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Nuclear mismatch in super-allowed Fermi beta-decay

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Abstract

If super-allowed Fermi beta-decay is to be consistent with the conserved vector current and with unitarity of the Cabibbo-Kobayashi-Maskawa matrix it is necessary to understand a mismatch, as between the parent and daughter members of the isotriplets involved, that is worth several tenths of a percent in the transition rates. Present theoretical estimates of this mismatch fall short of what is needed by a factor of order 2 or more. It has been suggested that this shortfall is due to inadequate allowance for the effects of the spectator nucleons but the present study shows that such effects are worth only a few hundredths of one percent.

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1. Introduction

It is now well recognized that our ability to extract the vector coupling constant G_V , and, hence, by comparison with G_μ , to access the V_{ud} element of the Cabibbo-Kobayashi-Maskawa matrix, from the eight accurately-measured $J^\pi = 0^+ \rightarrow 0^+$ transitions within isospin triplets, is limited not by the experimental accuracy with which the relevant ft -values are determined, nor by the confidence with which the various radiative corrections can be applied, but rather by the subversion of the isospin symmetry, along the multiplet, by the charge-dependence of the forces, Coulomb and specifically nuclear.*

The situation is exposed in Fig. 1. The experimental data have been adjusted by the radiative corrections only (using the mean of the quenched versions of Refs.[4] and [5] for the axial terms without any allowance for associated theoretical uncertainties) no correction for nuclear mismatch being applied. The line is the best fit quadratic in Z , it being evident that a fit linear in Z is not acceptable. The quadratic fit has $Q(\chi^2|\nu) = 0.26$, eminently acceptable, and the inferred extrapolation to $Z = 0$, at which presumably the nuclear mismatch has fallen away, is shown by the point to the right of the axis of $(ft)^*$. The point to the left of the $(ft)^*$ -axis, near $Z = 0$, is that inferred from G_μ together with validity of CVC and unitarity of the Cabibbo-Kobayashi-Maskawa matrix (using the Particle Data Group[6] values for $|V_{us}|$ and $|V_{ub}|$). The close agreement between the two points near $Z = 0$ means that if we wish to believe both in CVC and in unitarity of the CKM matrix then we must understand the differences between the $(ft)^*$ -values for the various experimental transitions of Fig. 1 and the value $(ft)_0^*$ at $Z = 0$, for which we use that inferred from CKM unitarity, as representing the slowing-down of the experimental transitions by the subversion of the isospin symmetry by the charge-dependent forces, i.e. the experimental nuclear mismatch δ_{Cexp} ; these δ_{Cexp} -values are listed in Table 1.†

2. Theoretical estimates of the nuclear mismatch

There have been many calculations of the nuclear mismatch δ_C . Those most appropriate for our present consideration are shell-model-based and use many-body nuclear wave-functions. The older of these treatments use wave-functions generated within some mean field, either Saxon-Woods[7,8] or Hartree-Fock[9]; a more recent treatment is R -matrix based[10]. The older treatments split δ_C into two parts, δ_{CM}

*It is not the purpose of this paper to review the matter of the super-allowed Fermi transitions in general terms; this has recently been done[1,2]; the data used in the present analysis are those presented in Table 1 of Ref.[2], slightly modified in the cases of ^{46}V and ^{50}Mn on account of weak Gamow-Teller transitions recently reported from Chalk River[3]; the radiative corrections follow the discussion in Refs.[1] and [2] with the exception that in the case of the C_{BORN} and C_{NS} terms of the axial part of the inner radiative correction the new treatment of Towner[4], in which quenched values for the relevant nucleonic constants are used, has been followed, those changes being also applied, *pari passu*, to the alternative treatment[5] of those quantities.

†We have conducted this exposé without any consideration whatsoever of theoretical estimates of δ_C such as we are about to discuss. It is, however, appropriate to comment here that more-refined approaches to $(ft)_0^*$, in which use is made of the theoretical δ_C in various ways[1,2] prior to quadratic extrapolation to $Z = 0$, all agree as to $(ft)_0^*$ and so strengthen our present conclusions as to δ_C as listed in Table 1.

and δ_{BE} , such that $\delta_C = \delta_{CM} + \delta_{BE}$. δ_{CM} , the smaller term, represents the mismatch due to the change of configuration-mix along the isospin multiplet; δ_{BE} , the chief term, represents the T_3 -dependence of the nucleonic binding energies and so the mismatch of the associated single-nucleonic orbitals as between parent and daughter bodies. Table 1 lists the δ_{CM} , δ_{BE} from the various sources; it also lists the δ_C -values from the R -matrix treatment which there do not separate cleanly into δ_{CM} and δ_{BE} components.

