CP violation at colliders

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Abstract

The prospects of experimental detection of CP violation at $e^+e^-$ and $pp/p\bar{p}$ colliders are reviewed. After a general discussion on the quantities which can measure CP violation and on the implications of the CPT theorem, various possibilities of measuring CP violation arising outside the standard model are taken up. CP violation in leptonic processes, especially polarization effects in $e^+e^- \rightarrow l^+l^-$ are discussed next. CP violation in $t\bar{t}$ and $W^+W^-$ production and decay is also described.

1. Introduction

1.1 CP violation in the standard model

CP violation in the standard model (SM), as is well known, arises due to complex Yukawa couplings, and finally shows up through quark mixing in the Cabibbo-Kobayashi-Maskawa matrix as a single phase. The reparametrization invariant quantity

$$J = \sin^2 \theta_1 \sin \theta_2 \sin \theta_3 \cos \theta_1 \cos \theta_2 \cos \theta_3 \sin \delta$$

(1)

is a measure of CP violation in SM, and though small, produces measurable effects in the $K$ meson system, and hopefully the effects in the $b$-quark states will be measurable. The effects in other sectors (as for example the neutron electric dipole moment) and in high-energy processes is generally predicted to be too small to be observed.
In the leptonic sector of SM the prospects of $CP$ violation being observable are worse, since there is no analogue of the CKM phase in the absence of neutrino masses. $CP$ violation in leptonic systems has to feed in from the hadronic sector through loops. For example, electric dipole moments (EDM) of charged leptons are induced due to the $W$ EDM, and are found to vanish up to three-loop order. The electron EDM is then estimated to be around $10^{-41}$ cm. [1]

Thus, any new observable $CP$ violation would be a signal of non-standard physics. It might be mentioned that perhaps there is already a hint towards non-standard $CP$ violation in current ideas on electroweak baryogenesis.

1.2 Scenarios for $CP$ violation beyond SM

$CP$ violation beyond SM can arise due to almost any extra Yukawa couplings which can be complex, and possibly also due to new Higgs self-interactions and complex scalar vacuum expectation values. Thus, introduction of extra fermions or scalars could give rise to new sources of $CP$ violation.

Retaining the gauge group to be $SU(2)_L \times U(1)$, $CP$ violation can arise due to extra fermion or Higgs doublets or singlets. Since the SM measure of $CP$ violation $J$ (eq.(1)) is small owing to the small mixing angles among quark generations as experimentally observed, larger $CP$ violating effects would arise if there are extra generations of quarks, whose mixing angles may be less constrained. If there are new fermions (quarks or leptons) in exotic representations (left-handed singlets and/or right-handed doublets of $SU(2)_L$) there are further complex flavour-changing couplings to $Z$ which violate $CP$.

Supersymmetry requires the addition of extra scalars and fermions, whose couplings violate $CP$. In left-right symmetric models, again, there further sources of $CP$ violation.

1.3 Use of effective Lagrangians for model-independent analysis

There is a large variety of sources of $CP$ violation beyond SM, and rather than discuss predictions of each model for each observable quantity, it is more economical to analyze $CP$-violating quantities in terms of the parameters of an effective Lagrangian. Examples of $CP$-violating terms in an effective
Lagrangian with which we will be concerned here are given below:

\[ \mathcal{L}_{\text{eff}} = -\frac{i}{2} \sum_i d_{\psi_i} \overline{\psi}_i \sigma^{\mu\nu} \gamma_5 \psi_i F_{\mu\nu} - \frac{i}{2} \sum_i \tilde{d}_{\psi_i} \overline{\psi}_i \sigma^{\mu\nu} \gamma_5 \psi_i (\partial_\mu Z_\nu - \partial_\nu Z_\mu) \]

\[ + \sum_{V=A,Z} \frac{i g_V}{m_W} \left[ \tilde{W}_\mu^\dagger W_\nu \tilde{V}^{\mu\nu} + \frac{\lambda_V}{m_W} W_\mu^\dagger W_\nu \tilde{W}_\nu \right] \]

\[ + g_f^V W_\mu^\dagger (\partial_\mu V^\nu + \partial_\nu V^\mu) \]  

(\tilde{V}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} V_{\alpha\beta}; \tilde{V}_\mu^{\nu} = \partial_\nu V_\mu - \partial_\mu V_\nu; W_\mu^{\nu} = \partial_\nu W_\mu - \partial_\mu W_\nu). \] These terms are of dimension \( \leq 6. \) In eq.(2), \( \psi_i \) refers to various fermionic fields (quarks and leptons), whose electric and “weak” dipole moment is given by \( d_{\psi_i} \) and \( \tilde{d}_{\psi_i}, \) respectively. It should be noted that all the parameters are in reality scale dependent “form factors”, and can be complex.

2. Some general considerations

2.1 Observable quantities which measure \( CP \) violation

There are basically two types of observables which can be used to characterize \( CP \) violation: asymmetries and correlations.

