2))

$$\begin{aligned}\hat{z}_0 &= -2i \frac{\partial}{\partial \bar{z}_0} - \frac{\eta + i\omega}{2} z_0, \\ \hat{z}_0^\dagger &= -2i \frac{\partial}{\partial z_0} - \frac{\eta - i\omega}{2} \bar{z}_0.\end{aligned}\quad (7)$$

The symbol dagger ($\dagger$) indicates the hermitian conjugation operation. We will use the operators defined in eq.(6) to construct the evolution, on large time scales, of the LLL wave function of the electron.

In order to do so let us observe that the operators $\hat{z}_\infty, \hat{z}_\infty^\dagger$, being the orbit center coordinates operators for the dissipative case, do not commute. More explicitly, their commutator is given by

$$[\hat{z}_\infty, \hat{z}_\infty^\dagger] = -\frac{2\omega}{\eta^2 + \omega^2}.\quad (8)$$

This non trivial algebra will help us, in the following, to construct the asymptotic states of the electron system.

2c. Construction of the asymptotic states

In describing the time evolution of the system in the presence of dissipation, we assume that the operators $\hat{z}_\infty$ and $\hat{z}_\infty^\dagger$, given by the eq.(6), being just the orbit center operators are good ones to construct the asymptotic states of the system (see also ref.[5] for the $\eta = 0$ case).

As a first step let us define $|z(t)\rangle_{t \rightarrow \infty}$ as the eigenstate of $\hat{z}_\infty$ belonging to the eigenvalue z . Then we construct the asymptotic wave function $\Psi_{t \rightarrow \infty}^\eta(z)$ as

$$\Psi_{t \rightarrow \infty}^\eta(z) = \lim_{t \rightarrow \infty} \langle z | U(t, 0) | \Psi_0 \rangle \equiv {}_{t \rightarrow \infty} \langle z(t) | \Psi_0 \rangle \quad (9)$$

where $|\Psi_0\rangle$ is the initial state of the system and $U(t, 0)$ is the evolution operator.

To construct explicitly the state $|z(t)\rangle_{t \rightarrow \infty}$ we define the following operators:

$$\begin{aligned}\hat{b} &= \frac{\omega + i\eta}{\sqrt{2\omega}} \hat{z}_\infty^\dagger, \\ \hat{b}^\dagger &= (\hat{b})^\dagger = \frac{\omega - i\eta}{\sqrt{2\omega}} \hat{z}_\infty,\end{aligned}\tag{10}$$

with the commutation relation:

$$[\hat{b}, \hat{b}^\dagger] = 1.\tag{11}$$

Then we can define the coherent state $|\xi\rangle$ such that:

$$\hat{b}|\xi\rangle = \xi|\xi\rangle\tag{12}$$

where $|\xi\rangle$ is given by:

$$|\xi\rangle = \exp\left(-\frac{|\xi|^2}{2}\right) \exp(\hat{b}^\dagger \xi)|0\rangle,\tag{13}$$

with the vacuum state $|0\rangle \equiv \sqrt{\frac{\omega}{\pi}} \exp(-\frac{\omega+i\eta}{4}|z|^2)$ annihilated by $\hat{b}$:

$$\hat{b}|0\rangle = 0.\tag{14}$$

From eqs.(10)-(12) we see that the eigenvalue ξ is related to the position eigenvalue z by:

$$\xi = \frac{\omega - i\eta}{\sqrt{2\omega}} \bar{z}\tag{15}$$

We want to evaluate $\Psi_{t \rightarrow \infty}^\eta(z)$ (see eq.(9)) in the case in which the initial wave function $\Psi_j(z_0)$ describing the electron system with no dissipation is given by the LLL state with orbital angular momentum j :

$$\Psi_j(z_0) = \sqrt{\frac{\omega}{\pi}} \frac{1}{\sqrt{j!}} (z_0 \sqrt{\frac{\omega}{2}})^j \exp(-\frac{\omega}{4}|z_0|^2)\tag{16}$$