To cover the present range of theoretical δ_C -values we now combine the δ_{CM} and δ_{BE} of Table 1 pairwise into their 6 totals so that together with the δ_C of Ref.[10] we have 7 theoretical estimates, δ_{Ctheo} . As is now evident from Table 1 the inferred δ_{Cexp} considerably exceed the δ_{Ctheo} ; there is a “missing mismatch”:

$$\delta_{Cmiss} = \delta_{Cexp} - \delta_{Ctheo}. \quad (1)$$

The situation is displayed in Fig. 2. It is clear that δ_{Cmiss} is large and systematic.

We pause at this point to review the overall situation. As we have seen, if we accept the present experimental data and the approach of dealing with them by extrapolation to $Z = 0$, either crudely as in Fig. 1 or by the more refined methods discussed elsewhere[1,2], we have agreement with CKM unitarity under the assumption of CVC but are then faced with the large and so-far-unexplained δ_{Cmiss} -values of Fig. 2. If, alternatively, we do not accept the method of extrapolation to $Z = 0$, and insist that present theoretical estimates of δ_C are at least indicative of the range of possibilities, then we certainly lose agreement with CKM unitarity and also violate CVC, particularly an account of the ^{14}O datum.[†]

3. The spectator nucleons

The theoretical estimates of δ_C that we have discussed above concern themselves, in detail, only with those nucleons that are normally considered as chiefly responsible for the bulk of low-lying nuclear properties, namely the valence nucleons outside the closed shells of the nuclear core; it is from these valence nucleons that the many-body wave-functions that are used for the theoretical estimates of δ_C derive. However, in the beta-decay process all the nucleons of the nucleus are involved in that all must relax from their initial into their slightly different final orbitals with an associated slightly-less-than-perfect overlap: an associated mismatch. Now this mismatch of the core nucleons, together with that of those valence nucleons other than the one that is transformed by the beta-decay, call them collectively the spectator nucleons, will be very slight; but there are many such spectators and their individual mismatches must be summed in their collective impact upon δ_C . It was therefore

[†]As has been emphasized before[1,2], the ^{14}O point is critically important to the present argument and also, unfortunately, the object of experimental conflict as to the phase-space factor (Q -value). The rather large error shown for the $(ft)^*$ -value in Fig. 1 is the result of having to reconcile the discrepant values by the standard (PDG[6]) method of error-inflation; it may also be remarked that although there is no experimental conflict as to the lifetime the (ostensibly very accurate and concordant) measurements are now quite old (about 20 years) and, in view of its importance, remeasurement is highly desirable. We also note here that an adequately accurate datum for ^{10}C at $Z = 5$ would be tremendously valuable.

suggested[11] that we must keep an open mind on this question of the spectator nucleons, more generally as to the reliability of the theoretical δ_C , and that we should, as far as possible, look to the data themselves to tell us about δ_C by some form of extrapolation to $Z = 0$; this was the genesis and motivation for this extrapolation method.

It is important at this point to remark that although the extrapolation method was suggested by concern for the spectator nucleons its validity in no way depends upon their being the cause of the δ_{Cmiss} of Fig. 2: it could well be that present theoretical estimates of the basic δ_C of the “transforming nucleon” are inadequate or that some other factor not at present in the equation is at work. But it is clearly desirable to quantify the possible contribution, δ_{Cspec} , of the spectator nucleons to δ_C whether or not that contribution is adequate to account for δ_{Cmiss} . An essay towards that quantification is the object of this paper.