An example of an asymmetry is the partial-width asymmetry for decay of particles \( i \) and \( \bar{i} \) which are \( CP \) conjugates of each other:

\[ A = \frac{\Gamma(\bar{i} \to f) - \Gamma(i \to \bar{f})}{\Gamma(\bar{i} \to f) + \Gamma(i \to \bar{f})}. \]  

(3)

If \( CP \) is a symmetry of the theory, \( A = 0. \) Non-vanishing \( A \) implies violation of \( CP. \) \( A \) is a convenient parameter because it is dimensionless and lies between \(-1\) and \(1.\) In particular, if \( i \) is an eigenstate of \( CP, \) \( \bar{i} = i, \) and \( A \) in (3) simplifies and measures the fractional difference in the decay rates of \( i \) to two \( CP \)-conjugate states \( f \) and \( \bar{f}. \) As we shall see later, the \( CPT \) theorem implies that \( A \) is zero even if \( CP \) is violated unless the amplitude has an absorptive part which can arise because of final-state interactions or loop effects in perturbation theory.

Another type of asymmetry is an asymmetry in a final-state variable like
energy or angle. It is defined in general (for \(i = \overline{i}\)) as

\[
g = \frac{\sum_{f \in S} d\sigma(i \rightarrow f) - \sum_{f \in \overline{S}} d\sigma(i \rightarrow f)}{\sum_{f \in S} d\sigma(i \rightarrow f) + \sum_{f \in \overline{S}} d\sigma(i \rightarrow f)}.
\]

(4)

Here \(S\) and \(\overline{S}\) are sets of states with \(CP\)-conjugate kinematic ranges, and \(f\) is a final state assumed to have particles conjugate to one another in pairs. An example is energy asymmetry between the energies \(E_+\) and \(E_-\) of \(CP\)-conjugate particles in \(f\):

\[
g = \frac{\int_{E_+ > E_-} d\sigma(i \rightarrow f) - \int_{E_+ < E_-} d\sigma(i \rightarrow f)}{\int_{E_+ E_-} d\sigma(i \rightarrow f)}.
\]

(5)

The other category of quantities consists of \(CP\)-odd correlations which are expectations values of \(CP\)-odd operators in a process with initial as well final states described by \(CP\)-even density matrices. Thus for an observable \(O\left\{p_{A_i}, s_{A_i}\right\}\) which is a function of momenta \(p_{A_i}\) and spins \(s_{A_i}\) of particles \(A_i\), and which transforms under \(CP\) as

\[
O\left\{p_{A_i}, s_{A_i}\right\} \rightarrow O\left\{-p_{A_i}, s_{A_i}\right\} = -O\left\{p_{A_i}, s_{A_i}\right\},
\]

the \(CP\)-odd correlation is

\[
\langle O \rangle = \frac{\int d\sigma O\left\{p_{A_i}, s_{A_i}\right\}}{\int d\sigma}.
\]

(6)

A non-zero value of such a correlation signals \(CP\) violation.

It may be noted that an asymmetry in the variable \(\mathcal{O}\) may be described as a special case of a correlation of \(\epsilon(O)\), where \(\epsilon\) is the antisymmetric step function:

\[
\langle \epsilon(O) \rangle = \frac{\int_{\mathcal{O} > 0} d\sigma - \int_{\mathcal{O} < 0} d\sigma}{\int d\sigma}.
\]

(7)

2.2 Statistical significance

Whether or not a measured asymmetry or correlation can really signal \(CP\) violation naturally depends on its statistical significance decided by the statistical fluctuation expected in the event sample.

For a rate asymmetry \(A\), the number of asymmetric events \(\Delta N\) is

\[
\Delta N = AN,
\]

(9)
where $N$ is the total number of events in the channel considered. The statistical fluctuation in these $N$ events is $\sqrt{N}$. Hence for discrimination of the signal, at the one standard deviation level, we require

$$\Delta N > \sqrt{N},$$  \hspace{1cm} (10)$$

or

$$A > \frac{1}{\sqrt{N}}.$$  \hspace{1cm} (11)$$

Thus, it would be possible to measure an asymmetry if its predicted value is larger than $1/\sqrt{N}$. To put it differently, the number of events should be larger than $1/A^2$.

In the case of a $CP$-odd correlation $\langle \mathcal{O} \rangle$, $CP$-invariant interactions can give individual events with $\mathcal{O} \neq 0$, averaging out to zero. Thus the mean square deviation

$$\Delta \mathcal{O} = \sqrt{\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2}$$  \hspace{1cm} (12)$$

is a measure of the background coming from $CP$-invariant interactions. For $N$ events in the channel, the $CP$-even events give rise to a fluctuation $\Delta \mathcal{O}/\sqrt{N}$. The signal $\langle \mathcal{O} \rangle$ should be larger than this to be measurable at the one standard deviation level:

$$\langle \mathcal{O} \rangle > \frac{1}{\sqrt{N}} \sqrt{\langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2}.$$  \hspace{1cm} (13)$$

There is a further experimental requirement for measuring $CP$ violation. All experimental cuts must respect $CP$ invariance. If not, they would introduce artificial asymmetries, diluting or obliterating the genuine signal of $CP$ violation.