In order to do so we must notice that the large time-scale evolution defined in eq.(9) can make sense only if we require that the zero angular momentum state ψ_0 is annihilated by $\hat{b}$. That is, when we turn on the dissipation at $t = 0$ the zero angular momentum state must acquire a space dependent phase factor $\exp(i\eta|z_0|^2/4)$ becoming:

$$\psi_0^\eta(z_0) = \sqrt{\frac{\omega}{\pi}} \exp\left(-\frac{\omega + i\eta}{4}|z_0|^2\right)\tag{17}$$

Then we immediately get for the evolution of the state given by eq.(17), in the coherent state representation:

$$\psi_{0,t \rightarrow \infty}^\eta(\xi) = \exp\left(-\frac{|\xi|^2}{2}\right) \langle 0 | \exp(\hat{b} \bar{\xi}) | 0 \rangle = \exp\left(-\frac{|\xi|^2}{2}\right).\tag{18}$$

By expressing ξ in terms of the coordinate operator eigenvalue z (see eq.(15)) we get:

$$\psi_{t \rightarrow \infty}^\eta(z) = N \exp\left(-\frac{\omega^2 + \eta^2}{4\omega}|z|^2\right) \quad (19)$$

where $N = \sqrt{\frac{\omega^2 + \eta^2}{\pi\omega}}$ is an overall normalization factor and for a generic “LLL state” ψ_j^η

$$\psi_j^\eta = \sqrt{\frac{\omega}{\pi}} \frac{1}{\sqrt{j!}} \left(\sqrt{\frac{\omega}{2}} z_0\right)^j \exp\left(-\frac{\omega + i\eta}{4}|z_0|^2\right) \quad (20)$$

it is straightforward to see that:

$$\psi_{j,t \rightarrow \infty}^\eta(z) = \sqrt{\frac{\omega^2 + \eta^2}{\pi\omega}} \frac{1}{\sqrt{j!}} \left(\frac{\omega - i\eta}{\sqrt{2\omega}} z\right)^j \exp\left(-\frac{\eta^2 + \omega^2}{4\omega}|z|^2\right), \quad (21)$$

having used in the intermediate step the identity:

$$|\psi_j^\eta\rangle = \frac{(\hat{b}^\dagger)^j}{\sqrt{j!}} |0\rangle \quad (22)$$

and the commutation relation between $\hat{b}$ and $\hat{b}^\dagger$ (see eq.(11)).

We will comment extensively on the physical interpretation of the result in eq.(21) in the following sections. We just observe here that the initial gaussian (see eq. (30)) describing the electron has shrunk as a consequence of dissipation. That is, the electron appears to be more localized in a dispersive medium.

In section 3 a consequence of this phenomenon will be described by discussing the transport properties of the system.

In order to do so let us also notice that by comparing eq.(20) with eq.(21) we see that the most relevant feature of this evolution is that the effect of dissipation simply causes the multiplication of the z “coordinate” by a factor:

$$\lambda \equiv \frac{\omega - i\eta}{\omega}, \quad (23)$$

that is:

$$z \rightarrow \frac{\omega - i\eta}{\omega} z. \quad (24)$$

More explicitly the transformation indicated in eq.(24) can be written as a transformation composite of a scale one and a rotation:

$$z \rightarrow \frac{\omega - i\eta}{\omega} z = \rho \exp(i\phi) z \quad (25)$$

where

$$\rho = \left(\frac{\eta^2 + \omega^2}{\omega^2}\right)^{\frac{1}{2}} \quad (26)$$

is the scale factor, and

$$\phi = -\text{atn}\left(\frac{\eta}{\omega}\right) \quad (27)$$

is the rotation angle. Let us observe that the effect of dissipation expressed in eq.(24) found for the special case of the symmetric gauge (Corbino disk) is valid for any gauge (in particular for the asymmetric one (cylinder)) if z is replaced by the coherent state coordinate ξ [7].

