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**X-RAY SPECTROSCOPY OF HIGHLY-CHARGED IONS
IN A STORAGE RING**

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X-RAY SPECTROSCOPY OF HIGHLY-CHARGED IONS IN A STORAGE RING

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1 INTRODUCTION

The purpose of the present lectures is to carry through the methods and procedures necessary for a meaningful spectroscopy of the heaviest few-electron ions in relation to present theories. Results achieved so far in accelerator-based x-ray experiments are highlighted with emphasis on recent developments on heavy-ion storage rings. Starting with a brief account of the basics of one-electron ions, the motivation for doing x-ray spectroscopy of the simplest atomic systems with a high nuclear charge is given.

In section 2 x-ray instrumentation and techniques are discussed including the precautions necessary when dealing with fast-beam sources. Peculiarities of heavy-ion storage rings are investigated in section 3 with regard to their use for spectroscopy. In section 4 are discussed results obtained so far on the measurement of the Lamb shift in very heavy ions. Section 5 gives some perspectives for the near future.

1.1 Motivation for the Spectroscopy of One- and Few-Electron Ions of High Nuclear Charge

Figure 1: Visible section of the atomic hydrogen spectrum showing the Balmer series. The H_α line has a detailed fine structure as shown in the insets above and below the main spectrum. The difference in resolution is evident when going from Doppler-limited to Doppler-free LASER spectroscopy. The spectra are reproduced with permission from Williams 1938 [2] and from Hänsch *et al.* 1972 [3] Copyright (1972) Macmillan Magazines Limited.

Historically, it was the study of x rays which led to the discovery of the shell structure of atoms. On the other hand, the wish to understand the spectra of the simplest atomic system, namely atomic hydrogen, has led to the development of quantum mechanics at the beginning of this century. For example, Fig. 1 shows the visible portion of the hydrogen spectrum with the most intense (red) H_α line at 6563 Å which was discovered by Ångström in 1853. In 1885 Balmer [1] found that the wavelengths of the members of this series can be described by the formula

$$\lambda = \frac{n_1^2}{n_1^2 - 4} G \quad (1)$$

where G is a constant. This constant can be directly related to the Rydberg energy, $1 \text{ Ry} = 13.606 \text{ eV}$, appearing in Bohr's model [4] of the hydrogen atom. In this model, the energy levels of the hydrogen atom or of a hydrogenlike ion is given by

$$E_n = -\frac{Z^2}{n^2} \text{Ry} \quad (2)$$

where Z is the charge of the nucleus.

The finestructure of the $H\alpha$ line [2, 3], also shown in Fig. 1, is not explained by Eq. (2). In 1916 Sommerfeld [5], by applying special relativity to Bohr's postulates, showed that electrons of the same principal quantum number n but with different angular momentum j are no longer degenerate in energy. Sommerfeld's formula agrees (up to terms proportional to Z^4) with the solution of the (relativistic) Dirac [6] equation for an electron in a Coulomb potential resulting in the following energies of the bound states

$$E_{nj} = -\frac{2Z^2}{N(N + n + \sqrt{(j + 1/2)^2 - (\alpha Z)^2})} \text{Ry}$$

with $N = \sqrt{n^2 - 2(n - j - 1/2)(j + 1/2 - \sqrt{(j + 1/2)^2 - (\alpha Z)^2})}$ (3)

The Dirac energies are assumed to describe, exactly, the electron binding energies of a one-electron atom. However, in 1947 Lamb and Retherford [7] demonstrated, in a series of their famous experiments, that the degeneracy of levels with the same principal and total angular-momentum quantum number j , as for the $2s_{1/2}$ and $2p_{1/2}$ states, is not exactly fulfilled. The discrepancy gave important impact for the development of Quantum Electrodynamics (QED) which at the time being is considered as the most precise theory in physics.

Figure 2: Level scheme for hydrogenlike U^{91+} according to the various atomic models.

A scheme of the low-lying states in one-electron ions is shown in Fig. 2. The binding energies are those for U^{91+} . For electrons bound in an atom the QED corrections to the binding energies can be written as

$$S = \frac{\alpha}{\pi} \frac{(Z\alpha)^4}{n^3} F(\alpha Z) mc^2 \quad (4)$$

where $F(\alpha Z)$ is a function slowly varying with Z . Eq. (4) shows that the size of the corrections increases rapidly with the nuclear charge of the atom and becomes most important for the inner shells of heavy atoms.

Table I: Relative contributions to the 1s Lamb shift of one-electron ions with nuclear charges between 1 and 92. The data are derived from the tabulation of Johnson and Soff [8]. The headings denote (SE) self energy, (VP) vacuum polarization, (FS) finite-nuclear-size correction and (small) small contributions mainly due to higher-order and relativistic-recoil corrections.

Z	SE	VP	FS	small
1	102.7%	-2.6%	0.013%	0.003%
36	108.7%	-11.6%	2.5%	0.095%
54	108.3%	-15.3%	6.8%	0.120%
92	78.4%	-20.4%	41.8%	0.094%

Table I details the major contributions [8] to the 1s Lamb shift for a number of nuclear charges between 1 and 92. Here and in the following we will use the pragmatic definition of the Lamb shift LS as the difference between the true binding energy and the Dirac eigenvalue for a point-like nucleus. The electron self energy SE is the largest contribution for bound electrons followed by the vacuum polarization VP. The correction due to the finite size of the nuclear charge is not of QED origin but is an important correction especially for heavy atoms. The last column summarizes all small contributions containing higher-order, relativistic-recoil and reduced-mass corrections.

Although it would be desirable to experimentally test the small contributions, there is already interesting structure within the largest contribution, the electron self energy. The electron self energy ΔE_{SE} can be written [8, 9] as

$$\begin{aligned} \Delta E_{SE} = & \frac{4\alpha(Z\alpha)^4 m_e c^2}{3\pi n^3} \left(L_n + \frac{3}{8} \frac{C_{\ell j}}{(2\ell+1)} \right. \\ & + \left(\ln \frac{1}{(Z\alpha)^2} + \frac{11}{24} \right) \delta_{\ell 0} + C_{50}(Z\alpha) + (Z\alpha)^2 \\ & \left. \times \left(C_{61} \ln \frac{1}{(Z\alpha)^2} + C_{62} \ln^2 \frac{1}{(Z\alpha)^2} + G(Z\alpha) \right) \right) \end{aligned} \quad (5)$$

The function $G(Z\alpha)$ may be regarded as the most critical one. As can be observed in Fig. 3 this term contributes only very little at $Z = 1$, therefore it is very difficult to test it in hydrogen even with the most precise techniques available. However, because of the strong increase with the nuclear charge it might be possible to make a meaningful comparison with theory at very high Z already at moderate measuring precision.

In a pure hydrogenlike system the binding energies are given as a simple sum of the analytical Dirac energies and the QED corrections. Therefore, it is conceptually easy to extract the latter out of measured binding energies or out of measured energy intervals. For two- and more-electron atoms exact analytical solutions are no longer available and because of the many terms contributing the effort for precise theoretical calculations is enormous. Theory has to account not only for the relativistic Coulomb interaction

Figure 3: The relative contribution of the term $G(Z\alpha)$, according to Eq. (6), to the self energy as a function of the atomic number.

between electrons but also for the two- or many-electron QED effects which, in lowest order, are the screening of the self energy and the screening of the vacuum polarization. Despite those difficulties theory has recently made progress towards precise calculations for two-electron atoms [10, 11, 12, 13] and even for the many-electron problem [14].

Table II: Approximate scaling with Z of the radiative decay rates for some of the low-lying states and the corresponding level widths in U^{91+}

state	scaling	level width
$1s_{1/2}$		0
$2s_{1/2}$	$\propto Z^6$ (2E1) $\propto Z^{10}$ (M1)	0.13 eV
$2p_{1/2}$	$\propto Z^4$ (E1)	30 eV

Though it seems advantageous to perform spectroscopic measurements on the innermost states of very heavy one- and few-electron ions there is a quite high price to pay. The traditional $2s_{1/2}$ - $2p_{1/2}$ Lamb shift interval in hydrogenlike ions is no longer accessible with lasers at high nuclear charge Z and such measurements are confined to ions below $Z \approx 20$ (see e. g. [15, 16]). Also the rates for radiative decay of the atomic states drastically increase with the atomic number. Table II gives the approximate scaling [17], with Z , of the corresponding level widths for some of the interesting states along with numerical data for U^{91+} . In U^{91+} the natural width of the 2p level for instance, of approximately 30 eV, is already a substantial fraction of the Lamb-shift interval amounting to only 75 eV. The 1s Lamb shift is about a factor of eight larger. However, it is only a small fraction of the 2p-1s transition energy that can be measured implying an accordingly high spectroscopic precision required.

In the following we will be concerned with the question of how one can succeed in the required x-ray spectroscopy of the heaviest ions. One aspect is how to produce

the ions in the desired charge state. Whereas fully stripped ions of intermediate Z are available from a rather large variety of different sources such as Tokamak, Laser-produced plasmas, Van-de-Graaff accelerators etc., there are only few places providing fully stripped ions up to bare uranium. Bare uranium has recently become available at the super EBIT at Livermore and this subject will be reviewed by D. Knapp at this summer school. Another possibility is the Experiment Storage Ring (ESR) within the heavy-ion accelerator facility at GSI Darmstadt on which I will concentrate. Once the ions are available, the next question is how to adapt suitable x-ray techniques to the peculiarities of the source. This usually comprises several steps which will be discussed next.

2 X-RAY INSTRUMENTATION AND SPECIAL TECHNIQUES

The interesting x-ray energy range is set by the ions nuclear charge and the relevant electronic transitions. The latter may include those from initial states near the continuum limit into the final K shell. If the radiation is emitted from an ion moving with a high velocity the energy observed in the laboratory can easily change by a factor of two depending on the angle of observation. Typical laboratory x-ray energies may range from a few keV for ions in the low- Z range up to ~ 200 keV for hydrogenlike uranium. For ultimate spectroscopic precision one might think of a double flat-crystal spectrometer as an ideal instrument. Such an instrument often referred to as ‘direct reading’ instrument reduces the wavelength measurement to an angular measurement provided the interplanar spacings of the crystals are known independently. Because of its small efficiency, however, the double flat-crystal instrument can not be directly used at the quite weak x-ray source represented by the highly-charged ions from an accelerator.

Instead, one has to divide up the measurement procedure into an off-line task, in which a suitable wavelength reference is established, and into an on-line task in which the x-ray line under study is compared with the reference. In the first part one can use strong x-ray and gamma-ray sources in combination with precision instruments of low throughput. Such an experimental program was carried out at NIST by Deslattes and coworkers [18] resulting in a number of precise secondary gamma-ray standards linked to the optical region of the electromagnetic spectrum. For the second part wavelength comparators are needed which do a wavelength interpolation with a rather high efficiency. Ideally, the wavelength difference between the reference line and the line to be measured is very small relaxing the requirements on the comparator.

As wavelength comparators may serve crystal-diffraction spectrometers [19] and non-dispersive instruments such as solid-state radiation detectors [20]. For comparison with theory, experiment has to provide its results on a wavelength or energy scale accessible to theory. Therefore, we will also have to discuss (see section 2.3) the standardization procedure necessary for an absolute wavelength scale in the x-ray region. The necessary reference wavelength may come from different kinds of measurements:

- *Direct* measurements of x-ray or (preferably) gamma-ray lines with high-resolution double-diffraction instruments employing crystal specimens that have been characterized by combined x-ray and optical interferometry.
- Wavelength *ratios* measured with high-resolution crystal spectrometers.
- Wavelength *differences* to previously established gamma-ray standards measured with curved-crystal optics or even with solid-state detectors.

2.1 Dispersive Instruments

2.1.1 Wavelength Measurements with Flat Crystals

Flat crystals can be used either individually or as pairs as schematically shown in Fig. 4 for transmission and reflection type of geometry. The first crystal serves as a kind of collimator which, if omitted, has to be replaced by a mechanical collimation. The wavelength determination is based on the Bragg-Laue relation

$$n\lambda = 2d \sin(\theta) \quad (6)$$

with d denoting the crystals interplanar spacing used, θ is the Bragg angle measured relative to the crystal planes and n denotes the order of diffraction. For precise work the instruments have to be equipped with selfcalibrating angular measuring devices making use of the symmetry as indicated in Fig. 4 by a 'left' and 'right' arrangement. In the Bragg case, an important index-of-refraction correction has to be applied to the interplanar spacing.

Figure 4: Double flat-crystal geometries with surface reflection on both crystals and with transmission through both crystals. The x-ray source is indicated by S and the detector by D; after Deslattes and Kessler [19].

2.1.2 Focusing Geometries

It seems natural to use curved and even ground crystals to increase the detection efficiency. As an example, the Johann and the Cauchois geometry are illustrated in Fig. 5. In the Rowland-circle plane, they provide a focus using a point or extended source, respectively. For the transmission-type of spectrometer, source and detector can be interchanged to realize the Dumond geometry.

Figure 5: Curved crystal geometries for (left) a Cauchois geometry in transmission and for (right) a Johann geometry in reflection.