### 2.3 $CPT$ theorem and all that

Since a combined $CPT$ transformation is a good symmetry according to the $CPT$ theorem, $CP$ invariance (or violation) implies $T$ invariance (or violation), and vice versa. However, it should be borne in mind that observation of a $T$-odd asymmetry or correlation is not necessarily an indication of $CP$ (or even $T$) violation. The reason for this is the anti-unitary nature of the time-reversal operator in quantum mechanics. As a consequence of this, a $T$ operation not only reverses spin and three-momenta of all particles, but also
interchanges initial and final states. This last interchange is difficult to meet
with in practice, and one usually has a situation where only momenta and
spins are reversed, with the initial and final states kept as such. In that case,
non-zero $T$-odd observables do not necessarily signal genuine $T$ violation.

There is, however, a case when $T$-odd observables imply $T$ violation, and
that is when final-state interactions and loop effects can be neglected. In
that case the transition operator $T$ obeys $T = T^\dagger$, since the right-hand side
in the unitarity relation

$$T - T^\dagger = iT^\dagger T$$

can be neglected. Then

$$\langle f | T | i \rangle \approx \langle f | T^\dagger | i \rangle = \langle i | T | f \rangle^\ast.$$  \hspace{1cm} (15)

Now if $T$ invariance holds, then

$$\langle f | T | i \rangle = \langle i_T | T \rangle f_T,$$  \hspace{1cm} (16)

where $|i_T\rangle$, $|f_T\rangle$ represent states with all momenta and spins inverted in sign.
Combining eq.(16) with (15) for time-reversed states, we have

$$|\langle f | T | i \rangle| = |\langle f_T | T | i_T \rangle|.$$  \hspace{1cm} (17)

In this case, if a $T$-odd observable is non-zero, it implies $T$ violation. Thus, $T$
invariance (and $CP$ invariance through the $CPT$ theorem) implies equality
of amplitudes with all momenta and spins reversed if and only if final-state
interaction (absorptive part for the amplitude) vanishes.

Put differently, this means that final-state interactions can mimic $T$
violation, but not genuine $CP$ violation. One should therefore use, as far as
possible, $CP$-odd observables to test $CP$ invariance, not $T$-odd observables
(unless they are also $CP$ odd).

For a genuine $CP$-odd quantity, there are two possibilities,
A. it is $T$ odd, and therefore $CPT$ even, or
B. it is $T$ even, and therefore $CPT$ odd.
In case B, there is no violation of the $CPT$ theorem provided the amplitude
has an absorptive part. (This is again due to the fact the $CPT$ operator is
antiunitary, and interchanges initial and final states). Thus non-vanishing of
$CPT$-odd operators necessarily requires an absorptive part of the amplitude
to exist.
The absorptive part of $CP$-odd $CPT$-odd quantities in perturbation theory usually comes from loop contributions where the intermediate state can be on shell. An interesting way of realizing this possibility in the case of an intermediate state of an unstable particle is through the Breit-Wigner form of its propagator. In this case the absorptive part is proportional to its width. This trick has been used in the case of the top, $Z$ and Higgs propagators [2, 3, 4]. One must however be careful to subtract out the part of the width corresponding to decay into the initial or final state for consistency with the $CPT$ theorem [5]. It has also been pointed out recently [6] that off-diagonal contributions to the self-energy of the virtual particles are also needed for consistency with the $CPT$ theorem.

3. $CP$ violation in the leptonic sector

3.1 Scenarios for leptonic $CP$ violation

In the standard model, no right-handed neutrinos are introduced. As a result, there is no mass matrix to diagonalize for the neutrinos. Hence the CKM matrix is the unit matrix, and no $CP$-violating phases can arise. However, in extensions of the standard model, $CP$ violation can arise either because of the presence of neutrino masses or because of extra leptons introduced (even though neutrinos may be massless), or both.

A. Massive neutrinos. Neutrinos can have Dirac or Majorana masses. $CP$ violation in the Dirac case is exactly analogous to that in the quark sector of the standard model. In case of Majorana masses, the freedom of phase redefinition of the neutral lepton fields is reduced because Majorana mass terms are not invariant under phase transformations. As a result there are more $CP$-violating phases in the CKM matrix than the corresponding Dirac case. It is thus possible to have $CP$ violation with even two generations of Majorana neutrinos.

B. Massless neutrinos with exotic leptons. It is possible to have $CP$ violation because of either charged or neutral leptons in exotic representations of $SU(2) \times U(1)$. The leptons then have flavour-violating couplings to $Z$ or Higgses, which can be complex and hence $CP$ violating.
We consider below some leptonic $CP$-violating processes at high-energy colliders which make use of the above mechanisms of $CP$ violation. The importance of leptonic processes stems from the fact that they are relatively clean from the experimental point of view.