Now one clearly cannot use a conventional Saxon-Woods mean field approach for estimating δ_{Cspec} since it is of the essence that the effective mean field differs as between the initial and final states and the usual parameterizations of Saxon-Woods-like potentials do not adequately concern themselves with such subtleties in a manner suitable for reference to all the nucleons of the nucleus as is essential for δ_{Cspec} ; some form of self-consistent treatment is needed. Now the Hartree-Fock approach to the mean field[9] is a self-consistent one and it might be hoped that it would therefore lead to a reliable estimate for δ_{Cspec} . Indeed, straightforward use of the Hartree-Fock mean field gave $\delta_{Cspec} \simeq 0.5\delta_{BE}$ for ^{54}Co (Ref.[9]) superficially suggesting that δ_{Cspec} might be comparable with the δ_{Cmiss} of Fig. 2. However, it was pointed out[9] that the relevant Hartree-Fock wave-functions have a spurious isospin mixing and that this must somehow be discounted in a better estimate of δ_{Cspec} . Ormand and Brown[9] sought to effect such a discounting by two stratagems: (i) taking δ_{Cspec} to be the difference between the values calculated with and without inclusion of the Coulomb force; (ii) using wave-functions derived from the $T_3 = 0$ mean field together with an appropriate effective nucleonic charge in the $T_3 \neq 0$ nucleus. Stratagem (i) suggested $\delta_{Cspec} \simeq 0.02\% - 0.05\%$; stratagem (ii) suggested $\delta_{Cspec} < 0.01\%$; it was concluded that δ_{Cspec} is probably negligible. However, these stratagems are just that, being plausible and attractive recipes without fundamental basis; they have been criticized[1] on such grounds although with no specific justification.

4. Nuclear dilation

A chief agent for the difference between the parent and daughter nuclei of the positron beta-decay is the Coulomb force which tends to dilate the former relative to the latter so that the nucleonic orbitals shrink overall in the beta-decay process and δ_{Cspec} is thereby engendered. How large is this effect?

4.1. ROUGH EXPECTATION

We wish to estimate d , the fractional radial dilation of the nucleus as a result of the transformation of one of its neutrons into a (mirror) proton. Most naively, we

have for a uniform spherical nucleus of radius R :

$$d \simeq \frac{6Ze^2}{5AKR} \quad (2)$$

where K is the nuclear compressibility. For $K = 200$ MeV, a conventional value, and taking R as $\sqrt{\frac{5}{3}}\langle r^2 \rangle^{\frac{1}{2}}$ where $\langle r^2 \rangle$ is given its experimental value, we find that d decreases from about 0.13% to 0.09% as we go from ^{14}O to ^{54}Co along the bodies of our concern.

In evaluating the implications of $d \simeq 0.1\%$ for the δ_{Cspec}^N of a single nucleon we might be tempted to continue with the model of a uniform sphere that has given this value for d namely rectangular nucleonic orbitals. We immediately find:

$$\delta_{\text{Cspec}}^N \simeq 3d \quad (3)$$

namely $\delta_{\text{Cspec}}^N \simeq 0.3\%$ which implies a total δ_{Cspec} ranging from about 5% for ^{14}O to 15% for ^{54}Co .

The above very large values of δ_{Cspec} are, however, a consequence of the unphysical assumption of a sharp cut-off for the radial wave-functions at the edge of the nucleus. In practice the radial dilation dR is very small compared with the distance at the edge of the nucleus over which the wave-function declines to zero from its central value. We may easily see that this has two consequences: (i) that δ_{Cspec}^N is very much smaller than $3d$; (ii) that δ_{Cspec}^N goes quadratically with d rather than linearly.

We now demonstrate these two expectations through the smallest departure possible in the direction of a more realistic model namely wave-functions that remain constant out to certain radial distances from the centre of the nucleus and then decline linearly to zero over a further distance t . We illustrate the effect of changing t from zero, corresponding to the patently unphysical case just considered, up to “realistic” values of 2 fm or so for $d = 0.1\%$ and for a fixed value of $\langle r^2 \rangle^{\frac{1}{2}}$ of 3.5 fm such as is about correct for ^{54}Co . The resultant mismatch is shown in Fig. 3 where we see that $\delta_{\text{Cspec}}^N \simeq 0.001\%$ or less for “realistic” values of t . We therefore expect that the total δ_{Cspec} of the nucleus may be as small as 0.05% or less for ^{54}Co . Our second expectation, that δ_{Cspec}^N should go as d^2 is also quantitatively borne out within the above model: $\delta_{\text{Cspec}}/d^2$ varies by only 0.22% as d changes from 0.02% to 0.15%: a variation by a factor of over 50 in δ_{Cspec} .