From eq.(21) we can formally define the evolution operator $U(\eta, \omega)$, which transforms initial states into asymptotic ones:

$$\Psi_{t \rightarrow \infty}^\eta = U(\eta, \omega) \Psi_{t=0}, \quad (28)$$

where $U(\eta, \omega) \equiv \lim_{t \rightarrow \infty} U(t, 0)$.

Consequently, if one considers the evolution of a scalar operator $O_0(z_0)$, acting on the initial wave function, $\Psi_0(z_0)$, it is simply given by:

$$O_{t \rightarrow \infty}(z) \equiv U(\eta, \omega) O_0(z_0) U^\dagger(\eta, \omega) = O_0\left(\frac{\omega - i\eta}{\omega} z\right), \quad (29)$$

while, for a vector operator $\vec{O}$, one has the transformation properties

$$\begin{aligned} O_{x,t \rightarrow \infty}(z) &= \frac{\partial x}{\partial x_0} O_{x,0} + \frac{\partial x}{\partial y_0} O_{y,0} = \\ &= \frac{\omega^2}{\eta^2 + \omega^2} O_{x,0} - \frac{\eta\omega}{\eta^2 + \omega^2} O_{y,0} \\ O_{y,t \rightarrow \infty}(z) &= \frac{\eta\omega}{\eta^2 + \omega^2} O_{x,0} + \frac{\omega^2}{\eta^2 + \omega^2} O_{y,0}. \end{aligned} \quad (30)$$

In the appendix the above transformation properties will be explicitly shown for the basic operators $\hat{z}$, $\hat{\bar{z}}$, $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial \bar{x}}$, when projected on the LLL states. Then we will construct the large time scale evolution by introducing the propagator $G(z, z_0)$ through the relation

$$\psi_{t \rightarrow \infty}^\eta = {}_{t \rightarrow \infty} \langle z(t) | \psi_0 \rangle = \int d^2 z_0 G(z, z_0) \psi_0(z_0),$$

where

$$G(z, z_0) = {}_{t \rightarrow \infty} \langle z(t) | z_0 \rangle$$

and the unity partition

$$I = \int d^2 z_0 |z_0\rangle \langle z_0|$$

has been inserted.

The introduction of $G(z, z_0)$ will turn out to be very useful in extending our formalism to the torus case [7].

3 Localization and transport properties

In this section we will give a physical interpretation of the shrinking of the gaussian by the scale factor ρ (see eq.(26)) and of the rotation by the angle ϕ (see eq.(27)), due to the presence of dissipation, by analyzing the conduction properties of our system. To this purpose, let us consider an electron moving on a cylinder of radius $\frac{L}{2\pi}$, with a constant magnetic field of strength ω normal to the surface. More precisely, let the electron be in the Landau state:

$$\Psi^k(x_0, y_0) = \frac{1}{\sqrt{\pi L \omega^{1/4}}} \exp(i \frac{2\pi k x_0}{L} - \frac{\omega}{2} (y_0 - \frac{2\pi k}{L\omega})^2), \quad (31)$$

where k is an integer.

It is well known that the expectation value of the current operator is given by:

$$\vec{J} = \langle \vec{J} \rangle = \int d^2 r_0 [\frac{1}{2i} (\Psi_E^{*k} \vec{\nabla} \Psi_E^k - \Psi_E^k \vec{\nabla} \Psi_E^{*k}) - \omega y \hat{z} |\Psi_E^k|^2] \quad (32)$$

where $\vec{E}$ is an external electric field $\vec{E} = \hat{y} E$ and the wave functions modified by the presence of the electric field are given by:

$$\Psi_E^k = \frac{1}{\sqrt{\pi L \omega^{1/4}}} \exp \left[i \frac{2\pi x_0 k}{L} - \frac{\omega}{2} (y_0 - \frac{2\pi k}{L\omega} + \frac{E}{\omega^2})^2 \right]. \quad (33)$$