Indeed, these instruments have a much higher efficiency than flat crystals. However, they are not very useful for absolute wavelength measurements. They are merely used as wavelength comparators requiring wavelength markers for calibration. If the curved-crystal instrument and its geometrical aberrations are well characterized it is a very suitable instrument for comparison of nearly degenerate lines. In the latter case it usually is sufficient to perform a single-sided measurement. In some applications the Johann [21] or Johansson [22] geometry, originally designed as a monochromator, is used with a position-sensitive x-ray detector covering the whole spectral region of interest thus avoiding mechanical step-scanning.

2.2 Non-Dispersive Instruments

2.2.1 Gaseous Detectors

Figure 6: The principle of a standard proportional counter.

The earliest non-dispersive detector is the proportional counter which is widely used in x-ray work as a sensitive detector at moderate energy resolution. The main features

are recalled in Fig. 6. It is based on the fact that the number of ion-electron pairs produced in a gas as a result of absorption of an x-ray photon is proportional to the incident photon energy.

Figure 7: The x-ray conversion efficiency in a xenon-filled proportional counter as a function of photon energy. The example is for a 1 cm stopping region and a gas pressure of 1 and 5 bar, respectively.

The usual way of operating the proportional counter is to make use of a highly non-uniform field in which ‘avalanching’ occurs when electrons are accelerated in the high fields in the vicinity of the anode wire carrying a positive voltage of typically 1 to 3 kV in a gas of atmospheric pressure. In the avalanche process an exponential growth of the number $n(x)$ of free electrons along a path x occurs according to

$$n(x) = n_0 e^{\alpha x} \quad (7)$$

where the Townsend coefficient α can be identified by the probability of an electron to acquire sufficient kinetic energy within its free path L in order to release another electron through a collision with a gas atom with ionization potential V_I :

$$\alpha = \frac{e^{-\frac{V_I}{U}}}{L} \quad (8)$$

where U denotes the effective detector potential. Noting that the mean free path of the electron is inversely proportional to the gas pressure p the gain factor $G = e^{\alpha x}$ may be written as

$$G = \exp \left(C_1 p e^{-C_2 p/U} \right) \quad (9)$$

representing a very sensitive function of the pressure-to-voltage ratio. In Eq. (9) C_1 and C_2 are constants.

The counter gas, usually argon or xenon, must contain a quenching-gas admixture consisting of typically 5-10% methane or carbon dioxide. The quenching gas absorbs ultraviolet photons, produced in the avalanche and at the detector wall, which otherwise would initiate a second pulse. In Fig. 7 is shown the efficiency of a proportional counter filled with xenon. The conversion efficiency is governed by the photo-electric absorption revealing the typical edge structure.

The energy resolution of a proportional counter is largely determined by statistical fluctuations in both the primary number of ion-electron pairs and in the avalanche. If ϵ denotes the ion-electron pair creation energy, the FWHM energy resolution is given by

$$\frac{\Delta E}{E} = 2.36 \{ \epsilon (F + f)/E \}^{1/2} = A E^{-1/2} \quad (10)$$

where $F \approx 0.4$ is the Fano factor [23] and f is the avalanche variance. The Fano factor F is different from one because the statistics of the electron-ion pair creation does not show a Poisson distribution. Typical values for the constant A are near $0.2 \text{ keV}^{1/2}$ or slightly less. While the energy resolution of the proportional counter is not sufficient for spectroscopy, it is often used in combination with crystal spectrometers. There, the resolution of the proportional counter is sufficient to discriminate between different diffraction orders.

A renaissance of gaseous detectors has been initiated by the fabrication of position-sensitive multi-wire proportional counters (MPWC) which are widely used for x-ray and particle detection. A further impact comes from the possibilities offered by the micro-electronics industry and by the recent improvement in detector fabrication technology. The latter has led to the development of the micro-strip gas counter (MSGC) [24] and, very recently, of the micro-gap counter (MGC) [25].

Two-dimensional position resolution is required for x-ray detectors used inside focusing x-ray optics if one wants to take full advantage of the increased light-collection capabilities of those instruments. Working with large crystals at wide acceptance angles results in a two-dimensional pattern which, if unresolved, would lead to a deterioration of the spectral resolution. Such effects are even much increased when x rays from a fast-moving ion beam are observed.

Figure 8: The curved spectral line of $\text{Cu K}\alpha_1$ radiation recorded in a backgammon-type proportional counter mounted in a Johann spectrometer.

A classical example for the need of position-sensitive detectors is the known curvature of the spectral lines due to geometrical aberrations. As an example, Fig. 8 shows the curved spectral line of $\text{Cu K}\alpha_1$ radiation measured with a crystal spectrometer in Johann geometry equipped with a position sensitive proportional counter [26]. This counter was developed at NIST [27, 28, 29] and uses a split zig-zag shaped cathode [30] for charge division. Fig. 9 shows the basic features of the detector. The position information along the direction of spectral dispersion is derived from the quantity $Q_R/(Q_L + Q_R)$ where Q_R and Q_L denote the image charge induced on the right and on the left side of the cathode. This can only work because, for each x-ray event, the charge is spread out inducing a fraction on the left and another fraction on the right side. With this kind of detector, FWHM position resolutions of $\sim 100\mu\text{m}$ have been obtained. In the transverse direction there was no need for a high position resolution. It was derived from the meandering anode wire by charge division identifying only what step of the wire is hit by the x ray. There are other schemes of cathode patterns for position sensing like a two-dimensional backgammon or strip [31] cathodes.

Figure 9: Sketch of a backgammon detector. The zig-zag shaped cathode is used for charge division. Reproduced with permission from reference [28].

Figure 10: Field and equipotential lines for a multi-wire proportional counter (MWPC), for a micro-strip gas counter (MSGC) and for a micro-gap chamber (MGC). Reproduced with permission from Ref. [25].

New designs make use of a much increased electric-field strength as illustrated in Fig. 10 comparing the distributions of the electric field in the MWPC, the MSGC and the MGC. The high field strength is obtained by micro-structuring technologies by which very small anode-cathode separations become possible. In particular, the MGC [25] might be thought of as a miniaturized version of a multi-wire detector. With this de-

sign, the avalanching is concentrated spatially to a very small area making the detector much faster and achieving a very good position resolution. This development has just started but one can foresee the benefits for the use in crystal-diffraction spectrometry and for alternative methods envisaged in the spectroscopy of fast ion beams.

2.2.2 Semiconductor Detectors

The principle of operation of silicon or germanium solid-state detectors (see Fig. 11) may be understood in close analogy to that of the proportional counter. The energy required to create an electron-hole pair in a semiconductor is roughly ten times smaller than the electron-ion pair creation energy in a gas resulting in a much better energy resolution. In order to minimize the leakage current in the detector, when no radiation is striking it, a reversed biased diode is always used operated at the temperature of liquid nitrogen (77 K). For use at higher photon energies high-purity germanium detectors, also referred to as intrinsic germanium detectors, Ge(i), are preferred because of their higher atomic number. Large depletion volumes are achieved in the detector material having a concentration of the order of one impurity atom per 10^{13} germanium atoms.

Figure 11: Schematic diagram of a p-n junction semiconductor used as radiation detector.

Besides the statistics of the charge-creation process the charge-collection efficiency and electronic noise contribute to the line width. Electronic noise is minimized by employing a charge-sensitive preamplifier that has a cooled input stage usually consisting of a field-effect transistor (FET). Practical FWHM line widths achieved are near 150 eV at 5.9 keV for a lithium-drifted Si(Li) detector and near 700 eV at 122 keV for a Ge(i) detector.

The detection efficiency can be estimated by the transmission through the entrance window and through the dead layer of the detector crystal times the absorption probability in the active volume. An efficiency curve for a Ge(i) detector is shown in Fig. 12.

Solid-state detectors usually need a large number of calibration lines if one wants to perform precise energy measurements over an extended energy range. This is due to a not well defined zero of the energy and a nonlinear energy response function of the detector and of the attached amplifier and ADC. Despite those difficulties state-of-the-art germanium-detector systems now in use have the capability of measuring small energy differences, which means peak positions on a pulse-height scale, with

Table III: Some properties of Si and Ge relevant for their use as x-ray detectors.

	Si	Ge
Atomic number	14	32
Energy gap (eV)	1.12	0.74
electron-hole energy (eV)	3.76	2.96
Fano Factor	0.084	0.057

Figure 12: A typical efficiency curve of a Ge(i) detector showing the effect of absorption in the entrance window for low x-ray energy and the incomplete stopping of high-energy x rays.

high accuracy. Quite a large number of secondary gamma-ray reference energies have been measured this way with quoted uncertainties of less than 10 eV at x-ray energies of several hundred keV [32]. In that work systematic effects have been thoroughly investigated including the source-detector geometry and the modeling of the spectral shape of the measured gamma lines [33].

2.2.3 Low-Temperature Devices

A new class of detectors has become accessible with the advance of cryogenic detectors. Low-energy excitations possible in solids when cooled down to sub-Kelvin temperatures has motivated a community of astrophysicists and elementary-particle physicists [34] to develop radiation detectors of high sensitivity and energy resolution. In particular two detector types have demonstrated their feasibility in the last years, the superconducting tunneling junction (STJ) [35] and the bolometer [36].

Basically, the STJ detector makes use of the fact that the lowest-lying quasiparticle states are separated from the superconducting ground state by an energy gap of the order of a meV. This is thousand times less than in conventional semiconductor detectors described above. The FWHM energy resolution expected solely from statistical

fluctuations would be

$$\Delta E = 2.36\sqrt{FE\epsilon} \approx 5 \text{ eV} \quad (11)$$

assuming a Fano factor of $F = 1$, an x-ray energy of $E = 6 \text{ keV}$ and an energy of $\epsilon = 0.6 \text{ meV}$ necessary to break a Cooper pair. Fig. 13 demonstrates the tunneling processes [37] when a photon breaks up a Cooper pair either on the left or right side of the barrier. In both cases a signal of the same sign is produced.

Figure 13: Tunneling processes in a junction with both films superconducting. Absorbed x rays lead to a flow of electrons. Cf. reference [37].

Figure 14: Mn $K\alpha$ spectrum measured with a Sn/SnO/Sn tunnel junction. The dotted line corresponds to the best resolution obtained with a Si(Li) detector cooled with liquid nitrogen. Figure reproduced with permission from reference [35].

By illuminating a Sn/Sn Oxide/Sn superconducting junction [35] with 6 keV x rays from a ^{55}Fe source a FWHM of 41 eV was demonstrated as shown in Fig. 14. A spectrum measured with a good conventional Si(Li) detector shows a line width of 150 eV. The limitation in the experiment is mainly attributed to the granularity of the thin films used. Typical dimensions of a STJ are $100 \times 100 \mu\text{m}^2$ with a thickness of less than $1 \mu\text{m}$. The constraints to the size of a STJ is dictated by the capacitance which has to be kept low in order to limit the noise from the leakage current. Also the tunneling time has to be kept below $\sim 1 \mu\text{s}$ which corresponds to a thickness of $\sim 1 \mu\text{m}$. A very low temperature is needed to reduce the density of thermal quasiparticles.

In the bolometer, the energy deposited by a single x ray or particle is measured by the temperature rise in a crystal (see Fig. 15). If the heat capacity C_V can be made small the temperature rise will be large and can be measured with a sensitive thermometer with a high precision against a low background of phonon fluctuations when operated at

Figure 15: The principle of a calorimetric x-ray detector. The absorption of an x ray leads to a temperature rise ΔT with a subsequent decay determined by the heat capacity C_V and the thermal conductivity G of the thermal link to a heat sink.

very low temperature. Calorimetric measurements of nuclear radiation has already been used in the early days of nuclear physics measuring the integrated decay of radioactive isotopes. At that time the advantage of cryogenic operation was already recognized [38].

For measuring single events, a very low temperature (typically below ~ 0.3 K) is essential because the heat capacity increases according to the Debye T^3 law. In practice, the temperature rise ΔT is measured by the voltage drop across a thermistor (doped semiconductor) which has to be in good thermal contact with the absorbing material. The thermal link to the heat sink is realized by the electrical leads to the thermistor.

Fig. 16 shows an x-ray spectrum obtained by Moseley *et al.* [36] with a device consisting of a silicon thermistor and a HgCdTe absorber illuminated by a ^{158}Tb source.

Figure 16: Spectrum of a ^{158}Tb source, showing x-ray lines from Gd and Dy L lines and K-fluorescence lines from Ni. Reproduced with permission from Moseley *et al.* [36].

The spectrum consists of L lines of Gd and Dy and of K-fluorescence lines of Ni. The FWHM energy resolution of this bolometer is 35 eV.

It is too early to draw a final conclusion as to the usefulness of the new developments for use in e.g. accelerator-based x-ray spectroscopy. At least, in one experiment [39] a calorimetric detector has successfully been used in such an environment. Both types of cryogenic detectors have limitations in the available detector size. Calorimetric detectors having a volume as large as 130 mm³ [40] have been constructed. Fabricating large arrays of such devices might be possible though technically demanding. Another limitation is the long recovery time of several milliseconds. This makes the detector suitable only for operation at low count rates and it excludes measurements of fast coincidences.