Although it appears most unlikely, from these crude arguments, that Coulomb dilation of the nucleus can be responsible for δ_{Cmin} we should explore more-realistic models both for the dilation itself and for the nucleonic orbitals. We should like to consider models that are as general as possible, and that have as little dependence as possible on details of nuclear structure, in our estimating of the dilation. We should similarly consider nucleonic orbital models that rely as little as possible on the specifics of the nuclei involved other than in the form of regular and unambiguous progression with A . For the dilation we consider a refined Thomas-Fermi model[12] and also a droplet model[13] recognizing that the latter in particular may be somewhat out of its range for the bodies of our concern. For the nucleonic orbitals we will use the harmonic oscillator and also, as a general surrogate for the nucleus as a whole, wave-functions as represented by (the square root of) the standard Fermi form for the nuclear density distribution.

4.2. THOMAS-FERMI EXPECTATION FOR THE DILATION

Modern Thomas-Fermi models for the nuclear charge distribution (Ref.[12]; see also Ref.[14]) work surprisingly well even for quite light nuclei. They treat neutrons and protons separately and yield overall values of $\langle r^2 \rangle_p^{\frac{1}{2}}$ and $\langle r^2 \rangle_n^{\frac{1}{2}}$, for the nucleus as a whole, as a function of A and Z . These models do not explicitly reflect the charge-independence of the specifically-nuclear forces and so it may seem best to deal with their predictions as to the $T_3 = \pm 1$ bodies of a given A when assessing the dilation, now separately d_n and d_p for neutrons and protons, along our $T = 1$ isomultiplets.

Write now $\langle r^2 \rangle_p^{\frac{1}{2}} = P$ and $\langle r^2 \rangle_n^{\frac{1}{2}} = N$ with subscripts $+$, 0 , $-$ for P and N signifying $N = Z + 2$, Z and $Z - 2$ respectively. We now define the actual Coulomb-related dilation D of the $\langle r^2 \rangle^{\frac{1}{2}}$ -values as one half of that found as between $N = Z - 2$ and $Z + 2$ so that:

$$P_+ = P_- - 2\Delta - 2D_p \quad (4)$$

$$N_+ = N_- + 2\Delta - 2D_n \quad (5)$$

In Eqs. (4) and (5) the term Δ represents the fact that the transformation of a neutron into a proton at the top of the Fermi-sea of itself increases $\langle r^2 \rangle_p^{\frac{1}{2}}$ and decreases $\langle r^2 \rangle_n^{\frac{1}{2}}$ without any account for dilation. Thus for a Fermi gas in an oscillator potential we have[§]:

$$\Delta/\langle r^2 \rangle^{\frac{1}{2}} \simeq \left(\frac{A+2}{A-2} \right)^{\frac{1}{2}} - 1. \quad (6)$$

In Eqs. (4) and (5) we set Δ the same for neutron-to-proton and for proton-to-neutron conversion since this is the charge-independent result relative to which we wish to assess the impact of the Coulomb force.

In order to disentangle the separate contributions of Δ and D to the changes in the $\langle r^2 \rangle$ -values we consider $P_0 - N_0$, the actual difference of the $\langle r^2 \rangle_p^{\frac{1}{2}}$ and $\langle r^2 \rangle_n^{\frac{1}{2}}$ for $N = Z$, a nucleus not necessarily a member of the isotriplet of our concern, although of the same A and so, within the model, of similar response to the Coulomb force. We now have:

$$d(P_0 - N_0)/dZ = D_p - D_n, \quad (7)$$

a quantity that varies very smoothly and slowly with Z . Eqs. (4), (5) and (6) together now yield Δ , D_p and D_n . The results are given in Table 2[¶] in terms of the proportionate effects d_p , d_n and $\Delta/\langle r^2 \rangle^{\frac{1}{2}}$. We see that they do not differ greatly from the naive expectation of Eq. (2). Figure 4 shows $\Delta/\langle r^2 \rangle^{\frac{1}{2}}$ over the range of A of our concern as given by Table 2 for the Thomas-Fermi treatment, as given by Eq. (6) for the Fermi gas in a harmonic oscillator potential, as given by a fully-quantized harmonic oscillator shell model and as given by the droplet model about to be discussed.