3.2 Leptonic flavour violating $Z$ decays

Leptons can have flavour-violating couplings to $Z$ giving rise to flavour violating $Z$ decays into charged leptons either at tree level or at one-loop level:

$$Z \rightarrow l_i \bar{l}_j \quad (i \neq j).$$

The corresponding $CP$-violating asymmetry is

$$A = \frac{\Gamma(Z \rightarrow l_i \bar{l}_j) - \Gamma(Z \rightarrow \bar{l}_i l_j)}{\Gamma(Z \rightarrow l_i \bar{l}_j) + \Gamma(Z \rightarrow \bar{l}_i l_j)}.$$  \hfill (19)

This is $T$ even and therefore $CPT$ odd. It therefore needs an absorptive part to be present.

Flavour-violating tree-level couplings of charged leptons to $Z$ arise in models with exotic charged leptons transforming as either left-handed singlets and/or right-handed doublets. In such a case the Glashow-Weinberg condition for flavour-diagonal couplings is not satisfied, and (18) occurs at tree level. For $A$ of (19) to be non-zero, one-loop correction to (18) is also needed, and only the absorptive part of that amplitude contributes. The asymmetry is then $\mathcal{O}(\alpha)[7]$.

On the other hand, models with exotic neutral leptons have flavour-violating couplings of neutral leptons at the tree-level giving rise to (18) at the loop level [8]. The absorptive part now comes from one of these loop diagrams. $A$ is now $\mathcal{O}(1)$. However, unlike in the previous case, the rate of the flavour-violating process (18) is $\mathcal{O}(\alpha^3)$. Thus, the minimum total number $N_Z$ of $Z$ events for an observable asymmetry is in both cases $\mathcal{O}(1/\alpha^3)$. However, constraints on leptonic mixing angles and masses of exotic leptons make this process too rare to observe at LEP.

3.3 $CP$ violation in $e^+e^- \rightarrow \gamma^*, Z^* \rightarrow l^+l^-$

In case of $CP$ violation in $e^+e^- \rightarrow \gamma, Z \rightarrow l^+l^-$ there are two general results [9]:

\[ \]
(i) No $CP$ violation can be seen without measuring initial or final spins. This follows basically because no $CP$-odd observable can be constructed without spins.

(ii) The only $CP$-violating couplings for the on-shell process are the dipole moment type couplings of $e$ or $l$ (electric or “weak” dipole moments). Since there are strong experimental limits on the electron and muon electric dipole moments ($d_e \lesssim 10^{-27} e\text{ cm}$, $d_\mu \lesssim 10^{-19} e\text{ cm}$), $\tau$ may be a good candidate for $l$. In fact, the weak moment of $\tau$ has been constrained using $\tau$ polarization in this reaction (see below).

Since it is clear from (i) that either initial or final spins have to be observed to detect $CP$ violation, we consider below both these cases for $e^+ e^- \to Z \to \tau^+ \tau^-$. 

### 3.3.1 $CP$ tests using $\tau$ polarization in $e^+ e^- \to \tau^+ \tau^-$

This possibility has been discussed by several authors [2,10-12]. For the process

$$e^-(p_-) + e^+(p_+) \to \tau^-(k_-, s_-) + \tau^+(k_+, s_+), \quad (20)$$

possible $CP$-odd quantities that can be constructed out of the momenta and spins in the centre-of-mass (cm) frame are $(p_- - p_+) \cdot (s_- - s_+)$ and $(p_- - p_+) \cdot s_- \times s_+$.

To measure these quantities, one must, of course, be able to measure $s_\pm$. This can be done by looking at decay distributions of $\tau_\pm$. In the rest frame of $\tau$, the angular distribution of an observed decay particle is

$$\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi} (1 + \alpha s \cdot \hat{q}^\ast), \quad (21)$$

where $\hat{q}^\ast$ is the unit vector along the momentum of the observed particle, and $\alpha$ is a constant called the analyzing power of the channel. For example,

$$\alpha = \mp 1 \quad \text{for} \quad \tau^\pm \to \tau^\pm \nu_\tau,$$

$$\alpha = \pm 1/3 \quad \text{for} \quad \tau^\pm \to l^\pm \nu_l \bar{\nu}_l,$$

as deduced from the theory of weak $\tau$ decay. Using (21), spin correlations can be translated to momentum correlations.