2.2.4 Differential-Absorption Spectroscopy

A method known as ‘critical absorption measurement’ or as ‘differential-absorption spectroscopy’ makes use of a near coincidence between the spectral position of an x-ray line and an absorption edge. If the transmission profile of a particular filter is known one can obtain the x-ray energy of the line from a single measurement of its transmission through the filter. Such measurements can maintain a high efficiency and simultaneously a high spectral resolution.

Figure 17: Unfolded absorption profile of the K-edge of metallic Er absorbers which were used to measure the attenuation of x rays emitted in the 6h-5g transition of kaonic potassium. The inset shows the expanded profile for the energy region in which the kaonic x rays are absorbed. The dashed lines indicate the measurement uncertainty. Reproduced with permission from reference [18].

They were first applied to the measurement of the muon mass by recognizing the spectral overlap of the 3d-2p transition in muonic phosphorus with the Pb-K absorption edge [41]. A similar coincidence was found for the 6h-5g transition in kaonic potassium with the Er-K edge [42]. For both experiments dedicated measurements of the K-edge profiles have been undertaken by Bearden [43] and by Deslattes *et al.* [18], respectively. The more recent and also more accurate result for Er [18] is reproduced in Fig. 17.

The method requires:

- smooth absorption profiles as realized by K edges of heavy metallic elements
- a simple preferably single-line spectrum to be measured
- the line to be located within the steep rise of the absorption coefficient versus energy
- absolute mass-absorption data, like those shown in Fig. 17, from which a transmission profile can be generated

Because the method is applied for rare events one usually has to account for a ‘poor’ geometry in which a large detector is placed close to the absorbing foil and to the x-ray source. A correction is necessary for inelastically scattered photons in addition to photoabsorption. When using a fast ion beam (as will be discussed below) it is easy to shift the lines to a desired absorption edge or to portray the whole profile. Then, the knowledge of a relative absorption profile is sufficient and it may also contain detailed structure.

2.3 The Wavelength Scale in the X-Ray Region

The scale on which x-ray and gamma-ray wavelength measurements have been presented has changed considerably over the years and can lead to some confusion. Although this problem is solved now by combined x ray and optical interferometry it might be necessary to consider the history of definitions used if one wants to use calibration wavelengths from literature. The subject has been reviewed by Deslattes [44] and by Kessler and Deslattes [19]. In the following is given a historical sketch:

- 1914 W. H. Bragg [45] proposes unit cell of NaCl as standard
- 1919 M. Siegbahn [46] invents local scale XU = $d(\text{NaCl}) \approx 10^{-3} \text{\AA}$
- 1925 M. Siegbahn [47] defines XU by $d(\text{calcite}(211)) = 3029.04 \text{ XU}$
- 1925 A. H. Compton and R. L. Doan [48] discover x-ray diffraction by ruled gratings.
- 1947 W. L. Bragg [49] and Wood [50] provide a link of x rays to optical wavelengths, $\Lambda = \lambda_g / \lambda_S \approx 1.00202$
- 1965 J. A. Bearden [51] defines $\text{\AA}^*$ by $\lambda(\text{W K}\alpha_1) = 0.2090100 \text{\AA}^*$ intending $\text{\AA}^* / \text{\AA} = 1$ with available $\rho(\text{Si})$ and N_A
- 1965 U. Bonse and M. Hart [52] show achromatic x-ray interference
- 1967 J. A. Bearden’s [53] compilation of x-ray wavelengths reducing discrepancy between wavelengths referenced against $\lambda(\text{W K}\alpha_1)$ or $\lambda(\text{Cu K}\alpha_1)$
- 1973 R. D. Deslattes and A. Henins [54] report combined x-ray and optical x-ray interferometry for accurate x-ray to optical wavelength ratios and $d(\text{Si}111)$ at NIST
- 1980 R. D. Deslattes *et al.* [18] publish many directly measured wavelengths of gamma lines including 412 keV ^{198}Au line
- 1981 P. Becker *et al.* [55] measure $d(\text{Si}220)$ at PTB; discrepancy between PTB and NIST of 1.8 ppm

Figure 18: Principle of simultaneous x-ray and optical interferometry. Figure reproduced with permission from reference [54].

1991 concordance between NIST and PTB results in $d(\text{Si})$ approaching the 10^{-8} level [56]

Especially in the early times of crystal spectrometry the angular measurements could be made much more accurately than those of the lattice spacing. The consequence was the use of reference lines defining the local scale. Today, the link to the optical scale comprises three steps, the measurement of a lattice repeat distance of a highly pure silicon crystal, the comparison of this crystal with a thick crystal suitable for use in a crystal spectrometer and the measurement of a gamma-ray line with that crystal. Such a programme was undertaken at NIST by Deslattes *et al.*, many details of the necessary procedures are reviewed in reference [18].

As one example out of the three steps is shown, in Fig. 18, the principle of the x-ray/optical interferometric measurement. The three crystals form a Laue-case (LLL) interferometer [57] where the first two crystals are made of a monolithic block and the third one can be shifted. The lattice parameter d can be measured by moving that crystal with respect to the moiré-generating pair of crystals by a known distance X and counting the number N of resulting moiré-fringe passages. Then $d = X/N$ independent from wavelength of the radiation used. The distance X is measured by the optical interferometer operating with a $^{129}\text{I}_2$ stabilized $^3\text{He}^{22}\text{Ne}$ laser. The large difference between the optical and the x-ray wavelengths imply a large number of cosine fringes recorded by the x-ray detector between two Fabry-Perot maxima. An extremely smooth translation, on the $0.01 \text{ \AA}$ level, of the movable crystal has to be guaranteed over a distance approaching 1 mm.

Spectroscopists have preferred to think in x-ray energies, in units of the eV, rather than in wavelengths. However, the more precise data have been gained by wavelength measurements which afterwards were converted into energies. The conversion factor as given by the “1986 Adjustment of the Fundamental Physical Constants” [58] is

$$hc = 12.398\,424\,4(37) \text{ eV\AA} \quad (12)$$

where h and c denote the Planck constant and the velocity of light, respectively. The uncertainty associated with the transformation amounts to 0.30 ppm. The numerical value has changed over the years with accordingly larger uncertainties in the past. Nevertheless, in many cases it is possible to retain the uncertainties associated with the

original wavelength measurements by retrieving the conversion factor applied in the original work.

2.4 The Doppler Effect

If a photon is emitted by a particle of velocity $\vec{v}$ its wavelength is subject to the Lorentz transformation

$$\lambda = \lambda_o \gamma (1 - \beta \cos \vartheta) \quad (13)$$

where λ_o denotes the wavelength in the emitter frame of reference and λ the wavelength observed by a detector, in the laboratory system, placed at an angle ϑ relative to the particle velocity $\vec{v}$. The relativistic parameters are defined in the usual way as $\beta = v/c$ and $\gamma = (1 - \beta^2)^{-1/2}$. If many particles in a light source have a velocity distribution $\partial^3 N(\vec{v})/\partial^3 \vec{v}$ the spectral line observed in the laboratory may be written as

$$\frac{dN}{d\lambda} = \int \frac{\partial^3 N(\vec{v})}{\partial^3 \vec{v}} \delta(\lambda - \lambda_o \gamma (1 - \beta \cos \vartheta)) d^3 \vec{v} . \quad (14)$$

In Eq. (14) it has been assumed that the natural linewidth is small compared to the Doppler shift. For thermal velocities given by an isotropic Maxwell distribution Eq. (14) yields a Gauß profile as

$$\begin{aligned} \frac{dN}{d\lambda} &\approx \frac{c}{\sqrt{\pi} v_o \lambda_o} e^{-\left(\frac{c}{v_o} \frac{\lambda - \lambda_o}{\lambda_o}\right)^2} \\ \text{FWHM } \Delta\lambda &= (\ln 2)^{1/2} \frac{v_o \lambda_o}{c} \\ &= (2 \ln 2 kT/m)^{1/2} \frac{\lambda_o}{c} \end{aligned} \quad (15)$$

where $v_o = \sqrt{2kT/m}$ is given by the particle temperature T and mass m . Eq. (15) is a good approximation even for hot plasmas where ion velocities are still small compared to the velocity of light usually below $\beta < 10^{-3}$. Therefore, second-order line shifts introduced by the thermal velocities are often negligible and only the broadening of the spectral lines is important.

2.4.1 Fast-Beam Characteristics

The situation changes greatly when radiation emitted by a fast ion beam is considered. For highly charged heavy ions, beam velocities well in excess of 0.1 c can be involved leading to huge Doppler shifts. However, the highly directional velocity distribution of an ion beam can still assure tolerable linewidths. The line profiles can be calculated with the aid of Eq. (14) using appropriate estimates of the longitudinal and transverse beam emittances determining the required velocity distribution $\partial^3 N(\vec{v})/\partial^3 \vec{v}$. For realistic estimates, the finite extensions of source and detector have to be included and the corresponding multidimensional integral can be solved by ray-tracing usually involving the Monte Carlo method.

The main properties of a fast-beam geometry can be obtained from the Doppler formula (13) by inspection of its first derivatives with respect to β and ϑ resulting in

$$\left(\frac{\Delta\lambda}{\lambda}\right)^2 = \{F(\beta, \vartheta) \Delta\vartheta\}^2 + \{G(\beta, \vartheta) \Delta\beta\}^2$$

with

Figure 19: The function $F(\beta, \vartheta)$ versus $G(\beta, \vartheta)$ according to Eq. (16), for $\beta = 0.1$ and for $\beta = 0.5$.

Table IV: The geometrical conditions for zero or utmost angular or velocity sensitivity. Given are the functions $F(\beta, \vartheta)$ and $G(\beta, \vartheta)$, defined in Eq. (16), at the angles indicated.

	Angle	Velocity
$\vartheta = 0^\circ/180^\circ$	0	$\gamma^2 \Delta\beta$
$\vartheta = \arccos \beta$	$\gamma\beta \Delta\vartheta$	0

$$\begin{aligned}
F(\beta, \vartheta) &= \frac{\beta \sin \vartheta}{1 - \beta \cos \vartheta} \\
G(\beta, \vartheta) &= \beta \gamma^2 - \frac{\cos \vartheta}{1 - \beta \cos \vartheta}
\end{aligned} \tag{16}$$

where $\Delta\lambda$, $\Delta\beta$ and $\Delta\vartheta$ may be interpreted either as widths or as uncertainties. The functions $F(\beta, \vartheta)$ and $G(\beta, \vartheta)$, determining the influence of the angle and of the velocity, respectively, are not independent of each other. This can be seen in Fig. 19 where G is plotted versus F for two different velocities. In Fig. 20 the functions $F(\beta, \vartheta)$ and $G(\beta, \vartheta)$ are illustrated in a polar diagram. Two special cases are defined by the zeroes of the two functions as summarized in Table IV. At 0° and 180° with respect to the ion-beam direction, the angular width/uncertainty vanishes and the sensitivity to the velocity is maximum whereas at an angle given by $\cos \vartheta = \beta$ the sensitivity to the width/uncertainty of the beam velocity vanishes and the angular sensitivity becomes a maximum. This behavior suggests an optimization depending on which of the two parameters, observation angle or beam velocity, is known with a higher precision.

2.4.2 Doppler Tuning

As already mentioned, the Doppler shifts can be very high. For $\beta = 0.5$, for instance, the backward/forward wavelength ratio amounts to $\lambda(180^\circ)/\lambda(0^\circ) = 3$. The large displacements may be used to shift the spectral line of interest into a desired spectral region or near a suitable calibration line by choosing the correct velocity and observation angle. In addition, the uncertainties introduced by the Doppler transformation can be minimized by performing a spectroscopic measurement at a variety of angles and/or velocities resulting in a sufficient overdetermination. For the special case of two detectors viewing simultaneously the source at an angle ϑ_1 and $\vartheta_2 = \vartheta_1 + 180^\circ$, respectively, the sum and difference of the corresponding wavelengths are given by

$$\begin{aligned}
\lambda_2 + \lambda_1 &= 2\lambda_0\gamma \\
\lambda_2 - \lambda_1 &= 2\lambda_0\gamma \cos \vartheta_1
\end{aligned} \tag{17}$$

Figure 20: The spectral sensitivity towards angular ($\Delta\vartheta$) and velocity ($\Delta\beta$) uncertainties or widths, plotted as a polar diagram for $\beta = 0.5$.

If the apparatus can be aligned in such a way that the colinearity of the line of sight of the two detectors is known more precisely than the individual observation angles an increased precision in the measurement of λ_0 follows. A dual-arm crystal spectrometer, serving exactly that purpose, was constructed by Hailey *et al.* [59]. They mounted two crystals in Johann geometry forming a common plane of dispersion which is aligned perpendicularly to the ion beam. Similar techniques are used and proposed for LASER spectroscopy on fast ion beams [16]. A LASER beam is reflected in itself with the reflected light being antiparallel to the forward beam. The ion velocity is tuned so that the red- and blue-shifted LASER beams are symmetric at equal distances from the maximum of an ionic resonance.

Figure 21: The principle of Doppler-tuned spectroscopy realized by scanning either the observation angle or the beam velocity over a known absorption feature of a thin filter which is placed between source and detector.