As a result one gets:

$$J_L = J_y = 0; \quad J_H = J_x = -\frac{E}{\omega}. \quad (34)$$

From eq.(34) one can see that the value of σ^H is:

$$\sigma^H = \frac{1}{\omega} \quad (35)$$

which can be rewritten as:

$$\sigma^H = n^H \tau^H, \quad (36)$$

where $n^H (= 1)$ is the density of Hall states and $\tau^H (= \omega^{-1})$ is a characteristic time (the cyclotron period).

Let us now turn on dissipation and consider its effect on the expectation value of the current operator $\vec{J}$ in eq.(32). By applying the transformation properties in eq.(30) to $\vec{J}$, after expressing $\vec{J}$ in terms of the coherent coordinate ξ , and using the transformed wave functions in the coherent representation [7], we obtain for the expectation value in eq.(32), in the presence of η :

$$J_x^\eta = -\frac{\omega}{\eta^2 + \omega^2} E; \quad J_y^\eta = \frac{\eta}{\eta^2 + \omega^2} E. \quad (37)$$

As a result we get that σ^H is now given by:

$$\sigma_{\eta \neq 0}^H = \frac{\omega}{\eta^2 + \omega^2} = \frac{\omega^2}{\eta^2 + \omega^2} \sigma_{\eta=0}^H, \quad (38)$$

and that the ohmic conductance σ^L is now different than zero and given by:

$$\sigma_{B \neq 0}^L = \frac{\eta}{\eta^2 + \omega^2} = \frac{\eta^2}{\eta^2 + \omega^2} \sigma_{B=0}^L \quad (39)$$

where $\sigma_{\omega=0}^L (= \eta^{-1})$ denotes the diffusive contribution to the conduction and η^{-1} is the characteristic time of collision.

Looking at eq.(38) we immediately notice that the number of Hall states (equal to 1 in the case $\eta = 0$, see eqs.(35),(36)), has gotten reduced by a factor $\frac{\omega^2}{\eta^2 + \omega^2}$ ($= \rho^{-2}$) which is just the shrinking factor in the gaussian, appearing in the transformed Landau states (eq.(21)).

On the other side, the number of the Hall states disappeared

$$1 - n_\eta^H = 1 - \frac{\omega^2}{\eta^2 + \omega^2} = \frac{\eta^2}{\eta^2 + \omega^2}, \quad (40)$$

is just the number of ohmic conduction states which appear in eq.(39).

It is interesting to notice that the ratio of the absolute values of the longitudinal current J_y^η to the Hall current J_x^η is related to the rotation angle $\phi(\eta, \omega)$ (see eq.(27)) by:

$$\left| \frac{J_y^\eta}{J_x^\eta} \right| = \frac{n_L^\eta \tau_L}{n_H^\eta \tau_H} = \frac{\eta}{\omega} = |\tan \phi|; \quad (41)$$

then, the parameters ρ, ϕ appearing in eqs.(26),(27) are directly linked to physical observable quantities given by eqs.(38),(39),(41).

In ref.[7] the wave functions on the infinite cylinder (torus), in the presence of dissipation, will be given explicitly, and the results expressed in eqs.(38)(39)(41) reproduced.