In terms of the observed momenta, possible $CP$-odd variables are $\hat{p} \cdot (q_+ \times q_-)$ ($CPT = +1$), $\hat{p} \cdot (q_+ + q_-)$ ($CPT = -1$), $\hat{p} \cdot (q_+ \times q_-) \hat{p} \cdot (q_+ - q_-)$ ($CPT = +1$), $\hat{p} \cdot (q_+ + q_-) \hat{p} \cdot (q_+ - q_-)$ ($CPT = -1$). Expectation values of the last
two were suggested by Bernreuther et al. [11, 12] for measuring respectively the real and imaginary parts of the $\tau$ weak dipole form factor $\tilde{d}_\tau(m_\tau^2)$. The suggestion has been carried out at LEP for $\text{Re} \tilde{d}_\tau(m_\tau^2)$ by the OPAL [13] and ALEPH [14] groups. OPAL looked at inclusive leptonic and hadronic decays of $\tau$, whereas ALEPH analyzed all channels exclusively. The results obtained are the 95% confidence-level upper limits $\text{Re} \tilde{d}_\tau(m_\tau^2) < 7.0 \times 10^{-17} \text{e cm}$ (OPAL [13]) and $\text{Re} \tilde{d}_\tau(m_\tau^2) < 3.7 \times 10^{-17} \text{e cm}$ (ALEPH [14]).

The theoretical prediction for the 1 s.d. limit obtainable in the measurement of $\text{Im} \tilde{d}_\tau(m_\tau^2)$ is $10^{-16}$ using the correlation $\langle \hat{p} \cdot (q_+ + q_-) \hat{p} \cdot (q_+ - q_-) \rangle$ and a sample of $10^7 Z$'s [12].

### 3.3.2 Longitudinal beam polarization

The Stanford Linear Collider (SLC), operating presently at the $Z$ resonance, has an $\epsilon^-$ polarization of about 62%, and is likely to reach 75% in the future. The present sample collected is of 50,000 $Z$'s, and the hope is to reach $5 \times 10^6$, or even $10^6$ $Z$'s.

Can this longitudinal $\epsilon^-$ polarization help in measuring the $\tau$ weak dipole moment? The answer is “yes” [15]. In fact, as we shall see, SLC can do better than LEP so far as $\text{Im} \tilde{d}_\tau$ is concerned.

The essential point is that the vector polarization of $Z$ gets enhanced in the presence of $e^+e^-$ longitudinal polarization. For vanishing beam polarization, $P_{\epsilon^-} = P_{\epsilon^+} = 0$, the $Z$ vector polarization is

$$P_Z^{(0)} = \frac{2g_{\nu\tau}g_{A\tau}}{g_{A\tau}^2 + g_{\nu\tau}^2} \approx 0.16,$$

where $g_{\nu\tau}, g_{A\tau}$ are vector and axial-vector couplings of $e^+e^-$ to $Z$. For non-zero polarization,

$$P_Z = \frac{P_Z^{(0)} - P_{\epsilon^+\epsilon^-}}{1 - P_Z^{(0)} P_{\epsilon^+\epsilon^-}},$$

where

$$P_{\epsilon^+\epsilon^-} = \frac{P_{\epsilon^+} - P_{\epsilon^-}}{1 - P_{\epsilon^-}P_{\epsilon^+}}.$$  

Thus, $P_Z \approx 0.71$ for $P_{\epsilon^-} = -0.62$ and $P_{\epsilon^+} = 0$, which is an enhancement by a factor of about 4.5.

It is therefore profitable to look for $CP$-odd observables involving the $Z$ spin $s_Z = P_Z \hat{p}$, where $\hat{p}$ is the unit vector along $p_+ = -p_-$ in the c.m. frame.
Examples of such observables are $\hat{p} \cdot (q_+ \times q_-)$ and $\hat{p} \cdot (q_+ + q_-)$. While both are $CP$ odd, the former is $CP T$ even and the latter is $CP T$ odd.

The above is not entirely correct in principle. $CP$-odd correlations give a measure of underlying $CP$ violation only if the initial state is $CP$ even. Otherwise there may be contributions to correlations which arise from $CP$-invariant interactions due to the $CP$-odd part of the initial state. In the case when only the electron beam is polarized, the initial state is not $CP$ even. In practice, however, this $CP$-even background is small because for $m_e \to 0$, only the $CP$-even helicity combinations $\epsilon_L^+ \epsilon_R^+$ and $\epsilon_R^+ \epsilon_L^+$ survive, making the corrections proportional to $m_e/m_Z \approx 5 \times 10^{-6}$. If one includes order $\alpha$ collinear photon emission from the initial state, which could flip the helicity of the $e^\pm$, then like-helicity $e^+ e^-$ states could also survive for vanishing electron mass. However, it turns out that this being a non-resonant effect, the corresponding cross section at the $Z$ peak is small. It is therefore expected that the correlations coming from $CP$-invariant SM interactions in such a case will be negligible.

The correlations $\langle O_1 \rangle \sim \langle \hat{p} \cdot (q_+ \times q_-) \rangle$ and $\langle O_2 \rangle \sim \langle \hat{p} \cdot (q_+ + q_-) \rangle$ have been calculated analytically for the single-pion and $\rho$ decay mode of each $r$. Also calculated analytically are $\langle O_1^2 \rangle$ and $\langle O_2^2 \rangle$ needed for obtaining the 1 s.d. limit on the measurability of $\tilde{d}_r$ obtained using eq.(13) [15].