Doppler-tuned spectroscopy using absorption edges was first applied by Schmieder and Marrus [60] to beam-foil spectroscopy. The principle, illustrated in Fig. 21, is to match the Doppler-shifted emission with a known absorption feature (like a K edge) of a thin filter inserted between fast-beam source and detector. The wavelength λ_0 in the emitter frame of reference can be calculated if the beam velocity v , the observation angle ϑ , at which the absorption feature is detected, and the absorption coefficient versus wavelength is known. The method makes directly use of the dispersion due to variable Doppler shifts as a function of emission angle and is entirely determined by the Doppler formula (13).

In practice, the absorption structure can be located by a variation of the angle at a

fixed beam velocity (angle-tuned spectroscopy), preferably by using a position-sensitive detector, or by scanning the velocity (velocity-tuned spectroscopy) across the absorption structure. Of course, requirements on an independent and absolute measurement of the beam velocity as well as on the absolute angular measurement can be relaxed when multiple detectors are used. Here, the same arguments apply as discussed above in context with an optimization of the detection geometry. In the optimization procedure one has to consider the variation of the angular and velocity dispersion with observation angle as already shown in Fig. 20. Maximum dispersion values are given in Table IV. Velocity tuning is most sensitive at $\vartheta = 0^\circ/180^\circ$ whereas angle-tuned spectroscopy has its largest sensitivity at $\cos\vartheta = \beta$. Both techniques are pursued at GSI with two approved proposals [61, 62] for the ESR storage ring. Availability of suitable absorption edges represents practical constraints when using simultaneously multiple angles with different absorption materials. The need for an accurate measurement of the absorption coefficients on an absolute wavelength scale has already been pointed out when discussing differential absorption spectroscopy in general.

2.4.3 ‘Can One Beat the Doppler Effect ?’

With respect to fast-beam spectroscopy, the Doppler-tuned techniques might be viewed as intermediate between solid-state detectors and crystal spectrometers. The latter are known to have the highest spectral resolution. But it is also known, that they can have extremely small efficiencies. Curved crystal instruments have experienced an increase of accuracy and efficiency for several reasons. Necessary wavelength standards are available for calibration. Position-sensitive x-ray detectors can be built. Highly accurate angular measurements using laser interferometers are possible. Large specimens of highly perfect crystals can be bent with good accuracy.

Figure 22: The geometry of a curved-crystal spectrometer in the Johann or Johansson version with a position-sensitive x-ray detector and a source that is moving fast [65].

However, geometrical aberrations due to the finite size of the crystal and of the source may cause a systematic shift and broadening of the spectral lines. Most of the aberrations can be made small by a careful design of an instrument or the result of a measurement can be corrected provided the type of aberrations and their magnitudes

are well known. This problem has been recognized for stationary sources. A detailed analysis was made by Schwitz *et al.* [63] for Laue-case transmission spectrometers and by Zschornack *et al.* [64] for Bragg-case spectrometers.

Even more severe are aberrations when, in addition, a fast-beam source is involved. A very careful analysis [65] has been made analytically and numerically for curved crystals, and computer codes are available for handling Bragg and Laue geometries. The geometrical situation for adapting a fast x-ray source to a curved crystal in the Bragg case is shown in Fig. 22 where also some of the variables used are explained. In the original Johann or Johansson geometry a vertical needle source on the periphery of the Rowland circle would be imaged symmetrically onto the Rowland circle. Here, the source with finite dimensions may be displaced off the Rowland circle and the emitting ions have a velocity defined by the vector $\vec{v}_{\text{ion}}$. A position-sensitive detector may also be placed at some distance from the Rowland circle.

Numerically, the intensity distribution in the detector may be written as

$$I(X, Y) = \frac{1}{V} \int F(\theta_i(\chi) - \theta_B(\chi, \lambda_{\text{ion}})) S(\lambda_{\text{ion}}) d\chi d\lambda_{\text{ion}} \quad (18)$$

where $F(\theta)$ denotes the diffraction pattern of the crystal, $\theta_i(x, y, z, r, t, h)$ the effective glancing angle and θ_B is the Bragg angle calculated via the Bragg-Laue relation taking into account the Doppler-shifted wavelength. The multidimensional integration runs over $\chi = (x, y, z, r, t, h)$ and over the spectral distribution $S(\lambda_{\text{ion}})$ of the source. The integral can be solved by the Monte-Carlo method. A number of special cases have also been calculated analytically which is very helpful when analytical fitting functions are needed for the analysis of an experiment (see below).

Figure 23: Intensity pattern in a position-sensitive detector mounted in a Johann spectrometer. The ion beam is perpendicular to the Rowland circle [65].

In Fig. 23 and 24 we summarize the result for the case in which the Rowland circle is oriented perpendicularly to the ion beam. The analytical result is

$$\begin{aligned} \sin \hat{\theta}_i &= \frac{n \lambda_{\text{ion}}}{2d(1 - \beta^2)^{1/2}} \\ &\times \left[(1 - \tan^2 \psi_x)^{1/2} - \beta (2 \sin(\tau_o/2)/\tau_o \right. \\ &\quad \left. \times \cos \psi_{\text{ion}} \cos \phi_{\text{ion}} + \sin \psi_{\text{ion}} \tan \psi_x) \right] \end{aligned} \quad (19)$$

where $\hat{\theta}_i$, denoting the effective glancing angle projected into the plane of the Rowland circle, determines the horizontal coordinate in the detector. The corresponding vertical

Figure 24: The difference between analytical and numerical results for the same data as in Fig. 23. Uncertainty bars represent the statistical uncertainty in the numerical calculation [65].

coordinate is determined by the out-of-plane angle ψ_x . The first term within the square brackets describes the curvature of the spectral image as can be realized in Fig. 8 for a stationary source. The second term proportional to β splits into two contributions. The first one is independent of ψ_x and describes a simple Doppler shift whereas the second one is proportional to $\tan \psi_x$ producing the slanting of the image which can be observed in Fig. 23. As shown in Fig. 24 the analytical formula 19 agrees with the Monte-Carlo simulation within the statistical accuracy which, in this example, is in the order of 1-2 μm . This example demonstrates that it is possible to use the full solid angle of a large curved crystal without losing resolution or intensity due to the Doppler effect.

For an ion beam moving in the plane of dispersion, i. e. $\psi_{\text{ion}} = 0$ there can be unfavorably large Doppler dispersions along the crystals aperture. There are two cases, though, that might have practical applications. In the first case the ion beam moves towards the crystal ($\psi_{\text{ion}} = \phi_{\text{ion}} = 0$) with a high Doppler shift. Because of the small Doppler breadth, however, a narrow focus can be sustained at negligible loss of intensity. In the second case the line of sight is perpendicular to the ion beam ($\psi_{\text{ion}} = 0, \phi_{\text{ion}} = 90^\circ$). In this configuration the large Doppler dispersion leads to a nearly complete loss of resolution for a detector placed at the Rowland circle. A refocussing, however, may be achieved by shifting the detector off from the Rowland circle. An analogue technique was invented by Stoner and Leavitt [66] for a grating spectrometer.

In Fig. 25 the two methods are compared. In case of the grating spectrometer [66] the exit slit is displaced by

$$d_{\text{off}} \approx \beta \left(\frac{\lambda_o}{k} \right) \quad (20)$$

where k is the reciprocal linear dispersion of the grating. For the crystal spectrometer the detector and the source are displaced by

$$\begin{aligned} d_{\text{off}} &\approx -R \frac{\beta \sin \theta \cos \theta}{(1 - \beta) \cos \theta - \beta \sin \theta} \\ x_{\text{off}} &\approx R \frac{\beta \sin \theta \cos \theta}{(1 + \beta) \cos \theta - \beta \sin \theta} \end{aligned} \quad (21)$$

where a positive value means a shift inside and a negative value a shift outside the Rowland circle and the orientation of the beam is as indicated in Fig. 25. If the beam

Figure 25: Refocussing of a grating and of a crystal spectrometer. The offsets have to be optimized for a particular wavelength and beam velocity.

direction is reversed the signs of d_{off} and x_{off} in Eq. (21) change. To the authors knowledge, the scheme of refocussing a crystal spectrometer has not been used in an experiment yet.

3 STORAGE-RING PROPERTIES FOR SPECTROSCOPISTS

3.1 Introduction to Cooler/Storage Rings

Figure 26: The range in nuclear charge Z and velocity β accessible by various ion-storage rings. Cf. reference [71].

Presently, there exist a large number of storage-ring facilities for heavy ions either already operational or in commissioning or planned. A more detailed comparison of their design parameters can be found in reference [67]. Fig. 26 shows the range of ions that can be accumulated in various storage rings. The range mainly depends on the kind of ion injector linked to the storage ring and on the magnetic rigidity of the specific bending-magnet design used. All those machines have some common features. They are built to accumulate high ion currents, they employ phase-space cooling to achieve excellent beam quality and they provide for in-ring experiments. Most of the concepts in use originate or are influenced from operational experience gained at the Low-Energy Antiproton Ring (LEAR) [68] at CERN. For the purpose of this lecture, examples from the ‘Experimentierspeicherring’ ESR will be presented because it is the only storage ring that can accumulate and cool bare ions up to U^{92+} .

3.2 Phase-Space Considerations

Figure 27: A coordinate system moving with an ideal particle on orbit is used to describe the motion of ions in a storage ring.

The motion of ions in a storage ring may be described relative to an ideal closed orbit as illustrated in Fig. 27. On the ideal orbit the particle energy $E_o = mc^2(\gamma - 1)$ and momentum $p_o = mc\beta\gamma$ is determined by the magnetic field and by the radius of the bending magnets used. To describe the distribution of particles in their six-dimensional phase space $\vec{r}, \vec{p}$ a special coordinate system is used the origin of which moves on the ideal orbit with the mean momentum p_o . The coordinate s along the orbit has an arbitrary origin. This coordinate system is referred to as the ‘mean-particle rest frame’ or the ‘co-moving system’. The horizontal and vertical transverse coordinates are denoted by y_h and y_v where the subscript will be omitted whenever a discrimination between the two directions is not important.

Figure 28: Betatron oscillations around the ideal orbit.

In order to keep the particles near the ideal orbit, a number of dipole magnets and transverse focusing elements, like quadrupole magnets, are arranged in a more or less symmetric way forming a lattice. The Lorentz forces in a quadrupole magnet lead to focusing only in one plane and to defocusing in a transverse plane. By periodically switching between planes in a set of quadrupoles an overall focusing is achieved. By the focusing, betatron oscillations around the ideal orbit are induced as schematically depicted in Fig. 28. In modern accelerators using ‘strong focusing’ more than one betatron oscillation is accomplished during one revolution. The equation of motion for the particles can be written as a Hill’s type differential equation

$$\frac{d^2y}{ds^2} + K(s)y = \frac{1}{\rho} \frac{\Delta p}{p_o} \quad (22)$$

where both the focusing strength $K(s)$ and the bending radius ρ are given by the particular lattice. A general solution of this equation is of the form

$$y(s) = C(s)y_o + S(s)y'_o + D(s)\frac{\Delta p}{p_o} \quad (23)$$

The functions C , S and D (Cosinelike, Sinelike, Dispersion) are called the principal trajectories. For $\Delta p = 0$ a real solution of Eq. (22) is

$$\begin{aligned} y(s) &= a_y \sqrt{\beta(s)} \cos(\psi(s) + \delta) \\ y'(s) &= -\frac{a_y}{\sqrt{\beta(s)}} \left\{ \sin(\psi(s) + \delta) - \frac{1}{2} \beta'(s) \cos(\psi(s) + \delta) \right\} \end{aligned} \quad (24)$$

where the prime denotes the derivative with respect to the longitudinal coordinate s . Eqs. (24) describe oscillations with a betatron amplitude $a_y \sqrt{\beta(s)}$ and with a betatron phase $\psi(s) = \int \frac{ds}{\beta(s)}$. $\beta(s)$ is called the betatron function and is an intrinsic property

of the lattice. Eqs. (24) are a parametric representation of an ellipse in the y - y' plane which is illustrated in Fig. 29 giving also the maximum values of y and of y' in terms of the emittance $\varepsilon = a_y^2$. The phase-space ellipse may be viewed either for a single-particle motion along the circumference or, at a fixed position, for many particles with different phases.

Figure 29: Phase-space ellipse as defined by Eqs. (24).

The betatron wavelength or tune defined as

$$Q = \frac{1}{2\pi} \oint \frac{ds}{\beta(s)} \quad (25)$$

is an important operating parameter of a storage ring. Because of unavoidable imperfections in the manufacturing of the lattice, disturbances occurring at a position along the ring could periodically build up if the ions encounter one of the betatron resonances

$$k Q_h + \ell Q_v = m \quad \text{with } k, \ell, m = 0, \pm 1, \pm 2, \dots \quad (26)$$

For stable operation, especially the lower resonances have to be avoided.

Particles with a deviation Δp from the mean momentum follow a dispersed orbit with a displacement $\Delta y = D(s) \frac{\Delta p}{p_o}$ given by the dispersion function $D(s)$. Changes in the momentum lead to different paths around the ring and to different revolution frequencies. The fractional frequency change is related to the fractional momentum change through

$$\frac{\Delta f}{f} = \left(\frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} \right) \frac{\Delta p}{p_o} \quad (27)$$

The first term within the brackets accounts for the relativistic velocity change with momentum and the second term relates the change of the orbital radius $\Delta \rho$ to the momentum change Δp . γ_t is known as the 'transition gamma' and $\gamma_t m c^2$ as the transition energy. From Eq. (27) one finds that the frequency increases with momentum if the particle energy remains below the transition energy ($\gamma < \gamma_t$). The transition point $\gamma = \gamma_t$ is very critical for momentum focusing especially in synchrotrons.