[§]I am grateful to Dr. W.J. Swiatecki for telling me of this result.

[¶]I am extremely grateful to Drs. W.D. Myers and W.J. Swiatecki for letting me have the latest results from their Thomas-Fermi model on which Table 2 is based.

4.3. THE DROPLET MODEL EXPECTATION FOR THE DILATION

We now briefly consider the expectations of the droplet model[13] which we evaluate in its recent version as presented in Ref.[15].^{||} We should not expect this to be the model of choice but we examine it in case it might suggest substantially larger values for δ_{Cspec}^N than the Thomas-Fermi model: it does not; its $d_{p,n}$ are smaller than those listed in Table 2, the differences decreasing as A increases. As seen from Fig. 4 the Δ of the droplet model is quite similar to that of the Thomas-Fermi model.

4.4. EVALUATION OF THE MISMATCH; HARMONIC OSCILLATOR

We must now apply the Coulomb dilations, d_p and d_n , of Table 2 to the evaluation of the δ_{Cspec}^N of the individual nucleons and to the overall δ_{Cspec} . To do this we need a nuclear model. As already mentioned, a natural choice is the harmonic oscillator. We have the standard result for the overlap of two oscillators $(\hbar\omega)_1$ and $(\hbar\omega)_2$ [16]:

$$\delta = 1 - \left(\frac{4(\hbar\omega)_1(\hbar\omega)_2}{[(\hbar\omega)_1 + (\hbar\omega)_2]^2} \right)^{2n+l-\frac{1}{2}} F^2. \quad (8)$$

In Eq. (8) F is a confluent hypergeometric function whose value is unity for nodeless wave-functions ($n = 1$) while for those with a single node ($n = 2$):

$$F = 1 - \frac{1}{4} \left(l + \frac{3}{2} \right) \frac{[(\hbar\omega)_1 - (\hbar\omega)_2]^2}{(\hbar\omega)_1(\hbar\omega)_2}. \quad (9)$$

Now $(\hbar\omega) \sim \langle r^2 \rangle^{-1}$ so for $n = 1$ orbitals:

$$\delta_{\text{Cspec}}^{1,l} \simeq \left(2n + l - \frac{1}{2} \right) d^2 \quad (10)$$

while for $n = 2$:

$$\delta_{\text{Cspec}}^{2,l} \simeq \left(2n + 3l + \frac{5}{2} \right) d^2 \quad (11)$$

Summing the $\delta_{\text{Cspec}}^{n,l}$ over all relevant shells (filled in their naive jj -sequence) we gain the overall δ_{Cspec} listed in Table 2.

4.5. EVALUATION OF THE MISMATCH; FERMI FORM

Our other, overall, model for the nucleonic wave-functions borrows from the Fermi form for the density distribution viz. we take:

$$\psi(r) \sim \{1/(1 + \text{Exp}[(r - R)/a])\}^{\frac{1}{2}}. \quad (12)$$

^{||}It is not useful to reproduce here the relevant formulae for the $\langle r^2 \rangle$: they are given explicitly in Ref.[13] - note only that in Eq. (17), for the redistribution term C' , the sign within the brackets is to be changed when neutrons are in question.

We evaluate δ_{Cspec}^N using Eq. (12) in two different ways: (i) fix a at 0.55 fm, appropriate to the region of the periodic table with which we are concerned, and adjust the R to fit the various $\langle r^2 \rangle$ and $d_{p,n}$ -values in question; (ii) fix the R at the values necessary to fit the $\langle r^2 \rangle$ -values for the lower Z -value, with $a = 0.55$ fm, and adjust a to fit the relevant $d_{p,n}$ -values. The results for the overall δ_{Cspec} are given in Table 2.

5. Conclusion and prospects

It is clear from Table 2 that, as Ormand and Brown maintained[9], the nuclear mismatch of the spectator nucleons fails by an order of magnitude to account for δ_{Cmiss} , the “missing mismatch”, exposed in Fig. 2** and that to preserve CVC together with unitarity of the CKM matrix that missing mismatch must be found elsewhere.