The results for only the single-pion channel are summarized in Tables 1a and 1b, which give, respectively for $O_1$ and $O_2$ and for electron polarizations $P_e^z = 0, \pm 0.62$, the correlations in units of $\tilde{d}_r m_Z/\epsilon$, the square root of the variance, and the 1$\sigma$ limit on $Re \tilde{d}_r$ and $Im \tilde{d}_r$ obtainable with $10^6 Z$'s. The enhancement of $\langle O_{1,2} \rangle$ and hence the sensitivity of $\tilde{d}_r$ measurement with polarization is evident from the tables.

<table>
<thead>
<tr>
<th>$P_e$</th>
<th>$\langle O_1 \rangle$ (GeV$^2$)</th>
<th>$\sqrt{\langle O_1^2 \rangle}$ (GeV$^2$)</th>
<th>1 s.d. limit on $Re \tilde{d}_r$ for $10^6 Z$'s (in $\epsilon$ cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.90</td>
<td>12.86</td>
<td>$1.5 \times 10^{-16}$</td>
</tr>
<tr>
<td>+0.62</td>
<td>-2.89</td>
<td>12.86</td>
<td>$4.6 \times 10^{-17}$</td>
</tr>
<tr>
<td>-0.62</td>
<td>4.01</td>
<td>12.86</td>
<td>$3.3 \times 10^{-17}$</td>
</tr>
</tbody>
</table>

Table 1a

$^1$I thank Prof. L.M. Sehgal for drawing my attention to this fact.
Table 1b

<table>
<thead>
<tr>
<th>$P_e$</th>
<th>$\langle O_2 \rangle$ (GeV) for $\text{Im} , \vec{d}_r = \epsilon / m_Z$ (GeV)</th>
<th>$\sqrt{\langle O_2^2 \rangle}$</th>
<th>1 s.d. limit on $\text{Im} , \vec{d}_r$ for $10^6$ $Z'$’s (in $\epsilon$ cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-0.16$</td>
<td>9.57</td>
<td>$6.2 \times 10^{-16}$</td>
</tr>
<tr>
<td>+0.62</td>
<td>0.51</td>
<td>9.57</td>
<td>$1.9 \times 10^{-16}$</td>
</tr>
<tr>
<td>−0.62</td>
<td>−0.70</td>
<td>9.57</td>
<td>$1.4 \times 10^{-16}$</td>
</tr>
</tbody>
</table>

Tables 1a and 1b: Correlations of $O_1$ and $O_2$ respectively, the standard deviation from SM interactions, and the 1 s.d. limit on the real and imaginary parts of the weak dipole moment, for different polarizations $P_e$.

The sensitivity can be further improved by looking at only the $P_{e-}$ dependent part of the cross section. This can be done by looking at a sample corresponding to the difference in the number of events for a polarization $P_{e-}$ and polarization $-P_{e-}$. The correlations are evaluated with respect to $d\sigma(P_{e-}) - d\sigma(-P_{e-})$. This reduces the number of events. However, correlations are enhanced by a larger factor giving a net gain in the sensitivity. The result for $P_{e-} = .62$ and $10^6$ $Z'$’s is given in Table 2.

Table 2: Quantities as in Table 1, but for a distribution asymmetrized between polarizations $+0.62$ and $-0.62$.

The 1 s.d. limit for $\text{Im} \, \vec{d}_r$ should be compared with the LEP expectation of $10^{-16} \, \epsilon$ cm for a larger sample of $10^7$ $Z$’s [12].

The use of correlations for measuring the $\tau$ edm at the proposed $\tau$-charm factories employing longitudinal polarization of electron and positron beams has also been studied in detail in [16].
3.3.3 Transverse beam polarization

Use of transversely polarized $e^+e^-$ beams for the study of $CP$ violation has been studied by several people (see for example [2, 17]). For a reaction

$$e^-(p_-, s_-) + e^+(p_+, s_+) \rightarrow f(k_+) + \bar{f}(k_-)$$

(25)

in the c.m. frame, where $f$ denotes a fermion, possible $T$-odd triple products are $s_- \cdot (p \times k)$, $s_\mp (p \times k)$, $p \cdot (s_- \times s_\mp)$, $k \cdot (s_- \times s_\mp)$, where $p_\mp = -p_\mp = p$ and $k_\pm = -k_\mp = k$. Of these triple products, the last two are purely $CP$ odd, whereas only the difference of the first two is $CP$ odd. Burgess and Robinson [17] have done an analysis of $e^+e^- \rightarrow \tau^+\tau^-, e\bar{e}, b\bar{b}$ in terms of operators

$$\mathcal{L}_W = \lambda_W \left[T P_{\mu} D D_{\mu} \phi \right] + H.c.,$$

(26)

$$\mathcal{L}_Y = \frac{i}{2} \lambda_Y \left[g_1 B_{\mu \nu} \left(T_{\gamma}^{\mu \nu} P H E \right) \phi \right] + H.c.$$

(27)

Their results for $\lambda_W = (400 \text{GeV})^{-2}$ and $L = 4.8 \times 10^5 \text{ pb}^{-1}$ at LEP are given in Table 3, where $A$ is the asymmetry for $s_- \cdot (p \times k)$ given by

$$A = \left[\int d\sigma(p_i, s_i) - \int d\sigma(-p_i, -s_i) \right] \frac{s_- \cdot (p \times k)}{N \text{ SM events}}.$$  

(28)

Though this is an interesting effect, a theoretical estimate for $\lambda_W$ is needed before concluding whether the effect would be observable. Assuming a systematic error of 0.1%, the 2-$\sigma$ limits possible on $\lambda_W$ are estimated to be (570 GeV)$^{-2}$ and (660 GeV)$^{-2}$ respectively for up- and down-type quarks.