4 Localization effect of the dissipative dynamics

One of the most relevant features of the introduction of dissipation in a quantum system is the reduction of the spread of the wave function, as shown, for the free particle, in ref.[6]. Our results are, as it can be seen from the form of the asymptotic wave functions (eq.(21)), in agreement with this statement. Infact, by taking for the electron state the simple Gaussian:

$$\psi_0(z_0) = \sqrt{\frac{\omega}{\pi}} \exp\left(-\frac{\omega}{4}|z_0|^2\right), \quad (42)$$

we obtain the asymptotic state

$$\psi_{0,t \rightarrow \infty}(z) = \sqrt{\frac{\eta^2 + \omega^2}{\pi \omega}} \exp\left(-\frac{\eta^2 + \omega^2}{4\omega}|z|^2\right). \quad (43)$$

As it can be seen from eqs.(42),(43) the factor $\sigma_0 = \omega^{-1}$, which measures the spreading at $t = 0$, gets reduced to, for $t \gg \eta^{-1}$:

$$\sigma_\infty = \frac{\omega}{\omega^2 + \eta^2}. \quad (44)$$

We may refer to this phenomenon as localization in the sense that the indetermination in the position of the particle is reduced, as an effect of dissipation.

Hystorically the concept of localization was first introduced by Anderson in ref.[8], where the randomness is the essential element to ensure localization that is due to impurities which act as scattering centers and are disorderly distributed on a 2d array.

The interaction potential allows electrons (or spins) to have a probability different than zero to tunnel from one site of the lattice to another.

In his model Anderson considers a form of "tight binding approximation" which uses for the electron wave function a combination of site orbitals. If the immaginary part of the self-energy for an arbitrary site is zero with probability one, the state is localized, that is, the wave function falls off exponentially away from the center of localization. A consequence of this result is that the transport properties of the system are modified. In particular, the Ohmic conductance gets reduced by the presence of electron impurity bound states.

It is instructive, now, to compare results of ref.[8] concerning the effect of localization on the conduction properties of the system with the analysis of section 3.

As it can be seen from eqs.(38),(39), the conduction properties of our system are deeply modified by the presence of η . On one side, infact, the Hall conductance σ^H gets reduced by a factor $\frac{\omega^2}{(\eta^2 + \omega^2)}$:

$$\sigma_{t=0}^H = \frac{1}{\omega} \rightarrow \sigma_{t \rightarrow \infty}^H = \frac{\omega}{\omega^2 + \eta^2}; \quad (45)$$

on the other side a process of Ohmic conduction is generated with conductance:

$$\sigma_{t \rightarrow \infty}^L = \frac{\eta}{\omega^2 + \eta^2}. \quad (46)$$

From eqs.(45),(46) we obtain, as seen in section 3, the asymptotic density of Hall and longitudinal conduction states:

$$n_{t \rightarrow \infty}^H = \frac{\omega^2}{\omega^2 + \eta^2}; \quad n_{t \rightarrow \infty}^L = \frac{\eta^2}{\omega^2 + \eta^2}. \quad (47)$$

We notice from eq.(47) that the total density of states:

$$n^T \equiv n^H + n^L \quad (48)$$

remains fixed to its initial value:

$$n^T = \omega |\sigma_0^H| = 1 \quad (49)$$

independently of the value of η . The physical meaning of these results is that, differently than in Anderson case, the localization induced by dissipation doesn't cause a lost of conduction states. Infact all the states not anymore available for the Hall (transverse) conduction are now converted into states which support Ohmic transport, absent in the case $\eta = 0$.

The reason why localization in our case is different than in Anderson picture is the different nature of the states on which localization acts. Infact, if the diffusive transport is reduced by localization of the Bloch wave function around its maxima (minima of the periodic potential) which reduces the tunneling between them, for a LLL state localization reduces the spreading in the direction normal to the Hall current.

On the other hand the introduction of a uniform dissipation representing a sort of average of the random distribution of impurities could account for the difference.

Thus the conduction properties of the Landau states in the presence of dissipation for our 2D system can be explained in terms of a scale transformation and a rotation, as described in sec.(2). In this simple way it is possible to understand the conservation of the total number of conduction states n^T (eq.(48)), which we believe is due to the two-dimensionality of our system as it is clearly expressed by eqs.(38),(39) for σ^H , σ^L .