Since the bending and focusing forces depend on the particle momentum there is also a tune shift related to a momentum shift

$$\Delta Q = \xi \frac{\Delta p}{p_o} \quad (28)$$

The factor ξ is called the chromaticity and is, like the dispersion, an intrinsic lattice parameter.

The principle of momentum focusing or bunching by an RF cavity operated at a harmonic frequency of the revolution frequency is demonstrated in Fig. 30. Fast

Figure 30: The principle of beam bunching in an RF cavity. Fast particles arriving early experience a positive (decelerating) electric field whereas slow particles experience a negative (accelerating) field.

particles arriving early experience a positive (decelerating) electric field whereas slow particles experience a negative (accelerating) field. Particles already in phase remain unaffected. By tuning the phase in an appropriate way a cavity can also be used for acceleration and for deceleration.

Figure 31: Layout of the ESR storage ring. Arrows indicate the positions along the ring where particle detectors can be installed.

Fig. 31 shows the layout of the ESR storage ring. The circumference of the ESR of 108 m is exactly one half of the heavy-ion synchrotron SIS used as injector. This

Table V: Maximum ion currents and corresponding number of ions accumulated in the ESR.

	Ion	$I_i(\text{mA})$	N_i
	Ne ¹⁰⁺ 250 MeV/u	7	2.5×10^9
	Xe ⁵⁴⁺ 250 MeV/u	6	4×10^8
	U ⁹²⁺ 330 MeV/u	2.2	8×10^7

facilitates beam transfer between both rings. The maximum magnetic rigidity of the ring amounts to 10 Tm. This allows storage of bare uranium ions with energies up to 560 MeV/u. Besides the main lattice components consisting of six 60° dipole magnets and six groups of quadrupole triplets or doublets there are two long straight sections. The latter include the internal gas-jet target and the electron cooler, respectively. These are also the places for in-ring experiments. Maximum ion currents and the corresponding number of particles accumulated so far in the ESR are listed in Table V.

3.3 Electron Cooling

Excellent reviews on the electron-cooling technique were published by Skrinskii and Parkhomchuk [69], Sørensen and Bonderup [70], and by Poth [71] giving also a comprehensive account on the electron-cooling experiments at LEAR. Here, only the basic ideas are summarized along with some recent results obtained at the ESR for very heavy ions.

Cooling an ion beam is a very attractive idea because it represents a way to compensate for various heating processes like intra-beam scattering [72], focusing errors and small-angle scattering and energy loss in an internal target. Budker [73] was the first to propose a cold electron beam as a refrigerant medium to cool an ion beam. A series of pioneering experiments with proton beams followed [74] and also theoretical studies were performed [75, 70].

Over a fraction of the circumference of the storage ring the circulating ions are merged with continuously refreshed cold electrons. The mean electron velocity in the laboratory system equals the mean ion velocity. The dissipative force introduced via Coulomb interactions between electrons and ions leads to heat exchange and after many revolutions to an equilibrium of the temperatures $T_i = T_e$. The ion temperature can be expressed in terms of the longitudinal and transverse velocities in the co-moving system

$$\frac{3}{2} kT_i = \frac{M}{2} (v_{\parallel}^2 + v_h^2 + v_v^2) \quad (29)$$

where M is the ion mass. Expressing the velocity components in terms of the momentum spread $\Delta p/p_o$ and emittance ϵ of the ion beam one finds

$$\frac{3}{2} kT_i = \frac{M}{2} \gamma^2 \beta^2 c^2 \left\{ \frac{\epsilon_h}{\beta_h(s)} + \frac{\epsilon_v}{\beta_v(s)} + \frac{1}{\gamma^2} \left(\frac{\Delta p}{p_o} \right)^2 \right\} \frac{\Delta p}{p_o} . \quad (30)$$

From the co-moving system of the cold electrons the cooling may be viewed as a damping of the relative motion of the ions with a corresponding friction force. This friction force may be estimated by considering the stopping of charged particles in a cold electron gas expressed by the Bethe-Bloch equation [76] :

$$F_c = \frac{dE}{dx} = - \frac{4\pi Z^2 e^4 n_e}{m_e v^2} L_c \quad (31)$$

Here, v denotes the relative electron and ion velocity and n_e the electron density. L_c is the Coulomb logarithm which is

$$L_c = \log \left(\frac{b_{\max}}{b_{\min}} \right) \approx 10 \quad (32)$$

defined in terms of the minimum and maximum impact parameters which may be identified with the distance of closest approach in central collisions and with the Debeye shielding length. From Eq. (31) a corresponding cooling rate

$$\tau_c^{-1} = \frac{dp}{p dt} = \frac{4\pi Z^2 e^4 n_e L_c \eta}{\gamma^2 m_e M v^3} \quad (33)$$

may be obtained. The quantity γ^2 appears because of the Lorentz transformation of the time and of the electron density. It is also taken into account that the cooling is effective only over a fraction η of the circumference of the storage ring. From Eq. (33) the important scaling of the cooling rate proportional to Z^2/A can be noted, where A denotes the atomic mass of the ion. Therefore, electron cooling should be much more efficient for highly charged heavy ions than for protons. Two effects have to be considered still that make cooling even more efficient:

- The velocity distribution of the electrons is strongly flattened after electrostatic acceleration.
- The strong magnetic field ($B \approx 0.1\text{T}$) necessary to confine and guide the electrons leads to a freezing of the transverse degree of freedom of the electronic motion referred to as ‘magnetized electron beam’

The flattening of the velocity distribution is a pure kinematical effect which can be understood from the transformation of the electron velocities. Whereas the transverse temperature is given by the temperature of the hot cathode in the electron gun, $T_\perp \approx T_{\text{cathode}}$, the longitudinal temperature is given by

$$T_\parallel = \frac{T_\perp^2}{2\beta^2 \gamma^2 m c^2} \ll T_\perp \quad (34)$$

In the combined magnetic field and space-charge field of the electron cooler the electrons move on epicyclic trajectories [77]. The motion consist of a rather slow rotation of the whole electron beam around its axis and a Larmor gyration of individual electrons. The Larmor radius, given by

$$r_L = \frac{\gamma m_e v_\perp}{eB}, \quad (35)$$

typically is several μm . For impact parameters larger than the Larmor radius, the ions interact with ‘Larmor discs’ traveling at the very low longitudinal velocity. This results in a much increased longitudinal cooling force. Cooling theory, therefore, discriminates between adiabatic and fast collisions at large and small impact parameters, respectively.

For limiting cases of small and high relative velocities a scaling of the cooling force is predicted [75] according to

$$F_\parallel \propto \begin{cases} v_{i\parallel}, & v_{i\parallel} < v_{e\parallel} \\ v_i^{-2}, & v_i > v_{e\parallel} \end{cases} \quad (36)$$

This is roughly confirmed by experiments. In Fig. 32 the measured [78] cooling force as a function of the relative velocity is shown for U^{92+} and for Ti^{22+} ions stored in

Figure 32: Longitudinal cooling force measured for two different ions stored in the ESR [78].

Figure 33: The layout of the ESR electron cooler. Electrons produced in the gun are guided by toroidal and solenoidal magnetic fields. The length of the straight cooling section is 2.5 m.

the ESR. For low relative velocities the measurement was performed by observing the response of the ion beam after stochastic heating and at higher velocities the drag force was measured when the cooler voltage was detuned by a preset amount.

The layout of the ESR electron cooler is shown in Fig. 33. Electrons produced in the gun are guided by toroidal and solenoidal magnetic fields usually near 0.1 T. The length of the straight cooling section is 2.5 m. The cooler was designed for an electron current of up to 10 A and a maximum voltage of 300 kV. Because of favorable cooling times electron currents of only $\sim 50 - 200$ mA are sufficient. Stable operation of the cooler has been obtained at voltages up to ~ 180 kV.

The momentum cooling can efficiently be observed by Schottky diagnostics [79] which makes use of the statistical current fluctuations. In the circulating ion beam of a storage ring they lead to peaks at the harmonics of the revolution frequency. If coherent effects can be neglected the electrical power contained in a band is proportional to the number of stored ions and the frequency spread is proportional to the momentum spread according to Eq. 27. As an example of the first electron-cooling results from the ESR Fig. 34 shows the Schottky spectrum for 90 MeV/u Ar^{18+} before and after electron cooling [80]. In this case, the relative momentum width is reduced from $\Delta p/p = 1 \cdot 10^{-3}$ down to $3 \cdot 10^{-5}$.

Figure 34: Schottky spectrum of 90 MeV/u Ar^{18+} before and after cooling with 1 A electrons. The relative momentum width is reduced from $\Delta p/p = 1 \cdot 10^{-3}$ down to $3 \cdot 10^{-5}$ [80].

Figure 35: Relative momentum spread of various ion species stored and cooled in the ESR as a function of the number of stored particles [81].

In Fig. 35 is shown the relative momentum spread of several different ion species as a function of the number of stored particles [81]. The results are in good agreement with an $N^{1/3}$ dependence typical for the equilibrium between electron cooling and intrabeam scattering. This inverse relation between beam quality and quantity has to be considered when planning an experiment.

3.4 Internal Gas Jet

Figure 36: Schematic of the supersonic gas-jet device of the ESR showing the multiple skimmers necessary for suppressing the background gas [82].

In one of the straight sections of the ESR a supersonic gas-jet device is installed

Table VI: Target thicknesses for various gas species derived from measurements of pressures in the jet dumping stages [83].

Gas Species	Thickness [cm^{-2}]
H ₂	$1 \cdot 10^{11}$
CH ₄	$5 \cdot 10^{12}$
N ₂	$3 \cdot 10^{12}$
He	$5 \cdot 10^{11}$
Ar	$1 \cdot 10^{13}$

[82]. The requirements for the gas jet are demanding because for experiments a high gas density in a localized gas stream is desirable and at the same time a good background vacuum in the order of 10^{-10} mbar has to be guaranteed. In order to meet these requirements a multi-stage gas inlet and dumping system was built as depicted in Fig. 36. In Table VI are summarized the gas thicknesses already obtained during experiments. Presently, the shape of the inlet nozzle and of the skimmers are being optimized.

3.5 Recombination and Capture Processes

For populating the electronic states in one and few-electron ions to be used for spectroscopy recombination and capture processes are attractive ways to proceed. In case of a hydrogenlike ion one would start with a bare ion and add one single electron either in the electron cooler or in the gas target. Single-collision conditions are easy to fulfill at the low gas and electron densities involved and multiple electron transfer in a single encounter can be largely avoided.

Fig. 37 schematically illustrates the electron-transfer processes dominant in the electron cooler or in the gas target for very heavy ions. For an electron initially bound in a target atom there is the possibility of radiative electron capture (REC) and of non-radiative capture (NRC) into various final states of the projectile. In case of free cooling electrons the transfer can *only* proceed via the emission of a photon or by three-body collisions. The relative importance of the various mechanisms strongly changes with the ion velocity and with the temperature and density of the cooling electrons or the nuclear charge of the target atom.

The dependence of REC and NRC cross sections on the nuclear charge Z_t of the target atom is demonstrated in Fig. 38. For light target ions and at high collision velocities ($\beta = v/c = 0.65$ in the example) the REC is the dominant capture mechanism.

At the electron-cooler side, the various contributions can be read from Figs. 39 and 40. The rate coefficient for REC is independent of the density and scales approximately with $T_e^{-1/2}$ [86]. For three-body recombination involving high- n states, the rate coefficient is proportional to the electron density and there are a number of radiative and collisional processes necessary in order to stabilize the ion [85]. For pure collisional recombination the rate coefficient decreases with temperature as $\sim T_e^{-9/2}$. A somewhat weaker dependence is obtained for collisionally transferred electrons that stabilize radiatively. Under practical conditions, with $T_e \approx 0.2$ eV and $n_e < 10^7 \text{cm}^{-3}$, the REC dominates. The recombination processes have important consequences for the ion-beam lifetime which is shown in Fig. 40. The steep rise at low temperature is caused by threebody-collisions and the slower rise by REC. Because of the high efficiency of

Figure 37: The various processes of electron-transfer into bare heavy ions out of a gaseous target or of free cooling electrons.

Figure 38: Measured cross sections for radiative (REC) and non-radiative electron capture for 295 MeV/u U^{92+} impact on various targets compared with theoretical calculations for the two processes [84].

electron cooling observed for the very highly charged ions, electron densities of only a few times 10^6 cm^{-3} are normally used securing beam lifetimes in excess of 1000 s.