There are at least two further investigations that should be carried out in a search for δ_{Cmiss} . The first concerns δ_{BE} itself. As seen in part from Table 1, δ_{BE} within mean-field approaches varies significantly with the way in which the estimate is made, being dependent upon the form and parameters chosen in the case of the Saxon-Woods treatments and upon the choice of the Skyrme force and other matters in the case of the Hartree-Fock. The choices involved in both cases are guided by experience with quite different problems; those choices may not be optimal for the delicate and dead-reckoning matter of δ_{BE} upon which it is not possible to make a check by reference elsewhere; a wider investigation is needed. Then there is the matter of the parentage spectrum which depends in the main upon estimation within the restricted shell-model bases $(1p)^{4-4}$, $(2s, 1d)^{4-16}$ and $(2p, 1f)^{4-40}$ using effective 2-body matrix elements (2bme) derived by fitting energy levels. Such approaches have had great empirical success but they are also known to absorb, through the 2bme, the effects of excitations into higher shells. Some open-shell studies of δ_{BE} with explicit account of such excitations would be valuable. We also have the problem of the explicit construction of the single-nucleonic wave-functions based on a given parentage spectrum; conventionally this is done by ascribing to the “transforming” nucleon an additional binding energy in its initial and final states reflecting the excitation of the parent state but this is clearly a fiction that is difficult to reconcile with the concomitant use of the overall mean field. It is again the delicacy of the effect that gives the difficulty: what are the nucleons involved “actually doing”?^{††}

**It could be feared that the present use of model single-nucleonic wave-functions that do not properly respect the asymptotic forms required by the range of binding energies found in practice within a single nucleus may gravely distort expectation as to δ_{Cspec} . This is not, however, the case as is shown by comparison of the δ_{BE} themselves evaluated using many-parent wave-functions involving a wide range of binding energies in each nucleus, as reported in Table 1, with the δ_{BE} evaluated using harmonic oscillator wave-functions that do not reflect the asymptotic binding-energy-dependent forms to which we have just referred[8]. If we use for the many-parent binding-energy-dependent δ_{BE} -values the means of the three reported in Table 1 for each body we find that these exceed those given by the harmonic oscillator approach by, summing the δ_{BE} over the 8 cases, a factor 1.24 while for the fp -shell cases above the corresponding factor is 0.86. For our present semi-quantitative purposes our simple surrogate wave-functions are therefore certainly adequate.

^{††}It is difficult to compare the mean-field approaches with the detailed R -matrix approach of Barker[10] since their pictures of what the relevant nucleons are “actually doing” are very different: for example Barker’s treatment involves beta-transitions “within the core”, with the same type

The second unexplored problem is that of nuclear deformation. Deformation will certainly vary along the isomultiplet on account of the differential play of the Coulomb force just as that differential play induces the δ_{CM} of Table 1.^{††} This is a difficult matter to estimate reliably since theoretical deformation may depend critically upon restrictions, such as that of axial symmetry, that may be placed upon the calculation; these restrictions can, in effect, themselves induce, in certain circumstances, apparent large differences in deformation, even as to sign, as between the $T_3 = \pm 1$ members of an isotriplet, differences that may largely disappear on relaxation of that restriction, into one of tri-axial deformation in the present example.^{§§} However, all such calculations are subject to some restrictions in order to make them tractable; it may not be possible to assess when one has gone far enough, in terms of the present problem, particularly in view of the rapid escalation of computer demand with progressive relaxation of restriction. Against the possible importance of T_3 -dependent deformation it may be argued that such dependence, if strong, will tend to come about as the result of some near-instability in the overall nuclear energy balance as a function of deformation that may be seriously affected by a single unit of nuclear charge; we might, therefore, expect any large effect to be strongly A -dependent which, as Fig. 2 shows, $\delta_{C_{min}}$ is not.

We finally comment that it may seem most likely that the present mean-field-based theoretical δ_C are “correct” but are missing a factor of order 2 on account of some deficiency, possibly included among the above considerations. This suggestion is based on the remarkable resemblance in form, as a function of A , between the theoretical δ_C and those empirically determined as described in this paper and discussed in more detail in Ref.[2] and displayed in Fig. 4 of that paper, the only difference being the “missing” factor of 2.