Acknowledgments

We would like to thank V. Marigliano Ramaglia, A. Tagliacozzo, G. Zucchelli of the group Struttura della Materia of our department and R. Musto, F. Nicodemi for useful discussions.

Appendix A

Large time scale evolution of the operators $\hat{z}$, $\hat{\bar{z}}$, ∂_z , $\partial_{\bar{z}}$.

In this appendix we give the transformation properties of the canonical operators z , $\bar{z}$, ∂_z , $\partial_{\bar{z}}$ projected on the LLL states, by using the “propagator” $G(z, z_0)$ defined by:

$$\psi_{j,t \rightarrow \infty}^\eta(z) = \int d^2 z_0 G(z, z_0) \psi_j^\eta(z_0) \quad (\text{A.1})$$

where:

$$G(z, z_0) \equiv {}_{t \rightarrow \infty} \langle z(t) | z_0 \rangle = \Omega \exp \left[-\frac{\omega - i\eta}{4} |z_0|^2 + \frac{\omega - i\eta}{2\omega} \bar{z}_0 z - \frac{\eta^2 + \omega^2}{4\omega} |z|^2 \right] \quad (\text{A.2})$$

with $\Omega = \sqrt{\eta^2 + \omega^2}$.

By using G it is possible to transform the action of the canonical operators on the initial wave functions as it will be shown below. Explicitly, if $\hat{z}_\infty$ is the operator $\hat{z}(t)$ at $t \rightarrow \infty$, we get the following relation between matrix elements evaluated on the LLL states:

$$\langle \psi_{j,t \rightarrow \infty}^\eta | \hat{z}_\infty | \psi_{k,t \rightarrow \infty}^\eta \rangle = \langle \psi_j^\eta | \hat{z}_0 | \psi_k^\eta \rangle \quad (\text{A.3})$$

We can write the left-hand side of eq.(A.3) by “propagating” $\hat{z}_0 | \psi_k^\eta \rangle$:

$$\langle \psi_{j,t \rightarrow \infty}^\eta | \hat{z}_\infty | \psi_{k,t \rightarrow \infty}^\eta \rangle = \langle \psi_{j,t \rightarrow \infty}^\eta | \int d^2 z_0 G(z, z_0) (\hat{z}_0 \psi_k^\eta) \rangle \quad (\text{A.4})$$

From this equation we obtain:

$$\langle \psi_{j,t \rightarrow \infty}^\eta | \hat{z}_\infty | \psi_{k,t \rightarrow \infty}^\eta \rangle = \sqrt{\frac{2(k+1)}{\omega}} \delta_{j,k+1}. \quad (\text{A.5})$$

Recalling the explicit representation of the asymptotic wave functions (eq.(21)) and, by simply requiring that:

$$\int d^2 z_0 G(z, z_0) (\hat{z}_0 \psi_k^\eta(z_0)) = \lambda z \psi_{k,t \rightarrow \infty}^\eta(z) \quad (\text{A.6})$$

we get, from eq.(A.5), the result:

$$\lambda = \frac{\omega - i\eta}{\omega} \quad (\text{A.7})$$

which reproduces eq.(23).

In the same way we can find the transformation properties of the operator $\hat{z}^\dagger$ obtaining:

$$\hat{z}^\dagger \rightarrow \frac{\omega + i\eta}{\omega} \hat{z}^\dagger \quad (\text{A.8})$$

which agrees with the result in eq.(24).

Finally, one can show that the derivative operators ∂_z , $\partial_{\bar{z}}$ transform in a way compatible with the previous results:

$$\begin{aligned} \frac{\partial}{\partial z} &\rightarrow \frac{\omega}{\omega - i\eta} \frac{\partial}{\partial z}, \\ \frac{\partial}{\partial \bar{z}} &\rightarrow \frac{\omega}{\omega + i\eta} \frac{\partial}{\partial \bar{z}}. \end{aligned} \quad (\text{A.9})$$

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