The REC process was treated by Stobbe [87] in a nonrelativistic dipole approximation. From this work the rate coefficient as a function of the final-states principal quantum number n is derived. In Fig. 41 it is compared with the simpler Kramers formula [88] expressed for a flattened velocity distribution as

$$\alpha^f(n) = 3.0 \cdot 10^{-13} \text{ cm}^3 \text{ s}^{-1} \cdot Z^2 T_e^{-1/2} n^{-1} \left\{ 1 - \frac{1}{2} \frac{T_e}{E_o} n^2 + \frac{3}{4} \left(\frac{T_e}{E_o} \right) n^4 \mp \dots \right\} \quad (37)$$

where E_o denotes the absolute value of the ground state binding energy. For higher n this formula agrees with the Stobbe theory. The dependence on the angular quantum

Figure 39: Rate coefficients for collisional-radiative recombination of bare uranium ions as a function of the electron temperature. The contributions are due to (---) pure collisional recombination, (···) collisional recombination stabilized radiatively and to (- · -) radiative electron capture, REC. The solid line represents the sum of the three contributions [85].

Figure 40: Calculated beam lifetime as a function of electron temperature at three different electron densities [85].

number ℓ is peaked as shown in Fig. 41. At high n , the peak gets rather narrow and shifts to low ℓ .

The total REC rate can be calculated summing up Eq. (37) truncated at an appropriate maximum quantum number $n_{\max}$ at which field ionization in the toroidal fields of the cooler becomes effective [89]. Experimentally, the REC is observed by recording the time decay of the stored ion beam with DC beam transformers or more directly by measuring ions that have lost one unit of charge in the electron cooler. The ions with the lower charge deviate from the mean orbit and can be measured with multi-wire proportional counters as can be seen in Fig. 42.

3.6 Rates and Measurement Efficiency

The capture processes, discussed above, lead to production of x rays suitable for spectroscopy and their rates may be estimated from the number of primary ions and from the relevant cross sections and rate coefficients. In case of the gas target with a thickness

Figure 41: The dependence of the REC rate coefficient on the final-state principal quantum number n and the angular-momentum distribution for $n = 10$ and for $n = 100$.

Figure 42: Beam profile of Ar^{17+} ions produced by radiative electron capture of Ar^{18+} ions in the electron cooler. The measurement was made with a multi-wire proportional counter mounted in a pocket of the ESR UHV beam pipe located behind the first dipole magnet after the electron cooler. See reference [106].

of $D = 10^{13}\text{cm}^{-2}$ and revolution frequency of $f = 1.6$ MHz the luminosity

$$L = NDf \quad (38)$$

can presently reach values of $\sim 2 \cdot 10^{27}\text{cm}^{-2}\text{s}^{-1}$ assuming $N = 10^8$ stored ions. With a capture cross section of $\sigma_c \approx 7 \cdot 10^{-21}\text{cm}^2$ a rate of 14 MHz results, a large fraction of which will be useful photons. The corresponding time constant by which the stored beam will decay is

$$\tau_{\text{gas}} = \gamma (\sigma_c D f)^{-1} \quad (39)$$

amounting to ~ 10 s using the same numbers as above.

At the electron cooler the time constant for REC is

$$\tau_{\text{REC}} = \gamma^2 (n_e \alpha_{\text{REC}} \eta)^{-1} \quad (40)$$

from which a time constant of ~ 1500 s is estimated for an electron current of 200 mA corresponding to a total rate of some 60 kHz. If necessary this can be decreased by increasing the electron current.

Figure 43: Accumulation of beam intensity during RF stacking and its degradation during measurement due to the influence of residual gas, electron cooler and gas target.

Equally important, however, is the question of how fast the storage ring can be refilled. In Fig. 43 a stacking and measurement cycle are illustrated by the relevant time dependence where, for beam accumulation, the following time dependence is assumed:

$$N(t) = N_0 \frac{1 - \exp(-t/\tau)}{1 - \exp(-t_0/\tau)} \quad (41)$$

Here, N_0 denotes the number of ions gained per injection and t_0 the repetition time. τ denotes the effective beam lifetime mainly determined by the REC in the electron cooler. In Fig. 43, t_0 is set to 10 s and τ to 1500 s. In this example, a large number of shots are necessary before one gets into saturation and further stacking will be uneconomic. After injection is stopped the number of ions decreases with time constants discussed above. The measurement can use up the beam in a much shorter time than necessary to restore it. Therefore an increase of the overall measurement efficiency may be gained by shortening the stacking time. This might be well feasible because there are still substantial losses in the transfer between the SIS synchrotron and the ESR.

4 RECENT PROGRESS IN THE MEASUREMENT OF THE LAMB SHIFT VIA X-RAY SPECTROSCOPY

The $1s$ Lamb shift has been measured through the spectroscopy of the $2\ell \rightarrow 1s$ transitions in a number of different heavy ions. It has been measured in hydrogenlike iron [90], krypton [91] and uranium [92] using fast-beam excitation in solid targets, in chlorine in a Tokamak [93] and in slow argon recoil ions [94]. For heliumlike uranium the $2s$ Lambshift was inferred from a lifetime measurement [96]. Problems mainly arise from the kind of violent collisions by which the highly charged ions are produced and from their high velocity. Therefore, population of spectator-electron states, solid-state effects and the Doppler effect are the main causes of uncertainties. Below, four examples will be reviewed where it is tried to overcome these problems and which have the potential of being further refined when employed to stored ion beams.

4.1 Accel/Strip/Decel at the UNILAC: Ni^{27+}

An experiment [26] was conducted at the UNILAC linear accelerator at GSI in which nickel ions were accelerated, completely stripped, decelerated and dressed with one electron in a thin target gas. The Lyman- α radiation was measured with a curved-crystal spectrometer and the beam velocity was altered in order to bring the Doppler effect under control. The specific advantages of this procedure are:

- single-electron capture in thin gas
- control of Doppler effect by variation of beam velocity
- independent velocity measurement (TOF)
- in-situ wavelength calibration
- well-characterized focusing x-ray optics

Figure 44: Schematic of the apparatus used in the accel/strip/decel experiment at the UNILAC [26].

The experimental configuration used is shown in Fig. 44. Nickel ions accelerated in the first sections of the UNILAC to a specific energy of 11.4 MeV/u were stripped

in a carbon foil and the last section of the accelerator, consisting of single resonators, was used to decelerate the beam to final energies of 8.4, 6.7, 5.2 or 3.1 MeV/u. The component of bare Ni^{28+} ions was magnetically selected and directed into a long straight collimation section followed by the experimental area shown in Fig. 44. The velocity of the ions could be determined to better than $\Delta\beta/\beta < 5 \cdot 10^{-4}$ by a time-of-flight measurement on a 15-m-long flight path. The beam traverses an open gas cell containing argon at a pressure of 0.2 mbar. The 2p states of interest in hydrogenlike Ni^{27+} are populated by one-electron capture, mainly into higher n states followed by cascade transitions.

The x rays of about 8 keV energy are measured with a Johann curved-crystal spectrometer employing a position-sensitive detector of the backgammon type. A Ge(220) crystal cylindrically bent to a radius of about 1 m was used to diffract the x rays in second order. The orientation of the Rowland circle was approximately perpendicular to the ion-beam direction ($\psi_{\text{ion}} \approx 90^\circ$ in Fig. 22). The imperfection of the alignment, however, does not enter into the wavelength transformed into the emitter frame of coordinate because of the velocity variation used. An online calibration was established by continuously observing the partly overlapping Cu-K α spectrum. This was achieved by placing the Cu anode of an x-ray generator close to the gas cell. A chopper wheel synchronized to the 50 Hz time structure of the pulsed ion beam transmitted copper x rays only during the 'off' periods of the ion beam.

Figure 45: Sample spectra of the Cu K α calibration standard and of the Lyman $\alpha_{1,2}$ lines of Ni^{27+} recorded in the position-sensitive proportional counter. The spectra were recorded at a specific energy of 5.2 MeV/u and at $Y \approx 0$. [26].

Sample spectra are displayed in Fig. 45. They were recorded at a specific energy of 5.2 MeV/u and represent a cut near the Rowland-circle plane. This means that the Y coordinate, perpendicular to the direction of dispersion, is approximately zero. For each ion-beam energy the line positions have been found relative to the copper standard at

12 different Y values corresponding to the 12 anode wires in the proportional counter. The results are plotted in Fig. 46 showing the slanted images of the Lyman- $\alpha_{1,2}$ lines. The dashed lines indicate the positions of the Cu $K\alpha_{1,2}$ lines which are not slanted but have a slight curvature not visible in this kind of plot (cf. Fig. 5).

Figure 46: The two-dimensional image of the Lyman- $\alpha_{1,2}$ doublet of Ni^{27+} at four different beam energies. The two dashed vertical lines indicate the positions of the Cu $K\alpha_{1,2}$ lines [26].

The shape of the lines is known analytically. It is given by Eq. (19) that has been tested by Monte-Carlo simulations to a precision of less than a μm . This function was fitted simultaneously to all data points shown in Fig. 46. The velocity β measured independently was inserted into the function as was the wavelength of the copper reference wavelength. The only free parameters were the wavelengths of the Lyman- $\alpha_{1,2}$ lines and the quantity $\cos \psi_{\text{ion}} \cos \phi_{\text{ion}}$ for the orientation of the Rowland circle relative to the ion-beam direction.

For the copper $K\alpha_1$ wavelength the value of Kessler *et al.* [95] was used corrected for remeasurements of the interplanar spacing of silicon [55, 56]. From the fit, the wavelength of the Lyman- α_1 line of Ni^{27+} emerges as

$$\lambda_{\text{Ni}, \text{Ly}\alpha 1} = 1.530\,336(20) \text{ \AA} \quad (42)$$

with an estimated uncertainty of 13 ppm dominated by nonuniformities in the bending of the cylindrical crystal. From the wavelength a 1s Lamb shift of 5.07(10) eV is deduced which is in good agreement with the calculation of Johnson and Soff [8]. The main source of uncertainty is not a principal limitation. After the experiment was completed, crystals have been prepared with 20-times smaller orientation errors than those present in the experiment.

4.2 Doppler-Tuned Spectroscopy at the Bevalac: U^{89+}

In this section, I want to review the experiment conducted by Schweppe *et al.* [97] at the Lawrence Berkeley Laboratory's Bevalac heavy-ion accelerator. This experiment, making use of the Doppler-tuned technique, may be viewed as a very good compromise using a less puristic three-electron system and a high experimental resolution and sensitivity. The authors chose lithiumlike U^{89+} because this ion has the long-lived $1s^2 2p_{1/2}$ state decaying in vacuum behind the solid target used for excitation and the $2s$ - $2p$ transition energy has large QED contributions. Feasibility of a three-electron QED theory is assumed.

Figure 47: Schematic diagram of the Doppler-tuned spectrometer used by Schweppe *et al.*. Figure reproduced with permission from Ref. [97].

In Fig. 47 the Doppler-tuned spectrometer is shown schematically. A beam of ground-state U^{89+} ions with a specific energy of 95 MeV/u ($\beta \approx 0.42$) is transported to the experimental area where a 1.7 mg/cm^2 aluminum foil excites the ions. To measure the $2p_{1/2} - 2s_{1/2}$ energy splitting in lithiumlike uranium near 281 eV, requires a known absorption feature for calibration. The $2p$ - $4s$ resonance in neutral argon gas occurring at 244.39(1) eV [98] is well separated from the rest of the $L_{2,3}$ absorption edge and can be matched with the uranium line when observed at an angle of about 95.5° . The x rays were viewed by six position-sensitive proportional counters arranged symmetrically around the beam axis. The distance between the target and the detectors was 72 cm. Tapered Soller-slit collimators in front of each detector were focused on a 1.2-cm-long section of the beam centered 1.3 cm downstream of the target foil and low-pressure argon-gas cells were positioned in front of each collimator.

The transition energy E_{ion} of the x rays in the rest frame of the lithium like uranium can be calculated via the Doppler formula (13) if the energy of the argon resonance E_{res} , the angle θ_{lab} at which the resonance is observed and the beam velocity β are known.

The angular measurement is performed by a careful optical alignment of each detector and by measuring their position responses. Through the use of six detectors, some cancellation of alignment errors occurs. The beam-velocity was measured through the time of flight on a 19-m-long path. The resulting spectrum is shown in Fig. 48 for one of the six detectors. Plotted is the ratio of the x-ray intensities obtained with and

Figure 48: The absorption profile obtained with one of the six detectors of the Doppler-tuned spectrometer. Plotted is the ratio of the intensities with and without argon gas. The solid line is a fit of a Monte-Carlo calculation to the data. Figure reproduced with permission from Ref. [97].

without argon gas in the cells. In each case the intensity was normalized to the beam current. The solid line is a fit of a Monte-Carlo calculation to the data. The calculation includes the absorption profile, the angular divergence and the energy spread of the ion beam and the decay length. Data were collected in six runs divided between two beam velocities resulting in a final value of $280.59(10)$ eV for the $2p_{1/2} - 2s_{1/2}$ energy splitting.

Systematic effects considered are the simultaneous population of neighboring charge states with similar transitions. In Be-like and He-like uranium the corresponding transitions are at 295 eV [99] and at 256 eV [96] and can energetically be discriminated with the ~ 1 eV instrumental resolution. From the measurement of the intensity as a function of the distance downstream from the target a lifetime of $61.8 \pm 1.2(\text{stat}) \pm 1.3(\text{syst})$ ps was derived for the $2p_{1/2}$ state in lithiumlike uranium in excellent agreement with the theoretical value of 60.7 ps [99]. Significant contributions from long-lived spectator-electron states $1s^2 2p, n\ell$ have been ruled out by the authors on the basis of estimated lifetimes and possible energy shifts. By way of illustration, the lowest s state with a mean life greater than 1 ps is at $n \approx 25$ and it would cause an energy shift of only 0.01 eV. The p and d states decay faster and produce smaller effects.