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of nucleon outside, such as do not figure at all in the mean-field picture. It should be noted that the R -matrix δ_C is strongly dependent upon choice of the channel radius although it appears that any reasonable choice for that radius must leave δ_C considerably short of the mean-field values. (It might be argued in favour of the R -matrix approach that it gives broadly the correct result[17] for the related problem of the ft -asymmetry found between mirror Gamow-Teller transitions. However, this is not strong support since in the Gamow-Teller cases we are dealing with large effects of the order of several percent against which the subtleties at the 0.1% level responsible for the Fermi effect may be obscured; indeed the asymmetry in the Gamow-Teller case is of first order in the symmetry-breaking while the Fermi effect is only of second order; we should note that the mean-field (Saxon-Woods) treatment also gives good agreement with experiment for the Gamow-Teller asymmetry[18].) Perhaps the considerable differences between the results of the mean-field and R -matrix approaches, as seen from Fig. 2, must leave us with the conclusion that the great theoretical delicacy of the super-allowed Fermi nuclear mismatch problem is unlikely to be rapidly resolved and that the best approach is indeed that of extrapolation to $Z = 0$ as is presently advocated.

^{††}It may be regarded that an element of δ_{CM} reflects a change of shape along the isomultiplet but this is within the spherical basis of the present calculations of δ_{CM} whereas we speak now, in terms of deformation, of moving into a non-spherical basis.

^{§§}I am grateful to Dr. J.M. Pearson for his elucidation of this calculational artifact.

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Table 1

δ_{Cexp} is the nuclear mismatch (%) inferred from the experimental data, radiatively-corrected, displayed in Fig. 1, together with the $(ft)_0^*$ -value shown near $Z = 0$ to the left of the $(ft)_0^*$ -axis which is derived from G_μ , $|V_{us}|$ and $|V_{ub}|$ under the assumptions of CVC and unitarity of the CKM matrix; the uncertainties shown for δ_{Cexp} are compounded of that attaching to the $(ft)_0^*$ -value derived from G_μ (due chiefly to the uncertainty in $|V_{us}|$) and those of the experimental data of Fig. 1. The columns under δ_{CM} and δ_{BE} are theoretical values: under δ_{CM} the column headed T is from Ref.[8] for ^{14}O , $^{26}\text{Al}^m$ and ^{34}Cl , by private communication from Dr. I.S. Towner for ^{42}Sc and from Ref.[3] for the remainder; the column headed O is from Ref.[9] for the first five entries and by private communication from Dr. W.E. Ormand for ^{42}Sc the entries in brackets being taken from the T column following discussion in Ref.[3]; under δ_{BE} the column headed W is from Ref.[7], updated for ^{14}O by the use of current parameterization for the Saxon-Woods potential employed and for ^{50}Mn and ^{54}Co by the use of present more extensive experimental data on the relevant parent states; the column headed T is from Ref.[8] and that headed O is from Ref.[9]. The column headed B is from Ref.[10].

	δ_{Cexp}	δ_{CM}		δ_{BE}			δ_{C}
		T	O	W	T	O	B
^{14}O	0.47 ± 0.10	0.00	0.01	0.29	0.28	0.18	0.12
$^{26}\text{Al}^m$	0.67 ± 0.09	0.06	0.01	0.35	0.27	0.23	0.05
^{34}Cl	0.92 ± 0.11	0.02	0.06	0.57	0.62	0.42	0.11
$^{38}\text{K}^m$	0.88 ± 0.13	0.10	0.11	0.34	0.54	0.38	0.10
^{42}Sc	0.92 ± 0.09	0.05	0.11	0.31	0.35	0.28	0.12
^{46}V	0.87 ± 0.10	0.09	0.09	0.31	0.36	0.20	0.05
^{50}Mn	0.86 ± 0.10	0.07	(0.07)	0.41	0.40	0.28	0.05
^{54}Co	0.98 ± 0.10	0.05	(0.05)	0.47	0.56	0.34	0.05