<table>
<thead>
<tr>
<th>$N$ (SM events)$\times 10^{-8}$</th>
<th>$\tau^+\tau^-$</th>
<th>$e\bar{e}$</th>
<th>$b\bar{b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \times 10^{-5}$</td>
<td>-6.1</td>
<td>97</td>
<td>-170</td>
</tr>
<tr>
<td>$A/N \times 10^{3}$</td>
<td>0.9</td>
<td>4.0</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Table 3: The transverse polarization asymmetry $A$ (defined in the text) compared with the standard model events for $e^+e^- \rightarrow \tau^+\tau^-, e\bar{e}, b\bar{b}$. 

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4. \( CP \) violation in top pair production

Evidence for a top quark of mass of about 174 GeV at the Tevatron has been reported by the CDF collaboration [18]. Even though the data is not conclusive, it is generally believed that the top quark will eventually be found with a mass of a similar magnitude. Top-antitop pairs can then be produced at large rates at future colliders and used for various studies. In particular, since a heavy top \( (m_t > 120 \text{ GeV}) \) decays before it hadronizes [19], information about its polarization is preserved in its decay products. Schmidt and Peskin [20] have suggested (elaborating on an old suggestion of Donoghue and Valencia [21]) looking for the asymmetry between \( t_L \bar{t}_L \) and \( t_R \bar{t}_R \) as a signal for \( CP \) violation (see also [22]). Note that this is possible only for a heavy particle like the top quark because for a light particle, the dominant helicity combination would be \( t_L \bar{t}_R \) or \( t_R \bar{t}_L \), each being self-conjugate.

The asymmetry \( N(t_L \bar{t}_L) - N(t_R \bar{t}_R) \) can be probed through the energy spectra of prompt leptons from \( t \rightarrow Wb; W \rightarrow l\nu \). This is understood as follows.

For a heavy top, the dominant \( W \) helicity in \( t \rightarrow Wb \) is 0. Now, due to \( V - A \) interaction, \( b \) is produced with left-handed helicity (neglecting the \( b \) mass). Hence in the \( t \) rest frame, \( W^+ \) momentum is dominantly along the \( t \) spin direction. It follows that \( l^+ \) in \( W^+ \) decay is produced preferentially in the direction of the \( t \) spin. In fact, the distribution is \( 1 + \cos \psi \), where \( \psi \) is the angle between the \( l^+ \) momentum direction and the \( t \) spin direction. In going to the laboratory frame, the \( t \) gets boosted. Thus \( l^+ \) from \( t_R \) is more energetic than \( l^+ \) from \( t_L \), and \( l^- \) from \( \bar{t}_L \) has more energy than \( l^- \) from \( \bar{t}_R \). Therefore, in the decay of \( t_L \bar{t}_L \), \( l^- \) from \( \bar{t}_L \) has higher energy than \( l^+ \) from \( t_L \), and the reverse is true for \( t_R \bar{t}_R \). Thus the energy asymmetry of leptons measures \( N(t_L \bar{t}_L) - N(t_R \bar{t}_R) \).

Schmidt and Peskin [20] looked at this asymmetry in hadron collisions in a \( CP \)-violating multi-Higgs model where the \( CP \) violation is described by the Lagrangian terms

\[
\delta \mathcal{L} = -\frac{m_t}{v} \bar{\phi} t [A P_L + A^* P_R] t,
\]

where only the effect of the dominant lightest Higgs field \( \phi \) is kept. \( v \) is the SM Higgs vacuum expectation value, and \( A \) is a complex combination of mixing angles and phases.
Since $N(t_L\bar{t}_L) - N(t_R\bar{t}_R)$ is $CP$ odd and $T$ even, the $CPT$ theorem requires the existence of an absorptive part for it to be non-zero. Looking at the tree and one-loop diagrams for $q\bar{q} \to t\bar{t}$, they conclude that

$$A = \frac{N(t_L\bar{t}_L) - N(t_R\bar{t}_R)}{N(t_L\bar{t}_L) + N(t_R\bar{t}_R)} \approx 10^{-3}$$

for $m_t = 150$ GeV, $m_\phi = 100-400$ GeV and $\text{Im}[A^2] = \sqrt{2}$. Thus the asymmetry $A$ would be observable with $10^8 t\bar{t}$ a year.