In theoretical calculations to be compared with the experimental result, uncertainties mainly arise from the many-body and screening part of the theory. In order to match the theoretical uncertainties with the small uncertainty of the experiment, inclusion of higher-order effects like the exchange part of the screening of the self-energy and of the vacuum polarization become important. At the time of the completion of the experiment those corrections were not yet implemented in the theory resulting in estimated uncertainties of ~ 1 eV [97]. In the meanwhile theoretical work on the Lamb-shift screening has grown and is still in progress [100, 101, 102]. In addition, two-photon Breit-Coulomb and mixed self-energy-vacuum-polarization (SEVP) corrections [12, 103] have been included in the theory. In Table VII theoretical calculations of Lindgren *et al.* [12, 103] and of Blundell [101] are compared with the experiment.

Table VII: Transition energy for the $2p_{1/2} - 2s_{1/2}$ transition in lithiumlike uranium in eV (cf. reference [12, 103, 101]).

Physical effect	Lindgren <i>et al.</i>	Blundell
Relativistic MBPT	322.33(15)	322.41
Valence self-energy, direct	-54.07(3)	-54.09
Valence self-energy, exchange	-0.17(5)	0.36
Core self-energy	-0.08(8)	-0.51(2)
Vacuum polarization (incl. screening)	12.59(1)	12.56(1)
Nuclear recoil and polarization	0.10	0.10
Breit-Coulomb reducible correction	0.04	
Second-order SEVP correction	-0.19	
Higher-order effects		0.01(4)
Total transition energy	280.55(20)	280.83(10)
Experimental	280.59(10)	

Although theoretical work might be regarded as not yet complete, at the present time, there is very good agreement between experiment and the theory of Lindgren *et al.*. Table VII also proves the importance of the SEVP corrections which might explain most of the difference in the two calculations.

4.3 ESR Gas Jet: Dy^{65+} , U^{91+}

4.3.1 Experimental Method

In this section first spectroscopic results [104, 105] from the ESR gas jet are considered in which the Lyman and Balmer series in hydrogenlike Dy^{65+} and U^{91+} , along with the radiative electron capture (REC) into bare projectiles have been observed.

Figure 49: Two Ge(i) x-ray detectors were placed at observation angles of approximately 132 and 48 degrees relative to the ion-beam direction. Ions which captured an electron in the argon gas jet were registered in the particle detector after the dipole magnet [104].

Fully stripped Dy^{66+} ions at a specific energy of 292 MeV/u were accumulated in the ESR with peak electrical currents approaching 1 mA. It was possible to maintain

an internal argon gas jet with a thickness of $6 \cdot 10^{12} \text{cm}^{-2}$. X rays emitted from the target region were measured in delayed coincidence with hydrogenlike ions, formed in the gas target. For the detection of the hydrogenlike ions a position-sensitive multi-wire proportional counter (MWPC) [106] was used located downstream of the target after the next ESR bending magnet (see Fig. refdyapp). The Two Ge(i) x-ray detectors were placed at observation angles of approximately 48° and 132° relative to the ion-beam direction. During part of the measurements a segmented Ge(i) detector with seven independent vertical stripes was placed at the 48° position. The latter angle is close to the magic angle $\cos \vartheta = \beta$ at which the velocity sensitivity vanishes at the price of a maximum angular dispersion.

Figure 50: Coincident x-ray spectrum measured at 132 degrees. Prominent lines arise from the $n\ell \rightarrow 1s$ Lyman series in hydrogenlike dysprosium and from the radiative electron capture into the low-lying states of Dy^{65+} [104].

Figure 51: X-ray spectrum measured under 48 degrees. The prominent lines arise from the Lyman series in hydrogenlike dysprosium [104].

4.3.2 REC in the Gas Target

Coincident Dy spectra are shown in Figs. 50 and 51. They reveal the Doppler-shifted characteristic lines and in case of the 132° spectrum also the REC into the K, L and M shell. Practically no background from random coincidences is observed. It would be a great advantage if one could simultaneously use the K REC and the Lyman lines

for a precise measurement of the 1s binding energy. This would require, however, a complete understanding of the broad line profile of the REC peak. In the REC process a radiative transition of an initial target electron into a final projectile state occurs. The centre of the line appears near an energy of $T_r + |E_p| - \gamma|E_t|$, where T_r denotes the kinetic energy of an electron traveling with the same velocity (βc) as the projectile, and E_t and E_p are the initial and final-state binding energies, respectively. The effect of the initial binding of the electron to the target is to broaden the REC x-ray line into a distribution with finite width. Within the impulse approximation, one assumes the initial states to be unperturbed target atomic states [107, 108]. The double-differential cross section for REC may be written as

$$\frac{\partial^2 \sigma^{REC}}{\partial \Omega \partial E_x} = \sum_{n\ell} \int d^3 \vec{p}_e |\Phi_{n\ell}(\vec{p}_e)|^2 2(2\ell+1) \frac{\partial \sigma^f}{\partial \Omega} \delta(E_x - (|E_p| - \gamma|E_t| + T_r - \gamma\beta c p_{ez})) \quad (43)$$

where the sum runs over all target $n\ell$ subshells, and $\partial \sigma^f / \partial \Omega$ is the differential cross section for the radiative capture of a free electron. E_x denotes the photon energy and $\Phi_{n\ell}(\vec{p}_e)$ the initial electron wavefunction in momentum space. p_{ez} represents the projection of the electron momentum onto the direction of the collision velocity. The integration is performed over all initial electron momenta $\vec{p}_e$ where the delta function takes care of the energy conservation by the photon emission. The factor $2(2\ell+1)$ is the number of electrons per subshell. Eq. (43) may be written as

$$\frac{\partial^2 \sigma^{REC}}{\partial \Omega \partial E_x} = \frac{1}{\gamma\beta c} \frac{\partial \sigma^f}{\partial \Omega} \mathcal{J}(p_{ez}) \quad (44)$$

where $\mathcal{J}(p_{ez})$ denotes the Compton profile of the target atom which is the probability for finding the momentum component p_{ez} .

Figure 52: Calculated double differential cross section for radiative electron capture into the dysprosium K shell. Displayed are the contributions of the argon subshells and their total sum [104].

The calculated double-differential cross section for radiative capture into the dysprosium K shell is plotted in Fig. 52 where the individual contributions from the argon-target subshells are given in addition to the summed cross section. The skewness of the summed REC line observable in Fig. 52 is attributable to two effects. First, the contributions from the different target orbitals are located at different positions according to their binding energies. Secondly, the individual contributions exhibit a skewness due to the decrease of the REC cross section with photon energy. The result is a noticeable

centroid shift to lower x-ray energies relative to the position $(T_r - \gamma|E_t|)$ predicted if the profile were symmetric. This is a critical point if one wishes to deduce inner-shell binding energies from the centroid of measured REC profiles. If neglected [109], measured binding energies can easily be in severe error.

Figure 53: Experimental REC spectrum (data points) and theoretical fit (full line) [104].

A simple estimate of the shift can be obtained using hydrogenic wave functions for the target atom resulting in

$$\delta E_{\text{REC}} = -\frac{5}{3} \frac{\gamma^2 \beta^2}{\gamma - 1} |E_t| \quad (45)$$

In Fig. 53 the REC spectrum is displayed logarithmically, where the measurement is represented by marker and the continuous curve shows the least-squares fit with the theoretical profiles. The absolute height and the energy position were the only free parameters. An excellent correspondence between experiment and theory is obtained.

4.3.3 The Lyman- α Energy of Dy^{65+}

Now, the Lyman α energy can be obtained without relying on the Doppler correction calculated from velocity and observation angle. The Doppler factor $\gamma(1 - \beta \cos \vartheta)$ can be obtained spectroscopically as the calculated energy difference between the Lyman α lines and the K-REC line divided by the corresponding energy difference of the measured spectrum. In the calculation only the small QED contributions for the L-shell binding energy are involved (for details see reference [104]).

Table VIII: Lyman- α_1 energy of Dy^{65+} in keV. The REC spectrum was used to determine the observation angle. The theoretical transition energy of Johnson and Soff [8] is given for comparison.

	Experiment	Theory
$2p_{3/2} \rightarrow 1s_{1/2}$	48.025(38)	48.038

The result of this procedure is shown in Table VIII where the 38 eV uncertainty is mainly determined by the counting statistics and by the energy calibration. A conservatively estimated uncertainty of 20 eV was included for the theoretical calculation of

the REC profile. Within the quoted uncertainties, the experimental transition energy agrees well with the theoretical value of Johnson and Soff [8].

The observation angle determined spectroscopically amounted to $132.19 \pm 0.06^\circ$ in perfect agreement with the less precise mechanical measurement of $132.02 \pm 0.15^\circ$. The final energy uncertainty could easily have been reduced by a factor of two to three by an advanced optical alignment of the detectors.

4.3.4 The 1s Lamb Shift in U^{91+}

The experiment with 295 MeV/u U^{92+} ions [105] stored in the ESR used the same apparatus with a slightly different measurement strategy. By laser-assisted trigonometry the detector positions with respect to the ideal ion-beam axis was determined with an angular precision of $\pm 0.01^\circ$. The geometry of an improved segmented detector (centered at $48.30 \pm 0.04^\circ$) was known very precisely. Therefore, the spectra from the seven segments could be individually corrected for Doppler shift and were summed up accordingly (for details see reference [105]). In Fig. 54 two individual spectra of the Lyman- α region are displayed taken from the most forward and from the middle segment, respectively. Also shown is the summed spectrum transformed into the emitter frame of reference.

Figure 54: (a) X-ray spectra coincident with electron capture in 295 MeV/u $U^{92+} \rightarrow$ Ar collisions taken by the most forward and middle (hatched area) segment of the multistriple detector at 48° . (b) Doppler-corrected sum spectrum transformed into the c.m. system. Figure reproduced with permission from Ref. [105].

As in the Dy spectra, the peak at lower energy is a blend originating from the E1 decay of the $2p_{1/2}$ state and of the M1 decay of the $2s_{1/2}$ state into the $1s_{1/2}$ ground state. Assuming a statistical population of the $2p_{1/2,3/2}$ states allows to calculate the relative contributions in the M1 plus Lyman- α_2 line. For Ar and N_2 targets the M1 transition contributes 70% and 80%, respectively. This is consistent with the REC population

Table IX: Experimental [105] Lyman- α_1 and (Lyman- α_2 + M1) energies of U^{91+} , in keV, compared to theory [8].

	Experiment	Theory
$2p_{3/2} \rightarrow 1s_{1/2}$	102.209(63)	102.180
$2s_{1/2}, 2p_{1/2} \rightarrow 1s_{1/2}$	97.706(61)	97.673

mechanism favoring low angular momenta. Using the statistical ratio and the theoretical energy difference between the $2p_{1/2}$ and the $2s_{1/2}$ state leads to a correction for the blend. In Table IX the experimental energies are compared with the theoretical calculation of Johnson and Soff [8] revealing a very good agreement within the claimed uncertainties of 63 and 61 eV, respectively. The corresponding experimental 1s Lamb shift amounts to 429 ± 63 eV.

4.4 ESR Cooler: Au^{78+} , U^{91+}

Figure 55: Level scheme for a hydrogenlike ion and a measured x-ray spectrum for bare Au^{79+} traversing the electron cooler. The line with the highest energy originates from a direct transition of cooling electrons into the 1s ground state. Because the electrons travel with precisely the same mean velocity as the ions with a very narrow distribution this transition energy equals the 1s binding energy. Other lines in the spectrum are due to the characteristic $n\ell \rightarrow 1s$ series induced by direct radiative capture into higher $n\ell$ states with subsequent cascade transitions.

As explained in section 3.5, spontaneous capture of free cooling electrons by bare high-Z ions leads to the emission of x rays. The distinctive features of this new x-ray source have been used in two experiments to measure the 1s Lamb shift in hydrogenlike Au⁷⁸⁺ [110] and in U⁹¹⁺ [111].

4.4.1 Excitation Mechanism

The REC process for free cooling electrons is illustrated in the level scheme of Fig. 55 which already contains a sample spectrum obtained with Au⁷⁹⁺ projectiles. The cooling electrons are situated just at zero energy as guaranteed by the drag force [71] between electrons and ions balancing the mean ion and electron velocity in the laboratory system. Due to the low electron temperature of approximately 0.2 eV the corresponding spectral width is small compared to the experimental width and uncertainties. This is a very important difference to the case where the electrons initially are bound to a target nucleus as presented in section 4.3. For the electron transition into the ground state ($\infty \rightarrow 1s$) the energy of the released photon equals the 1s binding energy.

The prominent lines observed, can be attributed to the direct transition of cooling electrons into the K shell of the projectile and to the characteristic $n\ell \rightarrow 1s$ transitions. The latter are also induced by capture into higher shells and subsequent cascade transitions. Principal quantum numbers up to $n_{\max} \approx 400$ (for uranium) are involved, beyond that the electrons get reionized by transverse magnetic fields in the cooler. For details see reference [89].