Table 2

The isotriplets covering the region of present study. The d_p and d_n are the percentage Coulomb dilutions of the respective $\langle r^2 \rangle^{\frac{1}{2}}$ -values due to an increase by one unit of Z along the multiplet. The $\Delta/\langle r^2 \rangle^{\frac{1}{2}}$ entries are the percentage increase in $\langle r^2 \rangle_p^{\frac{1}{2}}$ or decrease in $\langle r^2 \rangle_n^{\frac{1}{2}}$ due to the conversion of one neutron into one proton at the top of the Fermi distribution in a charge-independent manner. (These entries derive from the Thomas-Fermi model of Myers and Swiatecki: Ref.[12] and private communication.) $\delta_{\text{Capec}}^{\text{HO}}$ is the overall percentage nuclear mismatch evaluated using the d_p and d_n together with harmonic oscillator nucleonic orbitals; $\delta_{\text{Capec}}^{\text{FR}}$ uses the Fermi form for the nucleonic orbitals given by Eq. (12) of the text inducing the $d_{p,n}$ by changes of R , keeping a fixed while $\delta_{\text{Capec}}^{\text{Fa}}$ induces the $d_{p,n}$ by changes of a , keeping R fixed.

A	d_p	d_n	$\frac{\Delta}{\langle r^2 \rangle^{\frac{1}{2}}}$	$\delta_{\text{Capec}}^{\text{HO}}$	$\delta_{\text{Capec}}^{\text{FR}}$	$\delta_{\text{Capec}}^{\text{Fa}}$
14	0.26	0.20	2.59	0.014	0.012	0.007
18	0.23	0.17	1.89	0.015	0.013	0.009
22	0.22	0.16	1.47	0.017	0.014	0.010
26	0.19	0.15	1.20	0.019	0.015	0.011
30	0.18	0.14	1.00	0.020	0.016	0.013
34	0.17	0.13	0.86	0.024	0.016	0.014
38	0.16	0.13	0.75	0.025	0.017	0.016
42	0.15	0.12	0.66	0.024	0.017	0.016
46	0.15	0.12	0.59	0.027	0.019	0.019
50	0.15	0.12	0.53	0.031	0.022	0.022
54	0.15	0.12	0.49	0.034	0.024	0.026

Figure Captions

1. $(ft)^*$ -values for the eight accurately-measured super-allowed Fermi decays. $(ft)^*$ is the experimental ft -value modified only by the radiative corrections and without any correction for nuclear mismatch. The errors shown are those coming from the experimental ft -values only. The full line is the fit quadratic in Z ; the point, with its uncertainty, to the right of the $(ft)^*$ -axis near $Z = 0$ is the result of the fit. The point near $Z = 0$ to the left of the axis is that inferred from G_μ , from $|V_{us}|$ and $|V_{ub}|$ and from CVC under the assumption of unitarity of the Cabibbo-Kobayashi-Maskawa matrix.
2. The "missing mismatch" $\delta_{C_{\text{miss}}} = \delta_{C_{\text{exp}}} - \delta_{C_{\text{theo}}}$. The $\delta_{C_{\text{exp}}}$ are taken from Table 1. $\delta_{C_{\text{theo}}}$ in the cases of the mean-field-based estimates are given by the 6 combinations of the δ_{CM} and δ_{BE} from Table 1, the vertical bars covering their spread. The letters B indicate the $\delta_{C_{\text{miss}}}$ -values implied by the δ_C -values given by the R -matrix treatment of Ref.[10].
3. The mismatch, as defined in the text, between two trapezoidal wave-functions, constant out to different radial distances from the centre and then each declining linearly to zero over a further radial distance t . $\langle r^2 \rangle_1^{\frac{1}{2}}$ for the one wave-function is held constant at 3.5 fm while for the other $\langle r^2 \rangle_2^{\frac{1}{2}} = \langle r^2 \rangle_1^{\frac{1}{2}}(1+d)$ where $d = 0.1\%$.
4. The increase Δ in $\langle r^2 \rangle_p^{\frac{1}{2}}$, or decrease in $\langle r^2 \rangle_n^{\frac{1}{2}}$, due to the charge-independent transformation of one neutron into its mirror proton at the top of the nucleon distribution. The discrete points are for a fully-quantized harmonic oscillator; the heavy full line is for the Thomas-Fermi nuclei of Myers and Swiatecki; the light full line is for the droplet model; the dashed line is for a Fermi gas in an oscillator potential (following Swiatecki).

Fig. 1

Fig. 3

Fig. 2

Fig. 4