For $pp$ collisions, since the $pp$ state is not a $CP$ eigenstate, there is also a $CP$-invariant contribution present, but this is shown to be small [20].

As in the case of $pp$ or $p\bar{p}$ collisions, lepton energy asymmetry can be used to measure $CP$ violation in $e^+e^- \to t\bar{t}$ [23]. The authors of ref.[23] also consider another asymmetry, viz., the up-down asymmetry of the charged leptons with respect to the $t\bar{t}$ production plane in the laboratory frame. It is also possible to construct a “left-right” asymmetry of leptons with respect to a plane perpendicular to the $t\bar{t}$ production plane, but containing the $t\bar{t}$ momentum direction [24]. Certain combinations of up-down and left-right symmetry with forward-backward symmetry can also be considered. All these probe different combinations of $CP$-violating couplings [24].

The result of ref.[23] is that asymmetry is at the few per cent level for the top-quark electric and weak dipole moments $d_i, \tilde{d}_i \sim e/m_t$. For $\sqrt{s} = 300$ GeV and $m_t = 120$ GeV, $\sigma \approx 1400$ fb, which corresponds to 20,000 prompt leptons a year for a luminosity of $10^{33}$ cm$^{-2}$s$^{-1}$. In such a case, $d_i, \tilde{d}_i$ can be determined to a few percent level. This should be compared with the prediction of $10^{-2} - 10^{-3}$ from the Higgs model for $CP$ violation.

Apart from the above asymmetries, $CP$-odd correlations could provide a measure of $CP$ violation in $t\bar{t}$ production [25]. Certain correlations are more sensitive to $CP$ violation in $t$ decay, rather than production [25].

Another process which has been suggested is $e^+e^- \to t\bar{t}\nu\bar{\nu}$ through $W^+W^- \to t\bar{t}$ [4]. Two diagrams contribute to $W^+W^- \to t\bar{t}$, one corresponding to $t$-channel $b$ exchange and the other with an $s$-channel heavy Higgs $\phi$, which can be on shell for $m_\phi > 2m_t$. Then, the absorptive part needed for a $CP$-odd $CPT$-odd asymmetry is provided by the width of the Higgs. A sizable asymmetry can be obtained thus [4].

At linear colliders, there is a possibility of producing electron beams with longitudinal polarization. This may be exploited to enhance the sensitivity of
measurement of the top dipole moments as well as to measure the electric and weak dipole moments independently [24, 26]

5. CP violation in other processes

The process $e^+e^- \rightarrow W^+W^-$, which will be studied in the near future at LEP200, will be the first one to be able to test the SM couplings of the electroweak gauge bosons. The process is expected to put bounds on non-standard $\gamma$ and $Z$ couplings to $W^+W^-$. The non-standard couplings could be $CP$ violating ones, as in (2). These can be studied in a way similar to the one used for probing $CP$ violation in $e^+e^- \rightarrow \tau^+\tau^-$ and $e^+e^- \rightarrow t\bar{t}$ considered earlier. As in the case of $e^+e^- \rightarrow t\bar{t}$ with leptonic $t,\bar{t}$ decays, energy or up-down asymmetry of leptons may be used [27, 28]. Whereas Chang et al. [27] consider asymmetries, Mani et al. [28] gave estimated the energy correlation ratio

$$A = \frac{\langle E_- \rangle - \langle E_+ \rangle}{\langle E_- \rangle + \langle E_+ \rangle},$$

and suggest the angular correlation ratio

$$\delta = \frac{\langle \theta_- \rangle - \langle \theta_+ \rangle}{\langle \theta_- \rangle + \langle \theta_+ \rangle},$$

where $E_\pm$ are the energies of the leptons $l^\pm$ produced in the decay of $W^\pm$, and $\theta_\pm$ are the angles of $l^\pm$ momenta with respect to the $e^+$ beam direction. The general expectation is $A \approx 10^{-3}$ for $\kappa$ or $\lambda \approx 0.1$.

Some other processes considered in the literature are $t,\bar{t}$ decay asymmetries in $\phi \rightarrow t\bar{t}$, where $\phi$ is a heavy Higgs [29], decay lepton asymmetries in $e^+e^- \rightarrow \tilde{\chi}\tilde{\chi}$, where $\tilde{\chi}$ is a neutralino in the minimal supersymmetric standard model [30], decay correlations in $\gamma\gamma \rightarrow W^+W^-$ [31], and forward-backward asymmetry in $e^+e^- \rightarrow \gamma Z$ [32].

6. Conclusions

We have seen above the various points to be kept in mind when selecting processes and variables for detecting and measuring $CP$ violation, a number of processes and scenarios of $CP$-violating signatures which could be looked
for. The above discussion is mainly aimed at arriving at an idea of the sensitivities possible in different measurements. In general, the results in most popular models of $CP$ violation beyond SM indicate that $CP$ violation in the most optimistic theoretical scenario would be measurable only with some difficulty in the existing or presently envisaged experiments. Nevertheless, it would be good to keep one’s eyes open to these possibilities.
References