Figure 56: Lifetimes $\tau(n, \ell)$ and rate coefficients $\alpha(n, \ell)$ as a function of n and ℓ/n . For a better visibility, $\alpha(n, \ell)$ is multiplied by $n/\alpha(n)$ where $\alpha(n)$ is the rate coefficient summed over all ℓ substates for a given n .

For low n the decay times of the excited states are much smaller than the flight

time through the 2.5-m-long cooling section of about 13 ns. In case of hydrogenlike uranium this holds true up to $n \approx 100$. For $n\ell$ states above $n = 100$, the lifetimes and the REC rate coefficients $\alpha(n, \ell)$ are plotted in Fig. 56. Because states of very high ℓ are not strongly populated the very large decay times do not appreciably contribute. There is, however, an intermediate range from which a delayed cascade decay into the $n = 2$ states can be expected (also see Fig. 55). The summation of the $\alpha(n, \ell)$ and the consideration of the cascade transitions leads to a theoretical prediction of the Lyman- α intensity. It turns out that direct population is much weaker than the cascade population and that the intensity ratio between direct K REC and Lyman- α is correctly described by theory [89].

4.4.2 Experimental Details

Bare projectiles were stored in the ESR storage ring at specific energies near 300 MeV/u and x-rays emitted from the 2.5-m-long cooling section were measured in delayed coincidence with particles that have lost one unit of charge by capture of a cooling electron. The experimental arrangement is schematically shown in Fig. 57. The x rays were measured with an intrinsic Ge(i) germanium detector located 5.4 m downstream of the electron cooler in a $\sim 0^\circ$ geometry where angular uncertainties are not critical.

Figure 57: Schematic view of the experimental arrangement at the electron cooler. X rays emitted from the cooling section into the forward direction are measured with an intrinsic Ge(i) detector in delayed coincidence with ions that have captured one electron. The ions are registered by a multi-wire proportional counter (MWPC) [110].

Because of the phase-space reduction of the stored ion beam, Doppler broadenings are much reduced. The velocity spread of the beam was typically $\delta\beta/\beta = 10^{-5} \dots 10^{-4}$. For an observation angle ϑ near 0° and $\beta \approx 0.65$ the x-ray energy is as large as $E_x \approx 2.2E_0$. For the high- Z ions the K-series lines can be observed beyond the Bremsstrahlung limit defined by the terminal voltage U of the electron cooler $eU = mc^2(\gamma - 1)$, where e denotes the elementary charge. This condition plus the coincidence with the particles leads to an extremely small background. As explained in section 2.4.1, angular uncertainties are not critical near 0° . An uncertainty $\Delta\beta$ of the beam velocity, however, enters as $\Delta E_x/E_x \approx \gamma^2 \Delta\beta$ (cf. Table IV). Therefore the beam velocity has to be known very well.

4.4.3 Beam Velocity

The beam velocity has been determined by three different methods which agree within combined uncertainties as demonstrated in Fig. 58. The measurement via the cooler

Figure 58: The ion-beam velocity β determined by three different methods: (U) the terminal voltage of the electron cooler, (x) spectroscopically and (f) the revolution frequency of the stored ions.

voltage is the most precise method and was used for the data analysis. At the time of the completion of the Au experiment [85], the situation was different because it was not possible to estimate a confidence range of the cooler voltage. Therefore, a purely spectroscopic measurement of the beam velocity was adopted. In the meantime, the high voltage device has been calibrated resulting in an estimated uncertainty of ± 25 V at 180 kV or $\Delta\beta = \pm 3 \times 10^{-5}$. A small correction for the space charge of the electron beam [71] was also included.

Spectroscopically, the beam velocity β is calculated from the Doppler factor given in Eq. (13), $\gamma(1 - \beta \cos \vartheta)$, which can be inferred as the ratio of the calculated binding energy of the $2p_{3/2}$ state and the measured energy difference between the K-REC and the Ly- α_1 line.

The third method simply uses the revolution frequency f of the particles, which can be measured very precisely by Schottky diagnosis, and the circumference C of the storage ring. As a detailed measurement of the particle path has not yet been carried out, it is difficult to estimate a confidence range for the effective circumference. The uncertainty bars in Fig. 58 correspond to a guessed uncertainty of $\Delta C \approx \pm 2$ cm out of $C = 108.25$ m.

4.4.4 Calibration of the X-Ray Energy

The x-ray energies could be precisely measured relative to closely spaced gamma-ray lines the energies of which have previously been determined on an absolute scale with high precision.

Only very small energy differences had to be measured by fine (Doppler) tuning the interesting lines to overlap with suitable gamma-ray lines originating from ^{192}Ir [112] and ^{182}Ta [113] isotopes. An example is given in Fig. 59 where the K REC of uranium is tuned to the 295.956 keV line of ^{192}Ir . The lines were fitted with Gauß profiles for determination of the line positions. The line from the fast beam has the same width, of about 950 eV, as the calibration line confirming that Doppler broadening and influences from the electron temperature are both small.

Figure 59: The K-REC line of U^{91+} and the 295.956 keV gamma line of ^{192}Ir used for calibration [111].

4.4.5 Cascades and Lineshape

A theoretical calculation of the line profile for the Lyman lines is necessary because these lines are strongly influenced from cascades which can lead to a delayed population. In Fig. 60 the detection geometry is sketched showing the relative position of the detector to the electron and ion beams. The vertical dimension has been exaggerated as the detector is separated from the ion beam by only 42 mm. For prompt events the x rays are emitted from the 2.5-m-long cooling section with a mean observation angle of only $\sim 0.5^\circ$. For a single cascade with a time constant τ of about 10 to 50 ns much of the emission will take place close to the detector with an accordingly larger observation angle associated with a substantial Doppler shift towards low x-ray energy.

Figure 60: Schematic of the detection geometry. The vertical dimensions are exaggerated.

The number of x rays emitted along the flight path x (defined in Fig. 60) may be written as

$$\frac{\partial N_x(n, \ell)}{\partial x} = \gamma^{-2} n_e N_o \eta \alpha(n, \ell) \times \begin{cases} 1 - \exp \{-x/(\beta \gamma c \tau)\}, & x < L \\ (1 - \exp \{-L/(\beta \gamma c \tau)\}) \times \\ \exp \{-(x-L)/(\beta \gamma c \tau)\}, & x \geq L \end{cases} \quad (46)$$

where n_e , N_o and η denote the electron density, the Number of stored ions and the ratio of the cooling length L to the circumference of the storage ring. c denotes the velocity of light and $\alpha(n, \ell)$ is the rate coefficient for REC into an n, ℓ state.

The detected intensity distribution $\partial I_{n\ell}/\partial E_x$ is obtained by multiplying with the

Lorentz-transformed solid angle $\Omega(x)$,

$$\frac{\partial I_{n\ell}}{\partial E_x} = \frac{\Omega(x)}{4\pi} \frac{\partial N_x(n, \ell)}{\partial x} \frac{\partial x}{\partial E_x}. \quad (47)$$

The required relation between x and E_x is obtained from Eq. (13) taking into account the change of the observation angle $\vartheta(x)$ with x . For the summation of all contributing initial states a special grid in the n, ℓ space is used sorting by their decay times τ_i

$$\frac{\partial I}{\partial E_x} = \sum_i \sum_{n\ell, \tau_i \leq \tau(n, \ell) < \tau_i + \Delta\tau_i} \alpha(n, \ell) f_{n, \ell} \frac{\partial I_{n\ell}}{\partial E_x} \quad (48)$$

Here, $f_{n, \ell}$ denotes the cascade fraction which is close to 1 for high initial n and for the final 2p states considered. It is noted that the cascades into the final 2s state are much smaller [89]. A consequence of that is a rather small 2s admixture to the $2p_{1/2}$ peak for which a small energy correction has been applied.

The summation in (48) has been carried out numerically and has subsequently been convoluted with the detector response function. The result can be accurately parameterized as a Gauß function on top of a single-sided linear slope. The influence of this modeling on the extracted energy position was checked numerically and was found to be in the order of only $\sim 2 - 5$ eV (in the laboratory system).

Figure 61: The Lyman- α lines of U^{91+} . The intensity appearing on the low-energy side of the peaks is caused by delayed cascade feeding.

As can be seen in Fig. 61, displaying the Lyman- α lines of U^{91+} , the theoretical profile fits the experimental shape very well. For the Au^{78+} lines the effect is much less pronounced (see Fig. 55). This is attributed to a wider and consequently more shallow distribution when the contributing lifetimes are increased.

4.4.6 Results

From the simultaneous measurement of the three strongest lines in the spectra, i.e. the $2p_{1/2}, 2p_{3/2} \rightarrow 1s$ lines and of the $\infty \rightarrow 1s$ line, it is possible to extract three independent values of the 1s binding energy. By subtraction of the 1s Dirac energy, corrected for reduced-mass effect, the 1s Lamb shift is obtained. In order to check possible systematic effects, the data are subdivided as shown in Fig. 62 for the Au results. There, also the two different beam velocities, documented in Fig. 58, are involved. Individual uncertainties mainly determined by counting statistics are typically $\pm 20-30$ eV. All 25 data points resemble an internal variance of $(3.3 \text{ eV})^2$ and an external variance of

(3.1 eV)². There is no significant difference in the three groups of data. A similar procedure, though involving only one velocity, was used for the uranium data. They are given in Fig. 63, where the full horizontal line represents the weighted mean of the three data points and the hatched area represents the estimated confidence range of the final value. The systematic uncertainties caused by the beam velocity amount to 4.5 eV and to 6.9 eV for the Au and U ions, respectively. They were added linearly to the uncertainties mentioned before.

In Table X, the results are summarized having total uncertainties of 7.9 and 16 eV, respectively. They are compared with theory, as given by Johnson and Soff [8], by Soff [114] and by Mohr [115]. In case of uranium they are also compared with the experiments of Stöhlker *et al.* [105], obtained at the ESR gas jet (see section 4.3.4), and with the experiment of Briand *et al.* [92] conducted at the Bevalac. A very good overall agreement can be stated. The uncertainties of the theoretical results are smaller than those of the experiment. Differences in the theoretical calculations partly arise from recently [114], [115] considered smaller contributions, with estimated uncertainties near 0.1 to 0.3 eV. Also, new nuclear charge radii have been assumed for the finite-size correction. For uranium, the latter is comparable, in magnitude, to the self-energy correction.

Figure 62: The 1s Lamb shift of Au^{78+} as derived from subsets of the data which are grouped according to the three spectral lines from which the results have been obtained. The horizontal line indicates the weighted mean of all data.

Table X: Experimental and theoretical 1s Lamb shift (eV) in hydrogenlike gold and uranium.

Reference	Z=79	Z=92
Experiment		
Beyer <i>et al.</i> [110, 111]	202.3 ± 7.9	470 ± 16
Stöhlker <i>et al.</i> [84]		429 ± 63
Briand <i>et al.</i> [92]		520 ± 130
Theory		
Johnson and Soff [8]	205.3 ± 0.5	458.4 ± 2.7
Soff [114]	203.9 ± 0.5	463.4 ± 0.6
Mohr [115]		464.6 ± 0.6

Figure 63: The 1s Lamb shift of U^{91+} obtained from the three spectral lines indicated. The solid line indicates the weighted mean of the three data points and the hatched area represents the calculated confidence range including the systematic uncertainty. The dashed lines show the standard deviation dominated by the counting statistics. The arrow shows the theoretical value of reference [115].

5 SUMMARY AND PERSPECTIVES

In summary, a number of x-ray-spectroscopic techniques have been identified that can help to measure the inner-shell binding energies of the heaviest one and few-electron ions. Most of the techniques are well-established. However, adaption to fast-ion beams requires a careful optimization of detection geometry and spectrometers.

Clean one-electron spectra can be produced at the gas jet and at the electron cooler of a heavy-ion storage ring starting with completely ionized stored ions. This has been exemplified with the experiments conducted at the ESR storage ring at GSI which presently is the only storage ring that can store and cool bare U^{92+} ions.

Figure 64: The 1s Lamb shift plotted as (experiment - theory) divided by theory. For some of the data points the numerical values of their uncertainties are included.

First encouraging results on the 1s Lamb shift have been obtained making use of this new facility. Fig. 64 gives an overview of the status of such measurements. Illustrated is the relative deviation between experimental and theoretical 1s Lamb shift as a function of the nuclear charge Z . Besides the very precise data point at $Z = 1$ [116] obtained by Doppler-free Laser spectroscopy, all other data points are the result of x-ray spectroscopy [93, 94, 90, 26, 91, 104, 110, 92, 84, 111]. With the most recent experiments at the electron cooler, the experimental uncertainty at the high end of the Z scale approaches that reported for the intermediate region between $Z = 18$ and $Z = 28$.

Improvements can be anticipated from an optimization of the beam injection (see section 3.6). The time necessary for beam accumulation, presently more than half an hour, should be reduced drastically. This will increase the measurement duty cycle allowing higher spectral resolution to be employed. Doppler-tuned experiments [61, 62] are planned both at the gas target and at the electron cooler maintaining a high detection efficiency.

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